Foundations of Physics, Vol. 29, No. 4, 1999

Variations on the Theme of the Greenberger–Horne–Zeilinger Proof

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Received August 18, 1998; revised December 15, 1998

Three arguments based on the Greenberger–Horne–Zeilinger (GHZ) proof of the nonexistence of local hidden variables are presented. The first is a description of a simple game which a team that uses the GHZ method will always win. The second uses counterfactuals in an attempt to show that quantum theory is nonlocal in a stronger sense than is implied by the nonexistence of local hidden variables and the third describes peculiar features of time-symmetrized counterfactuals in quantum theory.

1. INTRODUCTION

Daniel Greenberger has uncovered numerous miracles of the quantum world. Reading his work on quantum experiments with neutrons(1) led me to adopt a revolutionary view on the reality of our universe.(2) But another of his discoveries, the Greenberger–Horne–Zeilinger (GHZ) nonlocality proof(3) influenced not just my view on quantum reality, but the views of many thousands of people. I myself have used this work to explain the power of quantum mechanics to hundreds of students as well as to many friends. In this paper I discuss three arguments based on the GHZ work.

The first is a version of the GHZ argument which can convert laymen into admirers of quantum theory by showing the miraculous power of quantum theory. This is a combination of Mermin's presentation(4) of the GHZ "paradox" and a story I heard from my students who took a course "Paradoxes in Quantum Probability" by Boris Tsirelson at Tel-Aviv

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0015-9018/99/0400-0615$16.00/0

University.[5] Mermin translated the GHZ result to peculiar correlations between the outcomes of some simple operations. Tsirelson constructed a certain gambling game for which a quantum team has an advantage relative to any "classical" team by using the setup given in the original Bell inequalities paper.[6] I have combined these two works, suggesting a gambling game based on Mermin's realization of the GHZ idea. This argument is presented in Sec. 2. Section 3 is devoted to a discussion in which this game is considered as a method for obtaining an experimental proof of nonlocality of quantum theory. The discussion includes also some speculations about possible local hidden variable theories which can explain experiments with non-ideal detectors.

The second argument was inspired by recent polemics triggered by Stapp's proof of the "nonlocality" of quantum theory.[7] He claimed to show, using Hardy's setup,[8] the nonlocality of quantum theory beyond the result of Bell, which is the nonexistence of a local hidden variable theory consistent with predictions of quantum theory. I have already reflected on this subject once[9] using an analysis of the EPR state of two spin-1/2 particles, but now I think that the GHZ setup is a better testing ground for Stapp's claim. In Sec. 4 I present an argument inspired by Stapp's work, based on the GHZ configuration, which shows a contradiction between certain counterfactual statements. However, in the next section I present my skepticism about the whole program. At this moment I cannot define for myself what is meant by a "local" theory which is not a local hidden variable theory. Therefore, for me, all that is shown in Stapp's and my proofs is that something which I am unable to define does not exist. Thus, the significance of the "proofs" is not clear to me.

The last argument, presented in Sec. 6, is also related to counterfactuals. Here I want to bring attention to an analysis based on the argument by Clifton *et al.*[10] about the question of existence of a realistic relativistically covariant quantum theory. I do not think that one can prove the nonexistence of a covariant realistic quantum theory,[11] but the example allows one to show a surprising property of time-symmetrized "elements of reality" which I suggest be defined in terms of certain counterfactuals. I show that such elements of reality do not obey the product rule. Clifton *et al.* preferred not to give up the product rule, and in this way they arrived at a contradiction with the existence of a realistic covariant quantum theory.

I conclude by explaining how the analysis of the GHZ work led me to accept the bizarre picture of quantum reality given by the many-worlds interpretation,[12] according to which all that we see around us is only one out of numerous worlds which all together comprise the physical universe described by quantum theory.

2. HOW TO WIN A GAME USING QUANTUM THEORY

When I have a conversation with a friend who is not a physicist and I want to show her the miraculous power of quantum theory, I start with a seemingly innocent puzzle.

I present the following game for a team of three players. The players are allowed to make any preparations before they are taken to three remote locations A, B, and C. Then, at a certain time t, each player is asked one of two possible questions: "What is X?" or "What is Y?" (see Fig. 1). Each player must give an answer which is also limited to only two possibilities: "1" or "-1." They have to give their answer quickly, i.e., before the time they might receive a message sent by another player after the time t.

