30 July 2001

PHYSICS LETTERS A

Physics Letters A 286 (2001) 241–244

www.elsevier.com/locate/pla

# Tests of Bell inequalities

Lev Vaidman [a,b]

[a] *Centre for Quantum Computation, Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK*

[b] *School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel*

Received 2 May 2001; received in revised form 15 June 2001; accepted 16 June 2001
Communicated by P.R. Holland

**Abstract**

According to recent reports, the last loopholes in testing Bell's inequality are closed. It is argued that the really important task in this field has not been tackled yet and that the leading experiments claiming to close locality and detection efficiency loopholes, although making very significant progress, have conceptual drawbacks. The important task is constructing quantum devices which will allow winning games of certain correlated replies against any classical team. A novel game of this type is proposed. © 2001 Published by Elsevier Science B.V.

Quantum mechanics predicts unusual correlations between outcomes of particular experiments in space-like separated regions. The peculiarity of these correlations is that they are stronger than any correlations explainable by a local theory. The quantum correlations, as was proven by Bell [1], break certain inequalities which have to be fulfilled if the results of every experiment are determined by some local hidden variable (LHV) theory. Recently, we have been witnessed an outstanding progress in the tests of Bell's inequalities, but a decisive experiment which would rule out any LHV theory has not been performed yet. The deficiencies of the experiments are described by *locality* and *detection efficiency* loopholes [2]. According to recent reports, Weihs et al. [3] closed the locality loophole and Rowe et al. [4] closed the detection efficiency loophole. I want to point out that the leading experiments claiming to close the loopholes [3,4], although clearly making very significant progress, have conceptual drawbacks. These drawbacks have to be removed before claims like "the last loophole closes" can be made.

Moreover, I want to argue that the experimental efforts should be turned to a slightly different task: instead of showing that the quantum correlations cannot be explained by local theories, it is more important to show that the quantum correlations can be used.

Today, there is a firm consensus that there is no real question what will be the outcome of this type of experiment: the predictions of quantum theory or results conforming with the Bell inequalities. Predictions of quantum mechanics have been verified in so many experiments and with such unprecedented precision that, in spite of the very peculiar and non-intuitive features that Bell-inequality experiments demonstrate, only a minute minority of physicists believe that quantum mechanics might fail in this type of experiment. However, the fact that we are pretty sure about the final result of these experiments does not mean that we should not perform them. One goal of such experiments is to change our intuition which developed from observing

*E-mail address:* vaidman@post.tau.ac.il (L. Vaidman).

0375-9601/01/$ – see front matter © 2001 Published by Elsevier Science B.V.
PII: S0375-9601(01)00427-3

classical phenomena. But more importantly, these experiments should lead to the stage in which we will be able to *use* these unusual correlations.

Conceptually, the most simple, surprising, and convincing out of the Bell-type experiments is the Mermin's version of the Greenberger–Horne–Zeilinger (GHZ) setup [5]. I find that it can be best explained as a game [6]. The team of three players is allowed to make any preparations before the players are taken to three remote locations. Then, at a certain time, each player is asked one of two possible questions: "What is $X$?" or "What is $Y$?" to which they must quickly give one of the answers: "1" or "$-1$". According to the rules of the game, either all players are asked the $X$ question, or only one player is asked the $X$ question and the other two are asked the $Y$ question. The team wins if the product of their three answers is $-1$ in the case of three $X$ questions and is 1 in the case of one $X$ and two $Y$ questions. It is a simple exercise to prove that if the answer of each player is determined by some LHV theory, then the best strategy of the team will lead to 75% probability to win. However, a quantum team equipped with ideal devices can win with certainty. Each player performs a spin measurement of a spin-1/2 particle: $\sigma_x$ measurement for the $X$ question and $\sigma_y$ measurement for the $Y$ question and gives the answer 1 for spin "up" and $-1$ for spin "down". Quantum theory ensures that if the players have particles prepared in the GHZ state, the team always wins. Actually constructing such devices and seeing that, indeed, the quantum team wins the game with probability significantly larger than 75% will be a very convincing proof of Bell-type inequalities.

The game need not be based on the GHZ-type "Bell inequality without inequalities" proof. A two-party game based on another proof is presented at the end of the Letter.

