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NONLOCAL MEASUREMENTS AND TELEPORTATION OF QUANTUM STATES

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Recent result of Bennett *et al.* on teleportation of an unknown quantum state is obtained in the framework of nonlocal measurements proposed by Aharonov and Albert. The latter method is generalized to the teleportation of a quantum state of a system with continuous variables.

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1. TELEPORTATION

When I first heard the word "teleportation," I imagined heroes of "Star Trek" entering a transmitting cabin in their star ship Enterprise: In a few seconds they disappear and immediately appear on a distant planet. It sounds, however, like bad science fiction: Too many laws are broken in this picture. For example, we tend to believe that the center of mass of a closed system should not move. Thus, when the heroes move far away, the spaceship has to move a little in the opposite direction, but this is not shown in the movies. Also, this story requires huge currents to fulfill the continuity equation, and it is very hard to understand how these currents can be created.

However, quantum theory teaches us that the essential feature of an object is not the *matter* out which it is made, but its shape. Indeed, all objects are "made" out of identical elementary particles, and what distinguishes one object from another is the state of these particles. Thus, we can consider a different kind of machine for teleportation. The receiving teleportation chamber is not empty before the transmission, but it contains elementary particles in a number equal to the number of particles of the object to be transmitted. Then, transmission results in building the Star Trek heroes out of these particles, while the heroes in the cabin on the Enterprise revert to an unstructured set of elementary particles.

One might be tempted to build the heroes in several locations, i.e., to

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produce several copies. This is certainly not teleportation. But the unitarity of quantum theory prevents this possibility. It is impossible to clone an unknown quantum state. It is also impossible to identify an unknown quantum state without significantly changing it. Therefore, the only option quantum mechanics leaves is destruction of the heroes in one place while creating them in another.

The procedure for such an operation was discovered recently. Bennett, Brassard, Crepeau, Jozsa, Peres, and Wootters (BBCJPW) [1] have shown how to teleport an unknown quantum state. They found a method to transmit an unknown quantum state of a spin-1/2 particle to another distant spin-1/2 particle without actually moving it from one place to another. We will show how this can be achieved using another method [2] based on the nonlocal measurements of Aharonov and Albert [3].

In the next section we will review the method of nonlocal measurements. Section 3 is devoted to teleportation of the state of a spin-1/2 particle. In Sec. 4 we generalize the method to systems with continuous variables. Section 5 deals with philosophical aspects of teleportation. A brief summary concludes the paper in Sec. 6.

2. NONLOCAL MEASUREMENTS

We call a measurement *nonlocal* if it cannot be reduced to a set of local measurements. An example is a measurement of a sum of variables A_1 and A_2 related to two separate locations 1 and 2. The method of Aharonov and Albert for nonlocal measurements uses only local interactions. The measurement is described by interaction Hamiltonian

$$H = g(t)P_1A_1 + g(t)P_2A_2, \tag{1}$$

where $g(t)$ is normalized function with a compact support at the time of the measurement; P_1, P_2 are conjugate momenta of the pointer variables of the two parts of measuring device which locally interact at locations 1 and 2. In order to perform a nonlocal measurement (and not two local measurements), the initial state of the measuring device has to be

$$Q_1 + Q_2 = 0, \qquad P_1 - P_2 = 0. \tag{2}$$

After the interaction is completed,

$$A_1 + A_2 = Q_1 + Q_2, \tag{3}$$

and local measurements of Q_1 and Q_2, therefore, yield the value of $A_1 + A_2$.

A set of measurements of nonlocal variables can serve as a verification of a nonlocal (entangled) state. The EPR-Bohm state of two spin-1/2 particles (completely anticorrelated state) can be verified using two consecutive measurements: first of $\sigma_{1x} + \sigma_{2x}$ and then of $\sigma_{1y} + \sigma_{2y}$; see Fig. 1. Here and below we will use the units of $\hbar/2$ for the spin components, such that each component can have values ± 1. If the outcomes are

$$\sigma_{1x}(t_1) + \sigma_{2x}(t_1) = 0, \qquad \sigma_{1y}(t_2) + \sigma_{2y}(t_2) = 0, \tag{4}$$

where $t_2 > t_1$, then, after time t_2, the system is in the EPR-Bohm state. Had we started at time $t < t_1$ with the EPR-Bohm state, we would be certain to obtain the outcomes (4).

