THE MEANING OF PROTECTIVE MEASUREMENTS

Yakir Aharonov[a,b], Jeeva Anandan[b] and Lev Vaidman[a]

[a] *School of Physics and Astronomy*
Tel-Aviv University, Tel-Aviv, 69978 ISRAEL

[b] *Physics Department, University of South Carolina*
Columbia, South Carolina 29208, U.S.A.

ABSTRACT

Protective measurements, which we have introduced recently, allow to measure properties of the state of a single quantum system and even the Schrödinger wave itself. These measurements require a protection, sometimes due to an additional procedure and sometimes due to the potential of the system itself. The analysis of the protective measurements is presented and it is argued, contrary to recent claims, that they measure the quantum state and not the protective potential. Some other misunderstandings concerning our proposal are clarified.

In this work we will analyze the meaning of our recent proposal of "protective measurements" which allow measuring the Schrödinger wave of a single particle [1,2]. There were several discussions of our proposal some of which challenging the validity of our results [3-7]. We shall briefly repeat our argument and will answer the questions raised by the critics. We, ourselves, believe that the objection to our proposal which is most difficult to refute is the one which was not raised by the critics. Since our measurement requires a long time, it is natural to think that what is measured is not the property of the Schrödinger wave at a given time, but the time average during the period of the measurements. In Refs. (1) and (2) we make a detailed analysis of this question and we will not discuss it here.

At present, the commonly accepted interpretation of the Schrödinger wave is due to Born. He proposed to interpret the wave intensity not as the density of distribution of actual matter, as Schrödinger first imagined, but as a probability density for the presence of a particle. Schrödinger, however, wanted to believe that his wave represents a single particle: the wave is an extended

object really moving in space. Born's interpretation was supported by the fact that nobody knew how to measure the density of the Schrödinger wave on a single system. There was a general belief that the Schrödinger wave can only be tested for an ensemble of particles. We have proposed a new type of measurements: "protective measurements" which allow direct measurement of the Schrödinger wave density on a single particle. We have shown that one can simultaneously measure the density and the current of the Schrödinger wave in many locations. The results of these measurements then allow to reconstruct (in an arbitrary chosen gauge) the Schrödinger wave.

In order to analyze the meaning of our proposal we will start the discussion not by considering a measurement of the whole Schrödinger wave, but of a single property of the quantum state: an expectation value of an operator. The essential novel point is that we can measure an expectation value of a quantum variable for a state which is not the eigenstate of the corresponding operator even when performing measurements just on a single system; or, what is even more important, that we obtain the expectation value directly for a single system and not as a statistical average of eigenvalues for an ensemble. In the standard interpretation the expectation value is defined as a certain statistical function of the eigenvalues. Therefore, since we can obtain the expectation value without knowing the eigenvalues, we can give a new interpretation.

In the standard interpretation the expectation values of variables are not considered as physical properties of a single system because in the outcome of the standard measuring procedure only one of the eigenvalues is observed. If the system, prior to the measurement of a variable A, is not in an eigenstate of A, then its quantum state is invariably changed due to the standard measuring procedure. A natural idea to prevent this change is to weaken the coupling with the measuring device. Indeed, in this case the state is not changed significantly, but then the pointer of the measuring device hardly moves. Its shift due to the measurement is smaller than its uncertainty, and therefore we cannot get information from the measurement. To remedy this we can increase the time of the coupling between the system and the measuring device. Then, if the state is constant during the measurement, the velocity of the pointer variable will also be constant and the total shift, which is proportional to duration of the interaction, will be large enough. However, under normal circumstances the state of the system is not constant during the measurement. Weak coupling leads to a small rate of change of the state. However, in order to obtain a distinguishable shift it requires a long time, and therefore, we invariably change the state. Thus, the reading of the measuring device will not correspond to the quantum state which the system had prior to the measurement, but to some time average depending on the evolution of the quantum state influenced by the measuring procedure. Therefore, in order to measure the quantum state, or a property of the quantum state such as expectation value of a variable (when the state is not an eigenstate of the corresponding operator) we need, in addition to the standard

weak and long measuring interaction, a procedure which will protect the state from changing during the measuring interaction.