According to the rules of the game, either all players are asked the X question or only one player is asked the X question and the other two are asked the Y question. The team wins if the product of their three answers is -1 in the case of three X questions and is 1 in the case of one X and two Y questions. My friend is asked what should the team do in order to win for sure.

Sometimes the problem interests friends immediately and sometimes they are urged to work on this seemingly children's puzzle by my promise that there is something unusual and profound in the solution. Usually, after less than a half an hour, I get an answer: "This is impossible!"

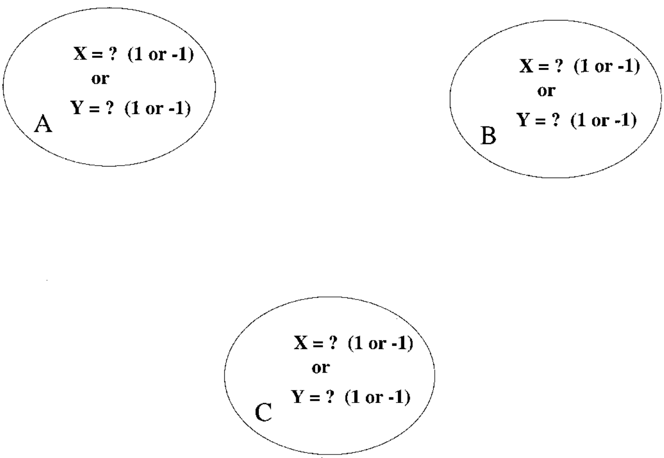

Fig. 1. A game which only the quantum team can always win. Three separated players should simultaneously provide a value for X and Y, which might be 1 or -1, such that the product of the values will satisfy Eq. (1).

The most effective "proof" for this is as follows. Since each player is not able to get any message from the other players about which questions they were asked before the time she has to give an answer, it seems that she cannot gain anything by delaying the decision of which answer to give for each question until the question is actually asked. Thus, an optimal strategy should correspond to prior definite decisions of each player which answers to give for possible questions. But it is easy to prove that any such strategy cannot ensure winning for all allowed combinations of questions. Indeed, if it does, then there must be a set of answers $\{X_A, Y_A, X_B, Y_B, X_C, Y_C\}$, where X_A is the answer of the player in A on question X, etc., such that the following equations are fulfilled.

$$\begin{aligned} X_A X_B X_C &= -1 \\ X_A Y_B Y_C &= 1 \\ Y_A X_B Y_C &= 1 \\ Y_A Y_B X_C &= 1 \end{aligned} \tag{1}$$

This, however, is impossible, because the product of all left-hand sides of Eqs. (1) is the product of squares of numbers which are ± 1 and therefore it equals 1, while the product of all right hand sides of these equation yields -1.

This is the shortest proof I know. Many people just show this by going through all possible strategies of deciding in advance the answers to the questions. After this elaborate exercise my friends have a firm belief that the task is impossible. At this stage I tell them that using a quantum experiment this can be done. Usually at first my claim is accepted with disbelief, but when I succeed to persuade them that this is true, their surprise is enormous.

The solution provided by quantum theory[3,4] is as follows. Each member of the team takes with her a spin-1/2 particle. The particles are prepared in a correlated state (which is usually called the GHZ state):

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|{\uparrow_z}\rangle_A\,|{\uparrow_z}\rangle_B\,|{\uparrow_z}\rangle_C - |{\downarrow_z}\rangle_A\,|{\downarrow_z}\rangle_B\,|{\downarrow_z}\rangle_C\right) \tag{2}$$

Now, if a member of the team is asked the X question, she measures σ_x and gives the answer which she obtains in this experiment. If she is asked the Y question, she measures σ_y instead. Quantum theory ensures that the team following this strategy *always* wins. Indeed, a straightforward calculation shows that the measurements on the system of three spin-1/2 particles prepared in the state (2) fulfill the following relations:

$$\begin{aligned} \{\sigma_{Ax}\}\{\sigma_{Bx}\}\{\sigma_{Cx}\} &= -1 \\ \{\sigma_{Ax}\}\{\sigma_{By}\}\{\sigma_{Cy}\} &= 1 \\ \{\sigma_{Ay}\}\{\sigma_{Bx}\}\{\sigma_{Cy}\} &= 1 \\ \{\sigma_{Ay}\}\{\sigma_{By}\}\{\sigma_{Cx}\} &= 1 \end{aligned} \tag{3}$$