The method of Aharonov and Albert is applicable also for measurements which are nonlocal not only in space but also in time. The interaction Hamiltonian has to be modified such that local interactions in separate locations will take place at different times. For example, for a measurement of the sum $A_1(t_1) + A_2(t_2)$, the Hamiltonian is

$$H = g(t - t_1)P_1 A_1 + g(t - t_2)P_2 A_2, \tag{5}$$

where $g(t)$ has compact support around zero. The sums and also modular

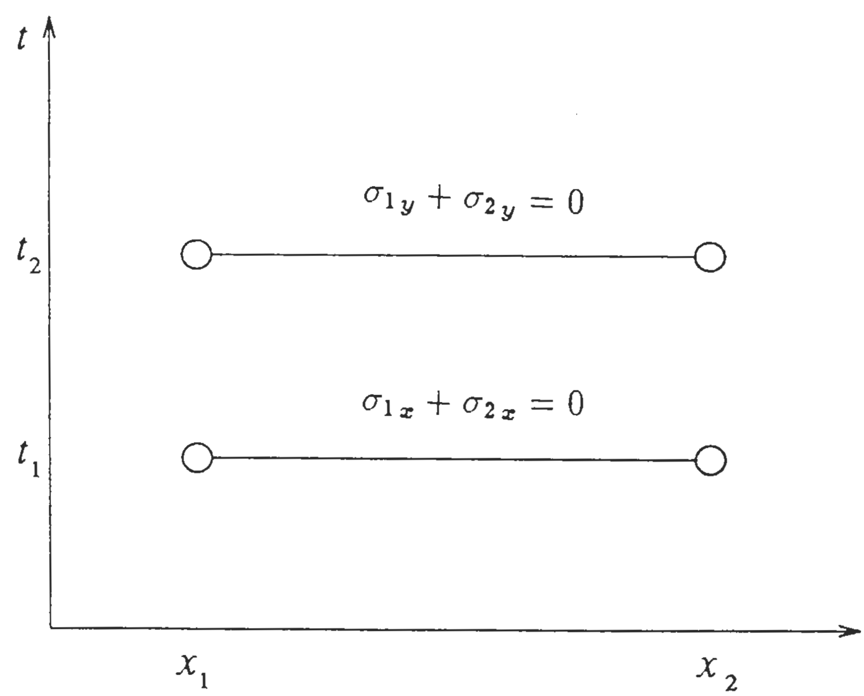

Fig. 1. If the system is in the EPR-Bohm state then the outcomes of the nonlocal measurements have to be as shown in the figure. Conversely, if these are the outcomes of the nonlocal measurements then, after the measurements, the system is in the EPR-Bohm state.

sums of local variables are measurable. For measuring a sum modulo a the measuring device has to be set in the following initial state:

$$(Q_1 + Q_2)\text{mod}a = 0, \qquad P_1 - P_2 = 0, \qquad P_1\text{mod}\frac{2\pi\hbar}{a} = 0. \qquad (6)$$

For more details see Ref. 4.

3. TELEPORTATION OF A STATE OF A SPIN-1/2 PARTICLE

Let us assume that the state of a spin-1/2 particle 1 is $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ and we have to teleport it to particle 2. To this end, consider the "crossed" measurements of $\sigma_{1x}(t_1) - \sigma_{2x}(t_2)$ and $\sigma_{1y}(t_2) - \sigma_{2y}(t_1)$, see Fig. 2. If the outcomes are