The simplest protection procedure is introducing a protective potential such that the quantum state of the system will be a nondegenerate eigenstate of the Hamiltonian. In fact, in many important cases this protection is given by nature: almost isolated systems will eventually decay to their ground state or to some stable excited state. In order to protect the state the protection potential need not act all the time. It is possible to switch it on for frequent very short periods of time such that most of the time the system evolves under the original Hamiltonian. The other proposed protection scheme - frequent testing that the state have not changed - is similar to this method.

As an example of a simple protective measurement, let us consider a particle in a discrete nondegenerate energy eigenstate $\Psi(x)$. The standard von Neumann procedure for measuring the value of an observable A in this state involves an interaction Hamiltonian

$$H = g(t)PA, \tag{1}$$

coupling the system to a measuring device, or pointer, with coordinate and momentum denoted, respectively, by Q and P. The time-dependent coupling $g(t)$ is normalized to $\int g(t)dt = 1$ and the initial state of the pointer is taken to be a Gaussian centered around zero.

In standard impulsive measurements, $g(t) \neq 0$ for only a very short time interval. Thus, the interaction term dominates the rest of the Hamiltonian, and the time evolution $e^{-\frac{i}{\hbar}PA}$ leads to a correlated state: eigenstates of A with eigenvalues a_n are correlated to measuring device states in which the pointer is shifted by these values a_n. By contrast, the protective measurements of interest here utilize the opposite limit of extremely slow measurement. We take $g(t) = 1/T$ for most of the time T and assume that $g(t)$ goes to zero gradually before and after the period T. We choose the initial state of the measuring device such that the canonical conjugate P (of the pointer variable Q) is bounded. We also assume that P is a constant of motion not only of the interaction Hamiltonian (1), but of the whole Hamiltonian. For $g(t)$ smooth enough we obtain an adiabatic process in which the particle cannot make a transition from one energy eigenstate to another, and, in the limit $T \to \infty$, the interaction Hamiltonian does not change the energy eigenstate. For any given value of P, the energy of the eigenstate shifts by an infinitesimal amount given by first order perturbation theory:

$$\delta E = \langle H_{int} \rangle = \frac{\langle A \rangle P}{T}. \tag{2}$$

The corresponding time evolution $e^{-iP\langle A\rangle/\hbar}$ shifts the pointer by the average value $\langle A\rangle$. This result contrasts with the usual (strong) measurement in which the pointer shifts by one of the eigenvalues of A. By measuring the averages of a sufficiently large number of variables A_n, the full Schrödinger wave $\Psi(x)$ can be reconstructed to any desired precision.

The adiabatic measurements described above can be performed on a single system only if the measured quantum state is protected. However, when the mathematical expression of protected and unprotected states are identical, the quantum system behaves identically for all standard (impulsive and strong) quantum measurements. Indeed, the probability for various eigenvalues are the same for protected and unprotected states. Here we assume that the strength of the impulsive measurements (which is infinite in ideal measurements) is much bigger than the strength of the protection procedure. Therefore, the state being protectively observed may be regarded as the same state if it were unprotected.

The identical probabilities can be tested only on ensembles of identically prepared systems. Thus, to compare protected and unprotected states we have to consider an ensemble of identical protected states and an ensemble of identical (and the same) unprotected states. It is interesting that these ensembles are identical not only for ideal measurements, but also for *weak measurements* [8]. If we have large ensemble of quantum systems in the same state, then the protection procedure is not necessary. We can have weak short coupling between the measuring device and all systems of the ensemble. This coupling does not change significantly the quantum state, but the pointer moves a distance much larger than its uncertainty due to the combined effect of all the systems of the ensemble, showing the expectation value of measured variable. Although we use here an ensemble, the measurement is very different from the standard procedure: in no stage of the measurement we obtain the eigenvalues of the measured variable. Each system in the ensemble contributes the shift of the pointer proportional to the expectation value (and not to one of the eigenvalues). It is interesting to note that for a large enough ensemble we can tune the coupling to the measuring device such that we will get a reliable outcome of the measurement of the expectation value of the measured variable practically without changing the states of the systems: if we will test all systems after our measurement, then the probability to find even one system not in its original state can be made arbitrary small.