Here $\{\sigma_{Ax}\}$ signifies the outcome of the measurement of σ_x by the player in A, etc. Let us show, for example, that the first equation is true. In the spin x bases for all particles the GHZ state [which in (2) is given in the spin z bases] is

$$\begin{aligned} |\mathrm{GHZ}\rangle = {} & \tfrac{1}{4}[(|\uparrow_x\rangle_A + |\downarrow_x\rangle_A)(|\uparrow_x\rangle_B + |\downarrow_x\rangle_B)(|\uparrow_x\rangle_C + |\downarrow_x\rangle_C) \\ & - (|\uparrow_x\rangle_A - |\downarrow_x\rangle_A)(|\uparrow_x\rangle_B - |\downarrow_x\rangle_B)(|\uparrow_x\rangle_C - |\downarrow_x\rangle_C)] \\ = {} & \tfrac{1}{2}(|\uparrow_x\rangle_A |\uparrow_x\rangle_B |\downarrow_x\rangle_C + |\uparrow_x\rangle_A |\downarrow_x\rangle_B |\uparrow_x\rangle_C \\ & + |\downarrow_x\rangle_A |\uparrow_x\rangle_B |\uparrow_x\rangle_C + |\downarrow_x\rangle_A |\downarrow_x\rangle_B |\downarrow_x\rangle_C) \end{aligned} \tag{4}$$

Therefore, we see explicitly that the GHZ state is a superposition of states for each of which $\{\sigma_{Ax}\}\{\sigma_{Bx}\}\{\sigma_{Cx}\} = -1$.

In the quantum solution of the problem the players do not decide in advance the answers they'll give for each out of the two possible questions. In the "proof" of the impossibility of this task presented above, it was erroneously assumed that delaying the decision which answer to give until the time the question is asked cannot help. The assumption sounds plausible since relativistic causality prevented sending signals after the time the questions were asked, but, nevertheless, the assumption is wrong because it does not take into account unusual correlations which quantum objects can exhibit.

3. THE GHZ PROOF OF NONEXISTENCE OF LOCAL HIDDEN VARIABLE THEORIES

Performing the game with spin-1/2 particles as described above might serve as an experimental proof of the nonexistence of local hidden variables which produce agreement with experiment. Suppose, however, that Nature *is* governed by local hidden variables. This means that the results $\{\sigma_{Ax}\}$, $\{\sigma_{Ay}\}$, $\{\sigma_{Bx}\}$, $\{\sigma_{By}\}$, $\{\sigma_{Cx}\}$, $\{\sigma_{Cy}\}$ exist prior to their measurements, and therefore, there are answers $\{X_A, Y_A, X_B, Y_B, X_C, Y_C\}$ prior to the time the questions are asked. These answers can fulfill at most three out of the four Eqs. (1), and therefore, in the game in which the four question patterns are chosen with equal probability, on average there will be at most a 75% success rate of any team playing this game. If a team shows on average a higher result, this is an experimental proof that Nature is not governed by local hidden variables.

The most common objection to experiments showing the nonexistence of local hidden variables is that, due to limited efficiency of detectors, a significant fraction of particles is lost. If every member of our team using the quantum strategy described above makes a random choice when she does not detect the spin component of the particle, then the success rate of the team is

$$\mathrm{Prob}(win) = \mathrm{Prob}(all\ detected) + \tfrac{1}{2}(1 - \mathrm{Prob}(all\ detected)) \tag{5}$$

Therefore, to ensure that $\mathrm{Prob}(win) > 0.75$ (the maximal value achievable by a classical team), so that the nonexistence of the hidden variables is shown, it is enough that $\mathrm{Prob}(all\ detected) > 0.5$, i.e., that the efficiency of each detector is bigger than $0.5^{1/3} \sim 0.8$. Note much more sophisticated studies of limitations posed by detector efficiencies by Greenberger *et al.*[13] and Larsson.[14]

It is interesting to think about possible local hidden variable theories that can produce results equivalent to those of quantum theory. For the experiment with limited efficiency detectors in which $\mathrm{Prob}(all\ detected) <$ 0.5 such a theory exists.[15] The spin-1/2 particles of a GHZ triple, when they were locally created, and then moved to their locations *A*, *B*, and *C*, carry with them "instruction kits" how to respond to various measurements. There are instructions for every possible direction of the spin measurement, which are "up," "down," or "be not detected." Indeed, it is always possible to construct instructions how each spin is to respond to σ_x and σ_y measurements such that two of Eqs. (3) are fulfilled and just one spin-1/2 particle has an instruction "be not detected" for one of the directions and therefore the other two Eqs. (3) are not tested. In this situation, on average, in half of the measurements there will be triple detections for all of which the results are in accordance with (3). (Note that in this model in every run at least two particles are detected, i.e., no single or null detections can occur. This is not expected according to quantum theory, which does not have predictions about correlations between detection or not detection of particles.)