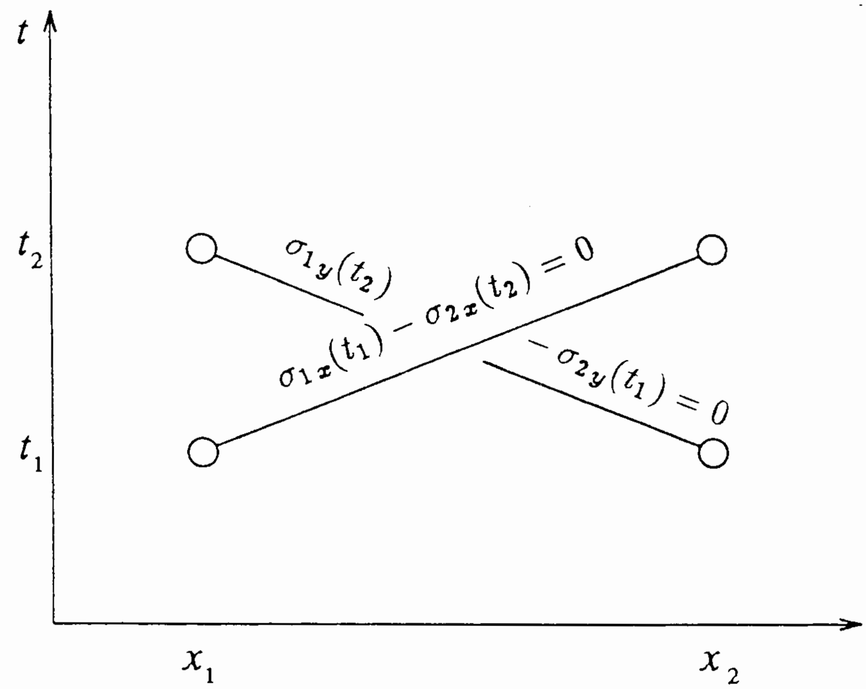

Fig. 2. Teleportation via "crossed" space-time nonlocal measurements. The measurements with the outcomes in the figure cause the state of particle 2 after t_2 to be the state of particle 1 before t_1 (and the state of particle 1 after t_2 to be the state of particle 2 before t_1). For *reliable* teleportation the nonlocal measurements to be performed are the measurements of the differences of the spin components modulo 4 accompanied by appropriate local rotations.

$$\sigma_{1x}(t_1) - \sigma_{2x}(t_2) = 0, \qquad \sigma_{1y}(t_2) - \sigma_{2y}(t_1) = 0, \tag{7}$$

then, taking into account the measurement interaction (5) with $A_1 = \sigma_{1x}$ ($A_1 = \sigma_{1y}$) and $A_2 = -\sigma_{2x}$ ($A_2 = -\sigma_{2y}$), the initial state of the measuring device (2), and the outcomes of the local readings of the measuring device (which are also described by Eq. (2)), we obtain, after straightforward calculation, that the final state of particle 2 is $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$, i.e., we have succeeded in teleporting the state of particle 1 to particle 2.

However, this procedure is not good enough, since the nonlocal measurements might not yield the specific outcomes (7). The difference between the spin components might equal ± 2 and in that case we destroy the state without teleporting it. In order to obtain *reliable* teleportation (such as the one suggested by BBCJPW), we must measure, instead, the following nonlocal observables:

$$\Big(\sigma_{1x}(t_1) - \sigma_{2x}(t_2)\Big)\mathrm{mod}4, \qquad \Big(\sigma_{1y}(t_2) - \sigma_{2y}(t_1)\Big)\mathrm{mod}4. \tag{8}$$

A null outcome reduces to the previous case. If, however, the outcome of one of the above is 2, then we can convert it to 0 by appropriate rotation of the coordinate frame of the second particle (for example, $\sigma_{2x} = -\sigma_{2x'}$, for $\hat{x}' = -\hat{x}$). Thus, for any set of outcomes of the nonlocal measurements (8) the spin state is teleported; in some cases the state is rotated, but the resulting rotation can be inferred from the nonlocal measurements. We can complete, then, the teleportation by the following transformations. In the case of two null outcomes no additional transformation is needed; in three other cases a transformation of rotation by the angle π is necessary: the rotation around the y axis for the outcome (2,0), around the x axis for (0,2), and around the z axis for the outcome (2,2).

The Aharonov-Albert method for nonlocal measurement contains the following elements: (i) a preparation of an entangled state of the measuring device, (ii) local interactions with separate parts of the system, (iii) local readings of the separate parts of the measuring device resulting in a set of numbers obtained in the respective space-time locations of the parts of the system. These numbers represent classical information which must be transmitted for completing the teleportation. (In our example, the information tells us which rotation must be performed). The initial entanglement of the measuring device, which is the core of the method, may employ pairs of spin-1/2 particles in the EPR-Bohm state (see Sec.IV of Ref. 4), making this method very similar to the BBCJPW proposal. In this case we also need to transmit just two bits of classical information, which is the minimal information for teleportation of a spin state, as has been proven in Ref. 1. The number of "nonlocal channels" in our method is two instead of just one in the BBCJPW method. This is because we have accomplished two-way teleportation. (Obviously, for teleporting also the state of particle

2 to particle 1 we need to send another two bits of classical information from the site 2 to the site 1.)