The main point of the majority of works criticizing our proposal was that we cannot measure an unknown quantum state of a single system since we cannot protect unknown quantum states. But we never claimed otherwise. More than this, we have claimed the opposite. If there was a procedure which allows to measure an unprotected unknown quantum state, then it is possible to distinguish between nonorthogonal states. The latter, however, contradicts unitarity of quantum theory: the scalar product between branches corresponding to these two states changes from

nonzero to zero when the measurement is completed. The answer is that there is no universal protection for all states. The nonorthogonal states require different protections; if we do not know the state we cannot protect it. What we have shown is that two nonorthogonal *protected* states can be distinguished. The unitarity paradox disappears since the quantum state of the environment in the two cases are orthogonal before the measurement [2]. Indeed, the states of the protection device for protection of two different nonorthogonal states must be orthogonal. Unprotected unknown nonorthogonal states cannot be distinguished.

We want to clarify the issue of the collapse of the Schrödinger wave in our procedure. Our adiabatic measurement of an expectation value of an operator performed on a protected state does not cause collapse. But, if the system is in a superposition of several quantum protected states (i.e., in the superposition of the eigenstates of the protection potential), then the state is not protected. Our measuring procedure will cause a collapse to one of the protected states. Indeed, the coupling to the measuring device will cause a correlation between the protected states and the states of the pointer of the measuring device. Then, in case that the expectation value for that eigenstate differs from that for the other eigenstates of the superposition, the process of macroscopical reading of the pointer variable will result in the collapse of the quantum state. Another collapse of the quantum state related to our proposal occurs if the initial state is unknown and we switch on the protection for a certain state (such as the strong magnetic field in the example of the spin measurement). So we have not "solved" the problem of the collapse of the state in quantum measurement, but we have never claimed otherwise.

We cannot measure an unprotected state. We cannot protect an unknown state. So, it seems that to measure the state we have to know it first, but then what is the purpose of the measurement? We believe that even if we measure a known property, it is still a measurement. Our measuring procedure is not some sophisticated method of reading known information. We use a coupling to the measured variable and look directly on the reading of the pointer of the measuring device. We use the information about the quantum state prior to the adiabatic coupling to fix the strength of the measuring coupling. In fact, this aspect is common to all physical measurements: we need to have some prior information before the measurement in order to choose an appropriate measuring apparatus. Although it is clear that in our method of the measuring of the expectation value the outcome is *measured* and not calculated from known quantum state, to make a decisive argument we can show that in our procedure we do obtain information which was not known before. To this end let us split our procedure into two stages. The first is a protection, made by one experimenter or even just by nature, and the second is the weak measuring procedure which yields the expectation value. The second stage is performed by another experimenter who does not know the state. She only knows that the state is protected and what is the degree of protection. If the protection is due to the energy conservation, i.e., the state is a nondegenerate

energy eigenstate, then the only information which is needed is a lower bound of the energy gaps to other levels. This is enough to fix the strength and the adiabaticity of the measuring coupling. Thus, the expectation value of any operator can be found, and even several such measurements can be performed, which together will yield the quantum state of the system. The precision of the measurement of the Schrödinger wave can be improved to any given limit by increasing the time of the interaction. What persuades us that the outcome of our procedure is the property of our single system is that it does not depend on the particular form of the measuring interaction. ***Any*** adiabatic measurement of a given observable performed on quantum system in protected state (with ***any*** valid protection mechanism) will invariably yield the same outcome. In order to demonstrate the new information we can obtain, consider an example of a system in an unknown potential for which we know approximately part of the spectrum of energies and we know, by leaving the system isolated for a long time, that the system has decayed to the ground state. Assume that we have just one such system and nobody knows the exact form of the potential. In this case we can perform adiabatic measurements of a set of variables which will yield the quantum state of the system. All this by measuring just one system and without obtaining the eigenvalues of the measured variables.

