Comment on "One-state vector formalism for the evolution of a quantum state through nested Mach-Zehnder interferometers"

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Bartkiewicz *et al.* [Phys. Rev. A **91**, 012103 (2015)] provided an alternative analysis of an experiment performed by Danan *et al.* [Phys. Rev. Lett. **111**, 240402 (2013)] which presented surprising evidence regarding the past of photons passing through an interferometer. They argued that the quantity used by Danan *et al.* is not a suitable which-path witness, and proposed an alternative. The weaknesses of the analysis are discussed, and it is argued that the alternative, analyzed properly, provides no new predictions.

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The experiment discussed in Ref. [1] continues to be in the center of a controversy. Bartkiewicz *et al.* [2] (BCJLSS) argued that they "shed more light on the ongoing discussion regarding the legitimacy of the experimental data and their interpretation." They write that "theoretical calculations and an interpretation of the experimental results using only the standard one-state vector quantum-mechanical approach are still lacking," and that they "establish a more reliable which-path witness and show that it yields well-expected outcomes of the experiment."

Danan *et al.* [1] "asked" photons where have they been inside an interferometer. BCJLSS considered the same interferometer, but proposed to ask the photons in a different way. BCJLSS claim that their analysis predicts the same results, reproducing the experimental data presented in Figs. 3(a) and 3(b) of Ref. [1] [BCJLSS Figs. 1(a) and 1(b)], but differs in the experiment presented in Fig. 3(c) [their Fig. 1(c)]. I will denote these experiments as cases (a), (b), and (c). I question the derivation of their results, and, more importantly, argue that these results are not relevant for the question of the position of photons in the interference experiment.

To describe the interaction of photons with vibrating mirrors, BCJLSS introduce "frequency modes" which in principle allow one to know the mirror with which the photon interacted. According to my understanding, their proposal can be implemented in the following way. Instead of placing the quad-cell detector at the output of the interferometer, the light emitted during the run (the duration of which was 1 s) is stored and then a quantum measurement of the mode is performed. The measurement should provide one of the existing modes which specifies the mirror with which the photon interacted.

This method is conceptually incorrect for analyzing the position of particles inside an interferometer. Danan *et al.* wanted to know where the photons were in an undisturbed interference experiment. A measurement telling us an exact path of the photon invariably destroys the interference, so it cannot tell us much about photons passing through a properly working interferometer. This is why Danan *et al.* used weak measurements which, in the limit, do not disturb interference. In such measurements the information comes from many photons. Danan *et al.*'s measurements provided almost no information about the location of individual photons.

But how can this difference arise when the setups are identical until the final measurements performed at the output port, after the interference took place? When the photons pass through the interferometer, the information is written in their transversal degree of freedom. How does it happen that the way this information is observed makes such a big difference? BCJLSS do not mention that most of the photons in the output port will not be detected in one of the frequency modes they mentioned, but in the zero-frequency mode corresponding to null information about the location of photons inside the interferometer. The amplitude of vibration in the experiment is very tiny, so in the output port almost all photons will have a zero-frequency mode. The measurements of BCJLSS provide no direct information about where these photons were.

A possible approach is to consider the full ensemble and to argue that fractions of photons with observed frequency modes tell us where the photons with an observed zero-frequency mode were present. Danan *et al.* also made a similar assumption: The information obtained from the ensemble was interpreted as relevant for every single photon. BCJLSS could argue that the photons with detected mirror frequencies were disturbed by their measurements, but they allowed one to know where the zero-mode photons were, which were the majority in this interference experiment.

Note, however, that BCJLSS are vulnerable to the following line of criticism: The photons, disturbed by the measurement, cannot provide reliable information about the undisturbed photons, since disturbance destroys interference. This criticism is less effective for the experiment discussed in Ref. [1] in which the information is taken from all photons, while the measurement disturbance only slightly spoils the interference of every photon

Besides the conceptual disadvantage of the BCJLSS method, I argue that they do not analyze it correctly. Note that case (a) of Ref. [1] and case (a) of BCJLSS are different by a relative phase of π of path C, such that the latter does not correspond to a full constructive interference of the interferometer as it was in the experiment discussed in Ref. [1]. However, in both cases the analysis of Ref. [1] makes the same predictions regarding the positions of pre- and postselected photons inside the interferometer, so we can follow the setup of BCJLSS.