The BBCJPW method can be presented in our language as in Fig. 3. The EPR-Bohm pair which is employed by BBCJPW can be created via two (successful) measurements (4). The measurement "in the Bell operator basis" (Eqs. 1 and 2 of Ref. 1) at the location 1, performed on the composite system consisting of particle 1 and one member of the EPR-Bohm pair, is equivalent to two consecutive measurements of the modular sums:

$$\Big(\sigma_{1x}(t_1) + \sigma_{2x}(t_1)\Big)\mathrm{mod}4, \qquad \Big(\sigma_{1y}(t_2) + \sigma_{2y}(t_2)\Big)\mathrm{mod}4. \tag{9}$$

The four different combinations of the outcomes of the nonlocal measurements (9) correspond to the four outcomes of local measurement of BBCJPW. The procedure of teleportation is completed by appropriate rotation according to these results. After the teleportation, particle 1 is in a mixed state and contains no information. This is in contrast with the "crossed" measurements method in which (after the appropriate rotation) the the final state of particle 1 is the initial state of particle 2.

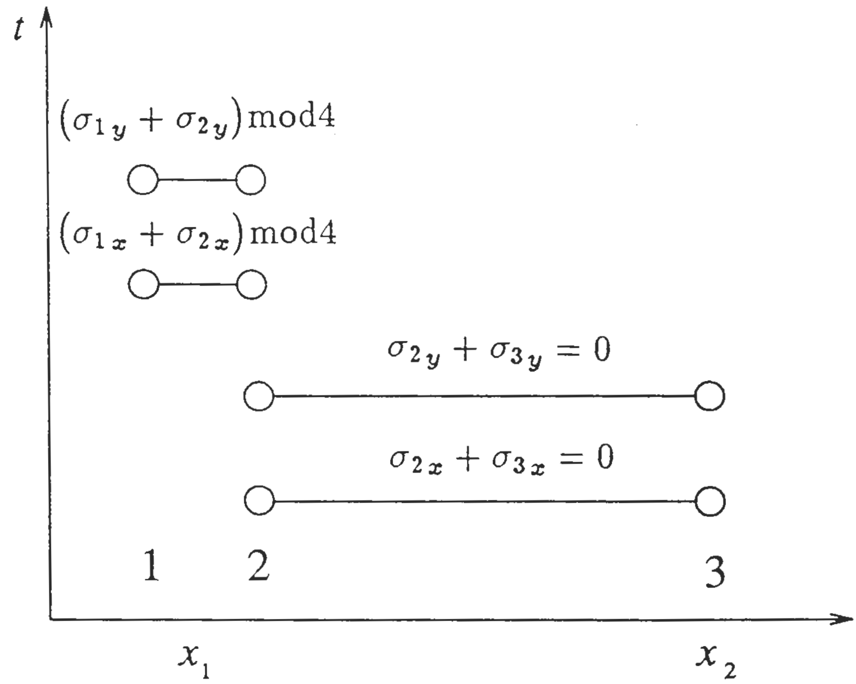

Fig. 3. Nonlocal measurements of the BBCJPW teleportation scheme. The state of particle 1 is teleported to particle 3. To this end the particle 3 is prepared in the EPR-Bohm state with particle 2 located near particle 1 using nonlocal measurements. Then the measurements of the composite system consisting of particles 1 and 2 causes the teleportation (possibly, with rotation).

