Nonlocality of a Single Photon Revisited Again

Recently Hardy [1] argued that the nonlocality of the quantum theory can be demonstrated for a single particle. The nonlocality means the impossibility of constructing a local hidden variable theory reproducing the predictions of quantum theory. However, Bohm [2] had constructed such a theory, i.e., hidden variable theory local at the one particle level, and therefore Hardy's claim cannot be true. (Bohm's theory is, however, nonlocal when applied to systems consisting of more than just one particle.)

Hardy proposed an experimental setup and correctly analyzed the possible outcomes of the experiment. However, I believe that its interpretation as a single photon experiment is misleading.

In the usual setup of Bell-type experiments [3] we have few systems at separated locations, one system at each location. Hardy's setup does not readily fall into this category, but if it does, the number of involved quantum systems is clearly larger than one. Indeed, he has three input channels s, a_1, and a_2 and essentially two separate locations in which the clicks of six detectors exhibit quantum (nonlocal) correlations. There is yet another sense of a single particle experiment (which is probably closer to Einstein's vision quoted by Hardy). In this setup there is a single nonrelativistic particle (which cannot be annihilated or created) with its Schrödinger wave spreaded in space. Obviously, Hardy's experiment does not belong to this category either.

If we do allow creation and annihilation of photons, then nonlocality can be demonstrated using a single photon state $|\Psi\rangle = \alpha|A\rangle + \beta|B\rangle$, which is a superposition of two separate wave packets localized at A and B. Aharonov [4] pointed out that there is an isomorphism between states of this type and states of two separate spin-$\frac{1}{2}$ particles: $|\Phi\rangle = \alpha|\uparrow\rangle_A|\downarrow\rangle_B + \beta|\downarrow\rangle_A|\uparrow\rangle_B$ for which nonlocality is well established [3]. The isomorphism alluded to above can be realized by a physical mechanism which creates locally a photon when the spin is "up" and absorbs a photon when the spin is down:

$$(\alpha|A\rangle + \beta|B\rangle)|\downarrow\rangle_A|\downarrow\rangle_B \leftrightarrow \alpha|\uparrow\rangle_A|\downarrow\rangle_B + \beta|\downarrow\rangle_A|\uparrow\rangle_B . \tag{1}$$

In fact, Hardy's work is, essentially, a translation of his other results on nonlocality for two particles without inequalities [5].

Hardy proceeds by presenting a "paradox." He considers his experiment in which the outcome was $F_1 = 1$ and $F_2 = 1$. He then points out that in this case the photon from the input s invariably has to be found in u_1 (if it were searched there by detector U_1) and, also, invariably has to be found in u_2 (if it were searched, instead, by detector U_2). He considers this as a paradox since in the input s we had at most one photon. Hardy resolves the paradox by introducing a genuine nonlocality. He claims that placing detector U_1 might influence the outcome of the measurement in the remote location and we might not get $F_2 = 1$. However, there is no reason for his unusual proposal, since there is no real paradox to resolve. The correct statement is instead that the photon invariably has to be found in u_1 if it was searched by U_1 *and was not searched by* U_2 similarly, the photon inveriably has to be Found in U_2 if it was seached by U_2 *and was not searched by* U_1. Clearly there cannot be a contradiction between these two correct statements.

Hardy considers here a preselected and postselected system and the feature he points out is typical for such systems. Probably the simplest example of this kind [6] is a *single* particle prepared in a superposition of being in three boxes A, B, and C: $|\Psi_1\rangle = (1/\sqrt{3})(|A\rangle + |B\rangle + |C\rangle)$ which was found later in the state $|\Psi_1\rangle = (1/\sqrt{3})(|A\rangle + |B\rangle - |C\rangle)$. If in the intermediate time, it was searched in box A it has to be found there, and if, instead, it was searched in box B, it has to be found there too. [Indeed, not finding the particle in box A would project the initial state $|\Psi_1\rangle$ onto $(1/\sqrt{2})(|B\rangle + |C\rangle)$ which is orthogonal to the final state $|\Psi_2\rangle$. In fact, Hardy has previously considered [7] another, truly surprising example of this kind; see Ref. [8] for our analysis of this example.

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