The naive local hidden variable theory proposed above fails to explain the GHZ correlations in a more sophisticated setup. Roughly speaking I propose "teleportation"[16] of the states of the GHZ particles to three particles in even more remote locations and testing the GHZ correlations on the remote particles. Quantum theory predicts such correlations but the naive local variables theory cannot. Indeed, the remote particles have no "common cause;" they were not locally created, and therefore they cannot carry with them instruction kits created in a single location. Thus, the naive hidden variable theory cannot explain why the three remote particles exhibit quantum correlations.

More precisely, I propose the following experiment. Three EPR pairs are prepared as for the teleportation of the states of the GHZ particles as shown in Fig. 2. Then, simultaneously, three Bell operator measurements are performed on the pairs of spin-1/2 particles consisting of one GHZ particle and one particle from the EPR pair at A, B, and C. At the *same* time spin-x or spin-y components of the three particles at A', B', and C' are measured according to the usual GHZ rule: either three x or just one x and two y components are measured.

In the original teleportation scheme,[16] the outcome of the Bell measurement is transmitted to the remote particle and a π rotation around one of the three axes (or no rotation) is performed according to that outcome. In the proposed experiment, no message is sent and no rotation is performed, because the spin component measurement is performed before the message about the result of the Bell measurement could have arrived. The significance of the Bell measurement is that it makes the connection between the result of the spin measurement on the remote particle of the EPR pair and the result of possible direct measurement of the same spin component of the GHZ particle. For two out of four possible results of the

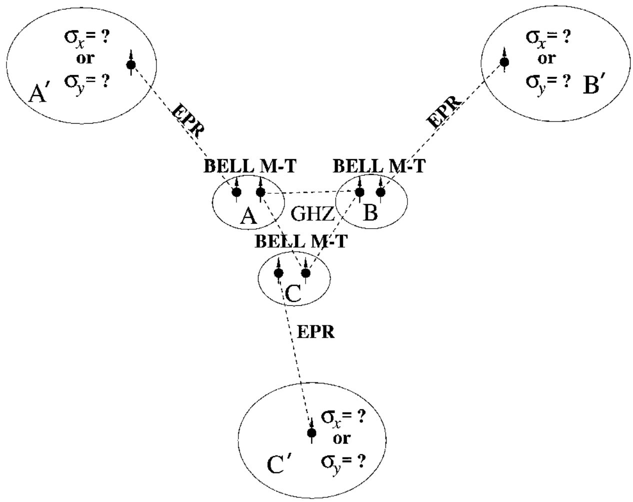

Fig. 2. A proposal for experiment for ruling out "naive" local hidden variable theories. There is a free choice of measurements in A', B', and C' and fixed (Bell) measurements in A, B, and C. Quantum theory predicts certain correlations between the outcomes of these measurements, but since the particles in A', B', and C' had no common origin, they cannot carry instruction kits to produce such correlations.

Bell measurement (corresponding to no rotation and the rotation around the axis of the direction of the spin component measurement) measurement of the remote particle is identical to the direct measurement performed on the GHZ particle. For two other results, the outcome of the measurement on the remote particle is "flipped" relative to the measurement performed on the GHZ particle. After taking into account these flips the correlations between the outcomes of the spin measurements on the remote particles turn out to be the GHZ correlations.

Since simultaneous detection of nine particles is necessary for the case when the correlation can be tested, there will be only a small fraction of such events. But in this setup the remote particles cannot carry with them instruction kits created at a single place because the particles never were together at a single location. Therefore, it is not clear how a hidden variable theory can produce *any* correlation between the outcomes of the measurement of the remote particles. I do not have a proof that a more complicated local hidden variable theory according to which the instructions for each particles will involve not just dependence on the experimental setup of the detectors, but also dependence on the local hidden variables of the other particles, cannot be created. However, it seems to be highly improbable to have such a theory which would not look artificial.