Another common line of the critique of our proposal was the claim that we are measuring the protection potential and not the quantum state of the system. Indeed, our measurement on a single system succeeds only if there is a protection, so it is natural to believe that it is the protection that has been measured. The example of a measurement of a spin-1/2 particle invites to accept this interpretation because of accidental one to one correspondence between protection (strong magnetic field) and the state (the spin polarized parallel to the magnetic field). On the other hand, in the example of a particle bound in a certain potential, there is no simple connection between the potential and the state. In order to find the density of the Schrödinger wave in a certain region, without protective measurements, one has to perform elaborate *calculations* based on the information about the potential everywhere. The density of the Schrödinger wave can manifest itself in numerous adiabatic measurements. We can make coupling to any kind of charge in any (although necessarily adiabatic and weak) form. All such measurements will yield the density of the wave. These measurements will yield the same results for infinite number of various protective potentials which are all characterized by having a nondegenerate eigenstate with this wave density in the chosen location. These persuade us that our procedure is the measurement of the density of the Schrödinger wave.

An even stronger argument against the interpretation of our procedure as a measurement of the protection potential rather than the protected state is that in general there are many possible

different protection procedures which can protect the same state. A general form of the protection Hamiltonian for a state $|\Psi_0\rangle$ is

$$H = G_0(1 - |\Psi_0\rangle\langle\Psi_0|) + \sum G_i|\Psi_i\rangle\langle\Psi_i|, \tag{3}$$

where the only requirements on the terms in the sum are: $\langle\Psi_0|\Psi_i\rangle = 0$ for all i, and the energies in the spectrum of the states $|\Psi_i\rangle$ are far from the energy of the state $|\Psi_0\rangle$. Thus, our measurement cannot be considered as a measurement of a protection Hamiltonian, but it can be considered as a measurement of a certain property of the protection Hamiltonian, i.e. the property to have as an eigenstate this quantum state characterized by the expectation values of various operators. We measure the property of the protection which is characterized completely by the quantum state, so it is just a matter of taste to call or not this property the quantum state.

The simplest protection Hamiltonian $H = G_0(1-|\Psi_0\rangle\langle\Psi_0|)$ can be immediately generalized also for the time-dependent quantum state $|\Psi_0(t)\rangle$:

$$H(t) = G_0(1 - |\Psi_0(t)\rangle\langle\Psi_0(t)|). \tag{4}$$

However, these protection Hamiltonians are, in general, nonlocal. In our discussion, in the framework of the nonrelativistic quantum theory, we can consider any Hamiltonian. Still it is an interesting question whether we can find a physical protection procedure for nonlocal states. A local potential $V(r)$ can give a protection to all its nondegenerate eigenstates. Note that for a Hamiltonian $H = p^2/2m + V(r)$ the energy eigenstate essentially defines the potential, so there is strong correspondence between the protected state and protection Hamiltonian. Also, local potential cannot protect the state which is nonvanishing on a few disconnected regions. To protect these states, and to restore the freedom of choosing various protections to the same state, we can use the following, rather artificial, but conceptually important method. As we mentioned before, the protection Hamiltonian does not have to act all the time. We can take the protection Hamiltonian which acts only for small but frequent periods of time. Then the procedure is to quickly bring all parts of the state to a single location, then switch on strong short protection Hamiltonian, then bring the state back and leave it for a period of time without protection Hamiltonian. The periods of time without protection interaction can be made much longer than the periods of time of the protection procedure. Then the disturbance due to these times of our measuring device can be neglected and the outcome of the adiabatic coupling to any variable of the system will be its expectation value in the protected state.