4. TELEPORTATION OF A CONTINUOUS VARIABLE STATE

In the framework of nonlocal measurements there is a natural way of extending the teleportation scheme to the systems with continues variables. Consider two similar systems located far away from each other and described by continuous variables q_1, q_2 with corresponding conjugate momenta p_1 and p_2. In order to teleport a quantum state $\Psi(q_1)$, we perform the following "crossed" nonlocal measurements (see Fig. 4*a*), obtaining the outcomes a and b:

$$q_1(t_1) - q_2(t_2) = a, \qquad p_1(t_2) - p_2(t_1) = b. \tag{10}$$

Straightforward calculation shows that these nonlocal "crossed" measurements correlate the state of particle 1 before t_1 and the state of particle 2 after t_2, thus teleporting the quantum state to the second particle up to a shift of $-a$ in q and $-b$ in p. These shifts are known (after the results of local measurements have been transmitted), and can easily be corrected by appropriate back shifts even if the state is unknown, thus completing a reliable teleportation of the state $\Psi(q_1)$ to $\Psi(q_2)$.

A generalization of the BBCJPW scheme to the case of continuous variables is also possible, see Fig. 4*b*. The method contains the following stages: first, the preparation of the EPR state of particles 2 and 3,

$$q_2 + q_3 = 0, \qquad p_2 - p_3 = 0, \tag{11}$$

second, the consecutive measurements performed on particles 1 and 2, yielding the outcomes a and b:

$$q_1 + q_2 = a, \qquad p_1 - p_2 = b. \tag{12}$$

Each pair of measurements (11), (12) causes an anticorrelation, thus the anticorrelation between particles 2 and 3 together with the anticorrelation between particles 1 and 2 lead to a correlation between particles 1 and 3. The only difference between the states is due to the shifts both in q and in p:

$$q_3 = q_1 - a, \qquad p_3 = p_1 - b. \tag{13}$$

If the initial state of particle 1 is $\Psi(q_1)$, then the state of particle 3, after the measurements (11) and (12) have been performed, is $e^{ibq_3}\Psi(q_3 + a)$, which is exactly the state obtained after the "crossed" measurements (10). The final stage of teleportation are the appropriate back shifts of the state in p and q. Note again that while the crossed measurements yield two-way teleportation, we have obtained now only a one-way teleportation: from particle 1 to particle 3.