A game-type experiment, if successful, will definitely close the detection efficiency loophole. If it is also arranged that the party asking the questions chooses them "randomly" (more about randomness below), then it will also close the locality loophole. Note, that an experiment which simultaneously closes both the detection efficiency and the locality loophole will not necessarily be suitable for winning games of the type I described here. Indeed, the current GHZ experiment [7], even if performed with ideal optical devices, cannot help to win the GHZ game. At no stage of this experiment is there a pure GHZ state. Only when the three photons were detected at three different detectors in coincidence with the fourth (trigger) photon, could we claim that the polarization state of the *detected* photons was the GHZ state. If a player in the team detects the trigger photon and makes a polarization measurement (the analog of the spin measurement) according to the question he is asked, he cannot be sure that the team will win since the other players would know the good answers only if they detect the photons too. However, the setup is such that there is a high probability for this not to happen even if they have 100% efficient detectors.

Let me now discuss the current experiments. In order to close the locality loophole Weihs et al. [3] used fast quantum experiments to choose between local measurements. The results of these experiments are "genuinely random" according to the standard quantum theory, but are *not random* in the framework of LHV theories. Their outcomes are also governed by some LHVs in each site. There is enough time for information about these LHVs to reach other sites before the measurements there took place and, therefore, the locality loophole is not closed. The experiment, nevertheless, is a significant step forward because its results can be explained only by a higher level LHV theory in which hidden variables specifying the behavior of one system are influenced by hidden variables of other systems.

A frequently discussed experiment in which a person at each site will have enough time to exercise his "free will" to choose between the measurements will be very convincing, but conceptually, not much better: we cannot rule out the existence of LHVs which are responsible for our seemingly "free" decisions. A better experiment for closing the locality loophole is to make the choice of the local measurements dependent on the detection of photons arriving from space-like separated events in distant galaxies. Then, the LHV-type explanation will be a "conspiracy" theory on the intergalactic scale.

Now I will discuss the latest experiment by Rowe et al. [4] who claimed to close the detection efficiency loophole. In this experiment the quantum correlations were observed between results of measurements performed on two ions few micrometers apart. The detection efficiency was very high. It was admitted that the locality loophole was not closed, but the situation was worse than that. Contrary to other experiments [3], not only the information about the choice, but also about the *results* of local measurements could reach other sites before completion of measurements there. The reading of the results was based on observing numerous photons emitted by the ions. This process takes time which is a few orders magnitude larger than the time it takes for the light to go from one ion to the other. Thus, one can construct a very simple LHV theory which arranges quantum correlations by "communication" between the ions during the process of measurement. It is much simpler to construct a LHV theory which employs also "outcome dependence" instead of only "parameter dependence" [8].

The purpose of closing the detection efficiency loophole was to rule out the set of LHV theories in which the particle carries, among others, instructions of the type: "if the measuring device has particular parameters, do not be detected". Such hidden variables cannot explain the correlations of the Rowe et al. experiment and this is an important achievement. However, the task of performing an experiment closing the detection efficiency loophole without opening new loopholes (the possibility for "outcome dependence" LHV in Rowe et al.) is still open.

Recently, the bizarre features of quantum mechanics have been explained through various games. Apart from the GHZ game described above there have been several other proposals: an interesting variation of the GHZ game by Steane and van Dam [9], a game based on the original Bell proof by Tsirelson [10], the "quantum cakes" game based on a non-maximally entangled state by Kwiat and Hardy [11] (see related experiment [12]). Note also the proposal of Cabello [14] for a two-party Bell-inequality proof which can be transformed into a game too. Let me present here one more game. My game is called an "impossible necklace" and it is based on the Zeno-type Bell inequalities proof [13].