The GHZ proof is the most clear and persuasive proof of nonexistence of local hidden variables. In the GHZ example we have *perfect* correlations which cannot be reproduced by any local hidden variable theory. This, however, does not mean that the GHZ-type experiment is the best candidate for experimental proof of the nonexistence of local hidden variables. There are serious technological difficulties with experiments involving GHZ type correlation. [Note, however, that the entanglement swapping (teleportation), discussed in the modified test of local hidden variables above, has been performed in the laboratory.(17)] Today, the best choices for experimental proof of the nonexistence of local hidden variables are still experiments based on the two-particle quantum correlations, either via the original Bell proposal or through ideas of Hardy(8) using which one can construct another game for a two-player team such that the success in the case of the existence of a local hidden variable theory must be less than the success predicted by quantum theory.

4. THE STAPP NONLOCALITY ARGUMENT APPLIED TO THE GHZ SETUP

Recently Stapp(7) suggested that the nonlocality of quantum theory goes beyond the nonexistence of a local hidden variable theory. In order to prove this, Stapp applied *counterfactual* reasoning to Hardy's setup.[8] I use here counterfactual arguments about the GHZ setup inspired by Stapp's work.

The general locality principle is as follows.

$\mathscr{L}$: *Action at a space-like separated region does not change the outcome of a local measurement.*

My understanding of $\mathscr{L}$ in the context of recent publications on this subject is as follows. Assume that in the actual world a quantum experiment has a certain outcome. Then, in a counterfactual world which differs from the actual one prior to the measurement only in some actions performed in a space-like separated region, the outcome of the measurement should be the same. I believe that this is, essentially, the assumption "LOC1" of Stapp's paper. I suggest extending the meaning of the locality postulate from comparing actual and counterfactual worlds to comparison between two counterfactual worlds. Consider two counterfactual worlds in which a certain measurement is performed. Assume that the two counterfactual worlds are identical prior to the measurement except for some actions performed in space-like separated regions. Then the outcomes of the measurements in the two counterfactual worlds must be the same.

Consider the GHZ setup of three spin-1/2 particles located in three space-like separated regions. Assume that in the actual world the outcomes are

$$\{\sigma_{Ax}\} = 1, \qquad \{\sigma_{Bx}\} = 1, \qquad \{\sigma_{Cx}\} = -1 \tag{6}$$

Now consider three counterfactual worlds.

CFW1: σ_{Ax}, σ_{By} and σ_{Cy} are measured.

CFW2: σ_{Ay}, σ_{Bx} and σ_{Cy} are measured.

CFW3: σ_{Ay}, σ_{By} and σ_{Cx} are measured.

Since in the actual and counterfactual worlds which differ only by actions in regions which are space-like separated from a certain space–time location, the outcomes of local measurements in this location should be the same, we may conclude that the results of σ_x measurements in the counterfactual worlds must be identical to those in the actual world, given in (6). Therefore, in order to fulfill (3) we must have

$$\begin{aligned} \{\sigma_{B_y}{}^{CFW1}\} &= \{\sigma_{C_y}{}^{CFW1}\} \\ \{\sigma_{A_y}{}^{CFW2}\} &= \{\sigma_{C_y}{}^{CFW2}\} \\ \{\sigma_{A_y}{}^{CFW3}\} &\neq \{\sigma_{B_y}{}^{CFW3}\} \end{aligned} \tag{7}$$

From the extended argument which considers two counterfactual worlds, we can deduce that the results of σ_y measurements must yield identical outcomes for each pair of the counterfactual worlds. Therefore, we obtain

$$\begin{aligned} \{\sigma_{C_y}{}^{CFW1}\} &= \{\sigma_{C_y}{}^{CFW2}\} \\ \{\sigma_{B_y}{}^{CFW1}\} &= \{\sigma_{B_y}{}^{CFW3}\} \\ \{\sigma_{A_y}{}^{CFW2}\} &= \{\sigma_{A_y}{}^{CFW3}\} \end{aligned} \tag{8}$$

It is easy to see, however, that Eqs. (7) and (8) are inconsistent. This inconsistency shows that predictions of quantum theory lead to failure of the general locality principle $\mathscr{L}$, i.e., that the quantum theory is nonlocal.