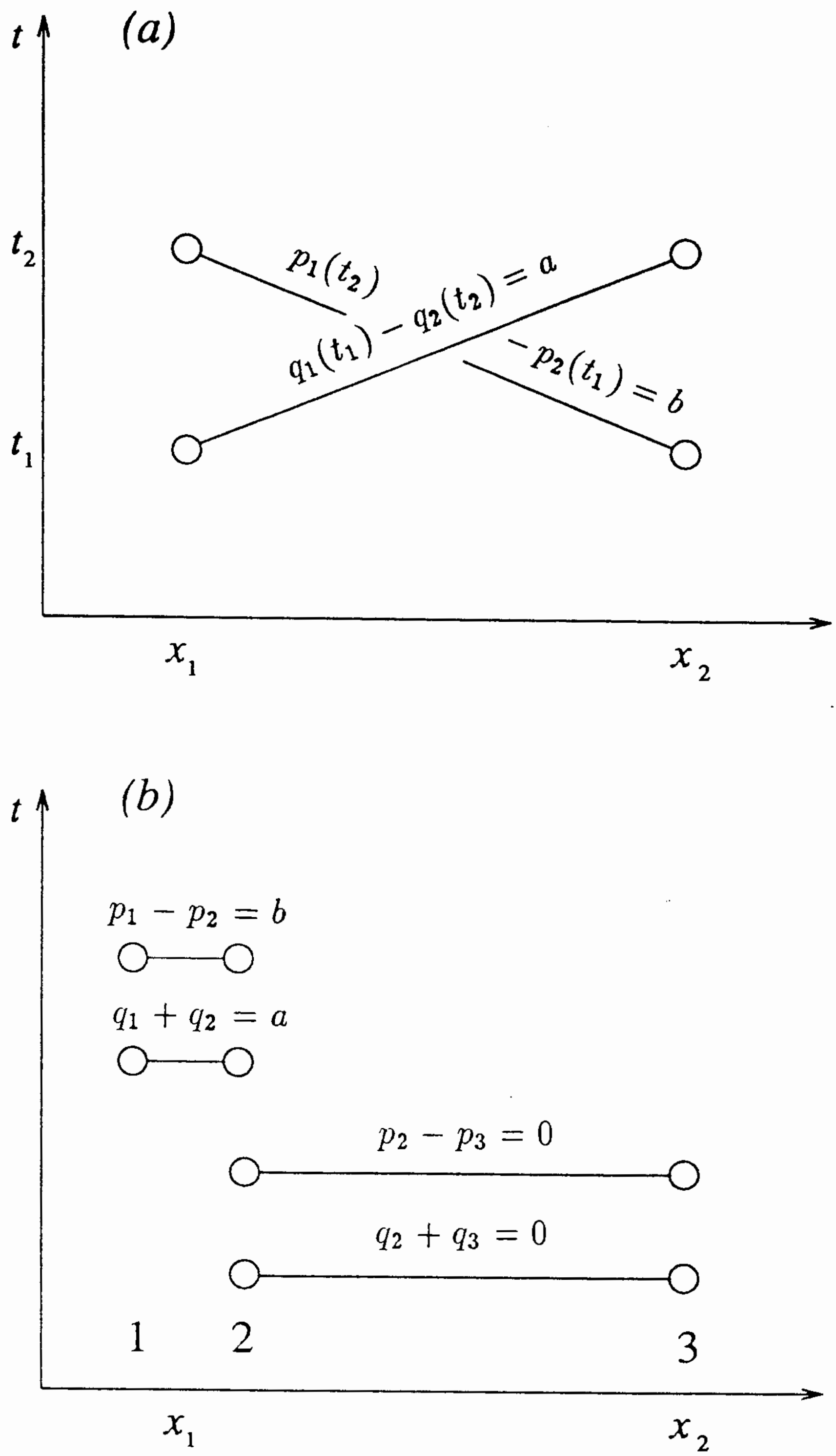

Fig. 4. Teleportation of an unknown quantum state of a system with continuous variables: (a) the method of "crossed" space-time nonlocal measurements, (b) the method analogues to the BBCJPW proposal. In both cases the final state (before the back shifts) is $e^{ibq}\Psi(q+a)$.

5. IS THERE A PARADOX WITH TELEPORTATION?

Consider teleportation, say in the BBCJPW scheme. We perform some action in one place and the state is *immediately* teleported (sometimes also rotated and/or shifted) to an arbitrary distant location. But relativity theory teaches us that anything which is physically significant cannot move faster than light. If we decide that an unknown rotation of the state is important, then we have to transmit classical information (which cannot be done with superluminal velocity) about the kind of back rotation to be performed for obtaining, after teleportation, exactly the same state. The classical information about the rotation is very small, but it is the only thing which is not transmitted immediately. Thus, it seems, that this classical information is the only essential part of the quantum state. Is the essence of a state of a spin-1/2 particle just 2 bits? I tend to attach a lot of physical meaning to a quantum state, not only because Aharonov and I found a method for measuring the state of a single quantum system [5], but also because I am a proponent of the many-worlds interpretation of quantum theory [6]. For me everything is a quantum state. But I also believe in relativistic invariance, so the only entities that cannot move faster than light have physical reality. Thus, teleportation poses a serious problem to my attitude.

A similar and even simpler example of this difficulty is a single spin measurement of one particle of an EPR-Bohm pair. It can also be be considered as some kind of teleportation. The state of the second particle of the pair immediately after the measurement is (up to inversion) the state of the first one. According to standard quantum mechanics there were no pure states of the particles before the measurement. The local measurement *created* the spin state of the first particle, and it was immediately transmitted and reversed (without destroying the state of the first particle) to a distant particle. This is a kind of superluminal teleportation.

I resolve this paradoxical situation in my interpretation of the many-worlds interpretation [7]. Anything superluminal is forbidden in a physical world (or universe as I call it). The universe incorporates all our worlds as members of a superposition which is the quantum wave (i.e., the universe). The act of measurement on one particle changes no physical property of the second particle from the point of view of an observer who can see the whole Universe, i.e. all worlds together. If, before the measurement, the second spin was correlated to the states of the first spin, then after the measurement, it will be also correlated to the measuring device and the observer of the first particle, but its density matrix remains unchanged. Thus, there is no change in any measurable property of the second particle. Why, then, do we believe that the state of the second particle has changed? Because our measurement locally splits the world of the observer of the first particle. When he (in one of the new worlds) later will look or ask