In the experiment, the correct transformation of the photon due to an interaction with a vibrating mirror, say, mirror C, is not $\hat{c}_{00000} \to \hat{c}_{00100}$ as it appears in Eq. (2) of BCJLSS, but

$$\hat{c}_{00000} \to \frac{1}{\sqrt{1+\epsilon^2}}(\hat{c}_{00000} + \epsilon\hat{c}_{00100}), \tag{1}$$

where $\epsilon \ll 1$. Then, the correct output state in the Fock space [instead of Eq. (9) of BCJLSS] is

$$|\Psi_{\text{out}}\rangle = \mathcal{N}\{e^{i\varphi}|1\rangle_{00000} + \epsilon[|1\rangle_{00100} + (e^{i\varphi}-1)(|1\rangle_{00010} + |1\rangle_{00001}) + e^{i\varphi}|1\rangle_{10000} - |1\rangle_{01000}]\} + \text{h.o.t.}, \tag{2}$$

where the normalization factor $\mathcal{N} = \frac{1}{3} + O(\epsilon^2)$ corresponds to a single photon in all output ports, and the high-order terms are

$$\text{h.o.t.} = \mathcal{N}\{\epsilon^2[e^{i\varphi}(|1\rangle_{10010} + |1\rangle_{10001}) - (|1\rangle_{01010} + |1\rangle_{01001})] + \epsilon^3(e^{i\varphi}|1\rangle_{10011} - |1\rangle_{01011})\}. \tag{3}$$

The probability of postselection in the output port on a particular frequency mode, say, mode A, is proportional to the weight of the output state projected on the space which has this mode,

$$\text{prob}(A) = \|\mathbf{P_A}|\Psi_{\text{out}}\rangle\|^2, \tag{4}$$

where

$$\mathbf{P_A} \equiv \sum_{i,j,k,l=0,1} |1\rangle_{1ijkl}\langle 1|_{1ijkl}. \tag{5}$$

This method provides the results of cases (a) and (b). For case (a), $\varphi = \pi$,

$$\text{prob}(A) = \text{prob}(B) = \text{prob}(C) = \frac{\epsilon^2}{9},$$
$$\text{prob}(E) = \text{prob}(F) = \frac{4\epsilon^2}{9}. \tag{6}$$

The probability for finding the zero mode is $\frac{1}{9}$.

For case (b), corresponding to phase $\varphi = 0$,

$$\text{prob}(A) = \text{prob}(B) = \text{prob}(C) = \frac{\epsilon^2}{9},$$
$$\text{prob}(E) = \text{prob}(F) = 0, \tag{7}$$

while the probability for finding the zero mode in this case is $\frac{1}{9}$, as in case (a).

The state in output port (2) is very different from the output state obtained by BCJLSS, as seen in their Eq. (9):

$$|\psi_{\text{out}}\rangle = \tfrac{1}{3}(|1\rangle_{00100} - |1\rangle_{01011} + e^{i\varphi}|1\rangle_{10011}). \tag{8}$$

In order to obtain the results of the experiment from their state, BCJLSS introduced a procedure "formally equivalent to the projection of the output state" in which they defined the postselected state in their Eq. (10),

$$|\Pi_A\rangle = \sum_{i,j,k,l=0,1} |1\rangle_{1ijkl}, \tag{9}$$

and then calculated the square of the value of the scalar product between (8) and (9), their Eq. (11):

$$|\langle \Pi_A|\psi_{\text{out}}\rangle|^2. \tag{10}$$

BCJLSS used (8) instead of (2), (9) instead of (5), (10) instead of (4), and obtained results which are proportional to what I have obtained, but I cannot see a justification for their procedure. Consider, for example, the absence of modes E and F in case (b) when $\varphi = 0$. When output state (8) has terms $|1\rangle_{10011}, |1\rangle_{01011}$, modes E and F must appear independently on the phase, since these are two orthogonal modes and thus they cannot interfere. BCJLSS obtained a cancellation of the modes by performing a scalar product with the postselected state

$$|\Pi_B\rangle = \sum_{i,j,k,l=0,1} |1\rangle_{i1jkl}, \tag{11}$$

which has particular phases between terms with different modes. What is the physical mechanism which fixes these phases? [In my analysis, modes E and F are not seen in case (b) because they appear only in the high order in ϵ in the output state; see (3).]

Another argument against the BCJLSS approach, which is not related to the method of observing output photons, is that in the experiment discussed in Ref. [1] no significant disturbance of visibility of the inner interferometer, due to vibration of the mirrors, was observed. The orthogonality of modes $\hat{c}_{10011}$ and $\hat{c}_{01011}$ appearing in (8) is in contradiction to this fact.