My argument might be attacked on the grounds that my extension of the locality principle to comparison of two counterfactual worlds instead of the actual world and a counterfactual world is not justified. In the framework of a hidden variables theory, the justification of my step is trivial, but without hidden variables the justification is not clear. I contend that without hidden variables even the unextended locality principle (such as Stapp's LOC1) is not clear. Therefore, it is not clear that my proof or even the original Stapp's proof shows anything new about the nonlocality of quantum theory. I discuss this issue in the next section.

5. DO THE STAPP ARGUMENTS SHOW THE NONLOCALITY OF QUANTUM THEORY BEYOND THE NONEXISTENCE OF LOCAL HIDDEN VARIABLES?

It seems to me that the answer to the question in the title of this section is negative, but I certainly cannot prove it because the truth of this statement depends crucially on the meaning of "nonlocality" which differs widely among physicists and philosophers. What I want to present here are some arguments relevant to *my* understanding of nonlocality in this context formed after reading an illuminating discussion by Mermin[18] of the previous version of Stapp's nonlocality argument. The title of Mermin's paper is, "Can You Help Your Team Tonight by Watching on TV? More Experimental Metaphysics from Einstein, Podolsky, and Rosen." The word "help" might be too strong: the question of nonlocality, as I understand it, is: "Can you *change* an outcome of the game by an action in a space-like separated region?" A one-sentence summary of my arguments below is: for counterfactual propositions in the EPR setup the word "change" is meaningless unless the existence of hidden variables is assumed.

Stapp's paper[7] generated a very intensive polemic including critical analysis by Unruh, Mermin, Finkelstein, Griffiths, Shimony, and myself[19–23, 9] and answers by Stapp.[24, 25] The ultimate goal of the project, as I understand it, is to show that quantum theory invariably leads to the failure of locality principle $\mathscr{L}$. Two relevant results are well known.

(i) According to quantum theory, action at a space-like separated region does not change the *probability* of an outcome of a local measurement.

(ii) If the outcomes of quantum measurements are governed by hidden variables, then there is an action at a space-like separated region which *does* change the outcome of a local measurement.

I argue that there is no meaning for $\mathscr{L}$ beyond (i) and (ii).

If we assume the nonexistence of hidden variables, then the outcome which does not have probability 1 is uncertain prior to the measurement. Therefore, the only things we can compare are *probabilities* for an outcome, strictly speaking, $\mathscr{L}$ is meaningless. $\mathscr{L}$ can be made meaningful only if we read "the outcome" in $\mathscr{L}$ as "the probability of the outcome:" then the meaning of $\mathscr{L}$ is (i) and $\mathscr{L}$ is true.

Under the opposite assumption of existence of hidden variables the meaning of $\mathscr{L}$ is (ii) and $\mathscr{L}$ is false. But this is nothing new; the non-existence of local hidden variables is known, and a nonlocal hidden variable theory means exactly the negation of $\mathscr{L}$ for some measurements.

The definition of hidden variables is that the outcome of any experiment is known prior to the experiment and the definition of *local* hidden variable is that the outcome of any local experiment is known prior to the experiment. (Frequently, the concept of *contextuality* is introduced in the discussion of this issue. This should not change the argument provided that the context of a local experiment is also local, i.e., relates only to the location of the experiment.)

A typical situation for which $\mathscr{L}$ is applied in the Stapp type proofs is that a local quantum measurement has several possible outcomes and a particular one takes place in the actual world. The locality principle says that in a counterfactual world, which differs from the actual world in some action performed in a space-like separated region, the outcome should not be *different*. Now, a counterfactual world should be as close as possible to the actual world. Since the question is about the result of a local measurement, the counterfactual world should be as close as possible to the actual world *prior* to the measurement. But since a hidden variable is not assumed, in the actual world prior to the measurement the outcome is not known. But if the outcome is not known, then the word "different" in the sentence above describing the locality principle becomes meaningless: different from what? There is nothing to compare with unless a hidden variable which determines the outcome is assumed. Therefore, it seems that there is no meaning for nonlocality of quantum theory beyond the nonexistence of local hidden variables.