A team of two players wants to persuade a third party, "the interrogator", that they found a secret of making an "impossible necklace". The impossible two-colored necklace has an even number of beads $N$ and all adjacent beads are of different colors except beads 1 and $N$ which are of the same color. The team does not want to reveal the "secret coloring", but the players are ready to reveal the colors of any two adjacent beads of the necklace. They claim to have identical necklaces of this kind, one necklace for each player. The interrogator arranges to ask one player the color of any single bead and ask the other player, at a space-like separated region, the color of one of the adjacent beads. If the team succeeds in giving the correct answers in many repeated experiments (with new necklaces each time), a naive interrogator might be persuaded that the team knows how to make such necklaces. Indeed, if it is a "classical team", and the players decide in advance what answer they will give for every question, then the probability to fail is at least $1/N$. (There are $N$ different pairs and there is no way to arrange that all have correct coloring.) Therefore, the probability to pass the test, say $5N$ times, is

$$prob_{\text{classical}} = \left(1 - \frac{1}{N}\right)^{5N} \sim e^{-5} \sim 0.01. \tag{1}$$

The quantum team can do much better. The players do not make any necklaces. Each player take with him a spin-1/2 particle from the EPR pair. When a player is asked the color of a bead $i$, he measures spin component in the direction $\hat{\theta}_i$ in the $x$–$z$ plane which makes an angle $\theta_i = \pi i/N$ with the $z$ axes. He says "green" if the result is "up", and "red" if the result is "down". His partner do the same. For all pairs, the measurements are in the directions which differ by the angle $\pi/N$ except for the pair $\{1, N\}$ in which case the angle is $\pi(N-1)/N$. Therefore, the probability to fail the test is $\sin^2(\pi/2N)$. The probability to pass $5N$ tests is

$$prob_{\text{quantum}} = \left(1 - \sin^2\frac{\pi}{2N}\right)^{5N} \sim \left(1 - \frac{\pi^2}{4N^2}\right)^{5N} \sim e^{-5\pi^2/4N}. \tag{2}$$

For $N = 100$ the quantum team has probability of almost 90% to succeed, compare with 1% of a classical team.

Technological problems will not allow an experiment with a large number $N$ in a near future. Putting aside the attempt to "fool" the interrogator that the team has impossible necklaces, the game can be defined as the competition of two-player teams to pass the interrogator tests a maximal number of times. For any number $N \geqslant 4$, the quantum team has an advantage over a classical team, so this game is a realistic proposal for demonstrating Bell-type inequalities.

(Certainly more realistic than the GHZ game which requires a three-particle source.)

The ideal situation (in the sense that it closes all loopholes) is that the questions which players are asked are decided by fast detectors obtaining signals from space-like separated distant galaxies and that the players give their answers quickly enough such that the communication between them is impossible. However, I do not think that the stringent requirement of closing the locality loophole is very important. It seems to me that an implementation of one of these games with players placed in separate sealed rooms which prevent them from sending out any signals will be a very dramatic and persuasive proof that the nature indeed has the peculiar features predicted by quantum theory. More importantly, it will show that quantum technology is capable of performing communication tasks which are impossible when classical devices are used.

## Acknowledgements

It is a pleasure to thank Jonathan Jones, Lucien Hardy, and Hans Christian von Bayer for helpful discussions. This research was supported in part by grant 471/98 of the Basic Research Foundation (administrated by the Israel Academy of Sciences and Humanities) and the EPSRC grant GR/N33058.

## References

[1] J.S. Bell, Physics 1 (1964) 195.
[2] P. Pearle, Phys. Rev. D 2 (1970) 1418.
[3] G. Weihs et al., Phys. Rev. Lett. 81 (1998) 5039.
[4] M.A. Rowe et al., Nature 409 (2001) 791.
[5] D.N. Mermin, Am. J. Phys. 58 (1990) 731.
[6] L. Vaidman, Found. Phys. 29 (1999) 615.
[7] J.W. Pan et al., Nature 403 (2000) 515.
[8] J.P. Jarrett, Nous 18 (1984).
[9] A. Steane, W. van Dam, Phys. Today 53 (2000) 35.
[10] B. Tsirelson, Lecture notes, Tel-Aviv University (1996).
[11] P.G. Kwiat, L. Hardy, Am. J. Phys. 68 (2000) 33.
[12] G. Brida et al., Phys. Lett. A 268 (2000).
[13] E.J. Squires, L. Hardy, H.R. Brown, Stud. Hist. Philos. Sci. 25 (1994) 425.
[14] A. Cabello, Phys. Rev. Lett. 86 (2001) 1911.