Curiously, the success of the BCJLSS method has a qualitative explanation in the two-state vector formalism (TSVF) [3]. It asserts that the photon was present in the overlap of the forward and backward evolving wave functions. A mirror can create a mode with its frequency if the forward evolving wave reaches it. If the backward evolving wave reaches the mirror, it means that the photon mode created there can reach the detector.

The quantitative analysis, however, shows that obtaining the same results using the BCJLSS method and Danan *et al.*'s type of measurement is accidental. In the experiment there are equal probabilities of finding the photon near mirrors A, B, and C and there is an equality

$$|(\mathbf{P_A})_w| = |(\mathbf{P_B})_w| = |(\mathbf{P_C})_w|, \tag{12}$$

which leads to equal size signals according to the TSVF.

In general, however, the results are different, for example, when the reflectivity of the beam splitters of the external interferometer are changed to 50 : 50, as in the example presented in Fig. 3 of Ref. [4]. In the notation of this reference,

$$|(\mathbf{P_B})_w| = |(\mathbf{P_C})_w| = \tfrac{1}{2}, \quad |(\mathbf{P_A})_w| = 1, \tag{13}$$

while the probabilities of finding the photon near the mirrors are related as

$$\text{prob}(B) = \text{prob}(C) = \tfrac{1}{4}\text{prob}(A). \tag{14}$$

Different ratios lead to different size signals calculated according to the two methods.

Let us turn to case (c). BCJLSS wrote that their "theoretical prediction does not match the experimental data from Ref. [1]." Their Fig. 1(c) tells us that in this case the photon was in A and/or in B. If their method is a reliable which-path witness, as they claim, then Fig. 1(c) also tells us that the photon was not present in C, E, and F. Indeed, the BCJLSS measurement method of detecting the frequency modes provides the following probabilities:

$$\mathrm{prob}(A) = \mathrm{prob}(B) = \frac{\epsilon^2}{9},$$
$$\mathrm{prob}(C) = \mathrm{prob}(E) = \mathrm{prob}(F) = 0. \tag{15}$$

But this is not a "well-expected" behavior. It apparently means that the photon did not exist for some periods of time before and after being in the nested interferometer. Assuming ideal devices, the probability for finding the zero mode in case (c) is 0. Therefore, the signal should be much larger than in cases (a) and (b) since the fraction of photons detected in the output port with frequency modes of mirrors is of the order 1, instead of ϵ^2. This signal, however, tells us nothing about the position of the undisturbed photons in the interferometer, since in such an experiment detector D observes only disturbed photons.

In practice, in case (c) of Ref. [1], without the zero-mode photons, the number of relevant detected photons was so small that no signal beyond the noise of the system was observed. In fact, case (c) was introduced in Ref. [1] not as an experiment which was supposed to tell where pre- and postselected photons were, but as a test that the experimental system, especially for case (b), worked properly. The location of the photons in this case was not considered in Ref. [1], but it can be analyzed according to the approach they adopted [4], in which the photon was where it left a weak trace. The gedanken experiment of case (c), performed on a pre- and postselected ensemble with a finite amplitude of vibration of the mirrors, the postselection of, say, mode A by the BCJLSS method, and a weak measurement of the trace with external devices, will show a continuous trajectory which includes E, A, and F. See Refs. [5,6] for further discussions about the position of the photons in case (c).

The past of a photon in an interferometer is a subtle issue both theoretically and experimentally. Quantum mechanics does not have an unambiguous definition for a position of a pre- and postselected particle [4,7]. The experiment discussed in Ref. [1] does show correctly where the photons were, when the definition of a location of a pre- and postselected particle is all places where it leaves a trace. It is not a direct measurement of the trace left by photons on other systems. It uses a degree of freedom of the photon itself, which makes the experiment much easier to perform, but more difficult to justify. Some variations of this experiment, such as different analyses of the output photons proposed by BCJLSS, or an apparently harmless insertion of a Dove prism [8], might not properly record and/or read this trace.

In light of the above, I argue that the BCJLSS analysis does not help to understand the past of photons passing through interferometers. Their which-path witness provides new predictions only in case (c) in which there are no undisturbed photons passing through the interferometer.

I also do not think that calculations of the experimental results of Danan *et al.* using only the standard one-state vector quantum-mechanical approach are lacking. The original Letter [1], with its Supplemental Material and preceding theoretical article [4], provide a correct one-state vector quantum description. It was repeated in detail by Saldanha [9], and recently repeated again with even more details by Potoček and Ferenczi [10]. However, the analysis of the physical meaning of the results and the consensus regarding their interpretation are still wanting.

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