Let me quote Stapp's recent paper:[24]

> With fixed initial conditions one can, by making a change in the Lagrangian in a small space-time region, shift from the actual world to a neighboring possible world, and prove that the effects of this change are confined to times that lie later than the cause in every Lorentz frame. The change in the Lagrangian in the small region can be imagined to alter an experimenter's choice of which experiment he will soon perform in that region.
>
> ...The question, more precisely, is this: Is it possible to maintain in quantum mechanics, as one can in classical mechanics, the theoretical idea that the one real world that we experience can be embedded in a set of possible worlds, each of which obeys the known laws of physics, if (1), the experimenters can be imagined to be able to freely choose between the different possible measurements that they might perform, and (2), no such free choice can have any effect on anything that lies earlier in time in some Lorentz frame.
>
> It was proved in Ref. 7 that with a sufficiently broad definition of "anything" the answer to this question is no.

In his proof Stapp considers an actual world with local measurements in two separate locations which have certain outcomes. Then, using (1) and (2), he shows that certain worlds with some alternative measurements should belong to the set of possible worlds. It seems to me that Stapp arrived at a contradiction (which he argued implies the nonlocality of quantum theory) by relying on a tacit assumption that in a possible world, identical to the actual world in *everything* prior to a measurement, the result of the same measurement which was performed in the possible world must be identical to that of the actual world. Thus, he excludes from the set of possible worlds those which have different results for the *same* measurements given fixed initial condition. However, the quantum theory predicts *random* outcomes and therefore this exclusion is not justified. Saying this again in the language of the previous consideration, this exclusion fixes the outcome prior to a measurement, i.e., this strategy assumes existence of hidden variables. Note that Shimony,[23] in analyzing Stapp's proof itself, reached a similar conclusion, i.e., that Stapp's nonlocality proof does not go through unless hidden variables are assumed.

The assumption of nonexistence of hidden variables, that is, for example, that the outcome of a spin component measurement performed on one particle out of the EPR pair (singlet state of two separated spin-1/2 particles) is not fixed prior to the measurement, leads to some "nonlocal influence": measurement on one particle fixes *immediately* the outcome of possible measurement on the other particle. This, however, does not correspond to the failure of $\mathscr{L}$. We cannot claim that the measurement on one particle *changes* the outcome of the measurement performed on the other particle. The latter is *random* with or without measurement performed on the first particle. The "randomness" is the same irrespectively of where it was generated, in the local measurement itself, or in the remote measurement.

Thus, it seems that without assuming hidden variables Stapp's arguments cannot show the failure of $\mathscr{L}$. According to my understanding, Stapp views "nonlocality" as something different from "the failure of $\mathscr{L}$" and he obviously believes that his proof shows something more than the existence of an instantaneous fixing of an outcome of a remote possible measurement—a trivial consequence of the assumption of the nonexistence of hidden variables and the existence of a "free" local agent which choses the measurement to be performed. After completing this section I found a similar worry about validity of Stapp's previous versions of the nonlocality argument in Section 4.2 of the book by Redhead.[26]

6. ELEMENTS OF REALITY IN THE GHZ SETUP

Several years ago an interesting argument based on the GHZ setup was advanced by Clifton *et al.*[10] They arrived at a contradiction assuming the existence of a realistic covariant description of quantum systems. Inspired by their work, I constructed time-symmetrized "elements of reality" in terms of which there is no contradiction. The price of removing the contradiction is a peculiar feature of these elements of reality: their product rule fails.

I have proposed the following *definition* of elements of reality:[11]

> If we can infer with certainty the result of measuring a physical quantity at time t then, at time t, there exists an element of reality corresponding to this physical quantity which has a value equal to the predicted measurement result.

This is a modification of what is usually considered to be a necessary *property* of an element of reality. The time-symmetrization is in the word "infer" which replaced the asymmetric term "predict."

Consider three particles prepared in the GHZ state at t_1 which were measured later at time t_2 and the results (6) were found. Now we can consider elements of reality which are counterfactual statements about measurements at time t, $t_1 < t < t_2$. Just from the fact that the particles are prepared in the GHZ state, it follows that

$$\begin{aligned}
\{\sigma_{Ax}\sigma_{Bx}\sigma_{Cx}\} &= -1 \\
\{\sigma_{Ax}\sigma_{By}\sigma_{Cy}\} &= 1 \\
\{\sigma_{Ay}\sigma_{Bx}\sigma_{Cy}\} &= 1 \\
\{\sigma_{Ay}\sigma_{By}\sigma_{Cx}\} &= 1
\end{aligned} \tag{9}$$

Note that these equations are different from (3), which relates to the product of the outcomes of separate spin component measurements at each location, while (9) relates to the measurement of the products of the spin components of the GHZ particles. The operators of the products commute when applied to the GHZ state, and therefore (9) are not "true" counterfactuals: they could be called "conditionals" because, in principle, they can be measured together without disturbing each other (not that I know how to do it). In contrast, Eqs. (3) are "true" counterfactuals because they cannot be measured together. However, Eqs. (3) are not exactly "elements of reality." [I named them "generalized elements of reality"[(27)] because each equation yields a certain relation between the outcomes of several measurements, and not just the value of a single measurement as appears in the definition of the element of reality.]