There is a position in which the only reality in quantum theory is a set of outcomes of measurements, and it is not allowed to discuss the reality of a system between the measurements. It is not that this attitude is incorrect – it is absolutely consistent, but we believe that in this

minimalistic approach we lose a lot of important physics. So we have to look for possible candidates for "reality". Unruh [4], following the standard approach, considers the dynamical variables as immediate natural candidates for the ontology of quantum theory. But as he himself points out, the dynamical variables frequently do not have certain values (which allowed to be only some eigenvalues). For example, a spin-1/2 particle in an EPR-Bohm pair has no definite spin value for spin components in any direction. Thus, the choice is, either to accept that it has no "spin" reality at all, or to accept that it has many different realities simultaneously. The quantum state however, does not have this difficulty: the Schrödinger wave is unique. Together with quantum state, the expectation values of all dynamical variables are well defined and we can perform a direct measurement of the expectation values using our adiabatic measuring procedure. Therefore, the quantum state and the expectation values of dynamical variables and not the eigenvalues are the entities which can be unambiguously considered as reality. In the example of a spin of a particle belonging to the EPR-Bohm pair, the expectation values of spin are defined in all directions (and equal 0).

Let us consider the example of the Schrödinger wave of an electron in a given potential. Our adiabatic measurement allows us to measure the expectation value of the projection operator on a certain region of space of unit volume. We interpret it as a density of the Schrödinger wave. We know that if we, in a similar adiabatic way, measure a Gauss flux of the electric field coming out of this volume we will get the density times the charge of the electron; if we were able to measure the flux of the gravitational field we would get the density times the mass of the electron; any other property which the electron might have will manifest itself for any adiabatic coupling as it is spread in the electron cloud. Do we want to explain all these facts as a property of the potential that is responsible for protecting the Schrödinger wave? We may have a continuum of different potential which all have the property of having eigenstates with the same density in the chosen volume. There is no contradiction in calling it just the property of the potentials, but we believe that there is no contradiction in calling it the density of the Schrödinger wave either, and the latter choice gives us very powerful intuition for analysis of various interactions and measurements, interactions which do not change the Schrödinger wave significantly.

Our measurements are not "measurements" defined in a standard approach to quantum theory, i.e. experiments which specify which eigenvalue the measured variable has. It seems our usage of the word "measurement" was the the main reason for the confusion generated by our work. We may follow the advice of Bell [9] to abandon the the word "measurement", and to call our procedure "observation". We can observe expectation values of operators, we can observe the density and the current of the Schrödinger wave. We can "see" in some sense the Schrödinger wave. This leads us to believe that it has physical reality. We hope that recent analysis of possible

realization of our ideas in a real laboratory [10, 11] will soon be implemented and this will serve as evidence of the fruitfulness of our idea.

This research was supported in part by grant 425/92-1 of the Basic Research Foundation (administered by the Israel Academy of Sciences and Humanities), by grant PHY-9307708 of the National Science Foundation, and by ONR grant R&T 3124141.

REFERENCES

1 Y. Aharonov and L. Vaidman, *Phys. Lett.* **A178**, 38 (1993).
2 Y. Aharonov, J. Anandan, and L. Vaidman, *Phys. Rev.* **A 47**, 4616 (1993).
3 C. Rovelli, *Phys. Rev.* ,**A 50**, 2788 (1994).
4 W. G. Unruh, *Phys. Rev.* **A 50**, 883 (1994).
5 D. Freedman, *Science*, **259**, 1542 (1993).
6 J. Schwinger , *Science*, **262**, 826 (1993).
7 J. Samuel and R. Nityananda, e-board: gr-qc/9404051.
8 Y.Aharonov and L. Vaidman, *Phys. Rev.* **A 41**, 11 (1990).
9 J. Bell, "Against "Measurement", in *Sixty-Two Years of Uncertainty*, A.I. Miller, Ed., Plenum Press, New-York, pp. 17-32 (1990).
10 J. Anandan, *Found. Phys. Lett.* **6**, 503 (1993).
11 S. Nussinov, "Realistic Experiments for Measuring the Wave Function of a Single Particle", UCLA preprint UCLA/94/TEP/2.