Taking into account the results of the measurements at t_2 given by (6), we can derive the following list of elements of reality related to the intermediate time t:

$$\begin{aligned}
\{\sigma_{Ay}\sigma_{By}\} &= -1 \\
\{\sigma_{Ay}\sigma_{Cy}\} &= 1 \\
\{\sigma_{By}\sigma_{Cy}\} &= 1
\end{aligned} \tag{10}$$

Note the difference between application of counterfactuals here and in the previous section, which allows derivation of (10). In the present section, the counterfactual worlds have fixed both initial and final conditions, i.e., the outcomes of the measurements at t_1 and t_2, in contrast to the situation in previous section in which only initial conditions were fixed.

Now we can consider the product of Eqs. (10):

$$\{\sigma_{Ay}\sigma_{By}\}\{\sigma_{Ay}\sigma_{Cy}\}\{\sigma_{By}\sigma_{Cy}\} = -1 \tag{11}$$

What is peculiar here is that the outcome of the measurement of the product of the operators appearing on the left-hand side of (11) is also known with certainty, i.e., we have an element of reality:

$$\{\sigma_{Ay}\sigma_{By}\sigma_{Ay}\sigma_{Cy}\sigma_{By}\sigma_{Cy}\} = \{\sigma^2_{Ay}\sigma^2_{By}\sigma^2_{Cy}\} = 1 \tag{12}$$

Thus, we have shown the failure of the product rule for time-symmetrized elements of reality. The product rule (which holds for time-asymmetric elements of reality with "predict" instead of "infer") is, "If $A = a$ is an element of reality and $B = b$ is an element of reality, then $AB = ab$ is also an element of reaiity."

7. CONCLUSIONS

I want to finish this paper by stressing again the lesson I learned from the GHZ proof and other works of Daniel Greenberger. As far as I know, he himself does not draw this conclusion, but I became a strong believer in the many-worlds interpretation (MWI)[12] of quantum theory. Only in this framework do the difficulties of the GHZ setup not lead to nonlocal physical action. There is no nonlocal action on the level of the theory of the whole physical universe which incorporates our world and many other worlds, while we can see that inside a particular world we obtain effectively a nonlocal action.

According to the MWI, the outcomes (6) take place in one world and there are, in parallel, three other worlds, corresponding to

$$\begin{aligned} &\{\sigma_{Ax}\} = -1, \qquad &&\{\sigma_{Bx}\} = 1, \qquad &&\{\sigma_{Cx}\} = 1; \\ &\{\sigma_{Ax}\} = 1, &&\{\sigma_{Bx}\} = -1, &&\{\sigma_{Cx}\} = 1; \\ &\{\sigma_{Ax}\} = -1, &&\{\sigma_{Bx}\} = -1, &&\{\sigma_{Cx}\} = -1. \end{aligned} \tag{13}$$

In each location the only change due to the measurement interaction is that each spin, originally correlated with the states of the two remote spins, becomes correlated also with a state of a measuring device. However, the complete local description of each spin, its density matrix, remains unchanged. On the other hand, "inside" a particular world when the particular outcomes of spin measurements of two particles are taken into account, the third particle is described by a pure state and therefore its density matrix is changed. Thus, no nonlocal actions take place in Nature, but, nevertheless, there is an explanation why *we* observe seemingly nonlocal phenomena.

ACKNOWLEDGMENTS

It is a pleasure to thank Philip Pearle, Sandu Popescu, and Abner Shimony for illuminating discussions and Jerry Finkelstein, Robert Griffiths, David Mermin, Henry Stapp, and Bill Unruh for helpful correspondence. This research was supported in part by Grants 614/95 and 471/98 from the Basic Research Foundation (administered by the Israel Academy of Sciences and Humanities). Part of this work was done during the 1998 Elsag–Bailey Foundation research meeting on quantum computation.

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