WEAK MEASUREMENT OF PHOTON POLARIZATION

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An optical procedure for measuring the recently introduced *weak value* of a photon polarization variable is suggested and some of its experimental aspects are discussed. The deflection of a beam passing through a birefringent prism is considered. It is shown that if the beam also passes through preselection and postselection polarization filters, it may have a deflection which is greater than the deflection of the beam without the filters. The magnitude of the deflection is give by *weak measurement* theory.

The concept of the *weak value* of a quantum mechanical variable A has recently been introduced by Aharonov, Albert, and Vaidman (AAV) [1] (see also ref. [2]), and discussed by other authors [3–7]. This value is the statistical result of the standard measuring procedure carried out on a preselected and postselected ensemble when the coupling of the system with the measuring device is sufficiently weak, a procedure called *weak measurement.* The weak value obtained from such a measurement can lie outside the range of the eigenvalues of A, in sharp contrast to the expectation value.

Such surprising outcomes of weak measurement represent a peculiar interference phenomenon of quantum wave functions. We shall show that the same phenomenon occurs in electromagnetic waves, and that it can be readily observed in the optical region. In making such a measurement, we are, in fact, making a weak measurement of a photon polarization variable. Since considerable experimental difficulty can be expected in any attempt to determine the weak value of a spin component of a spin-$\frac{1}{2}$ particle, as described in ref. [1], the optical measurements described here provide the simplest means of verifying the feasibility of such a determination.

The observations we discuss are to be made on a classical intense optical beam, but since they are motivated by a consideration of the properties of a single photon, and since they directly reflect those properties, it is proper to refer to them as "weak measurement of photon polarization". The optical polarization filters employed provide an especially precise means of performing the preselection and postselection which play an essential role in weak measurement. In addition to the interest of this experiment as an example of weak measurement, it may suggest practical optical applications of the underlying AAV interference phenomenon.

A weak measurement is carried out by allowing a quantum system to pass successively through a pair of filters which select states $|\psi_1\rangle$ and $|\psi_2\rangle$. A measurement of some variable A is performed, with poor resolution, when the system is between the filters. As shown in AAV, this provides a convenient method for exploring the properties of the system that are compatible with both selected states. The result of this measurement, the weak value of A, is given by

$$A_w = \frac{\langle\psi_2|A|\psi_1\rangle}{\langle\psi_2|\psi_1\rangle}. \tag{1}$$

When the two selected states are nearly orthogonal, this value can be much greater than the largest eigenvalue of A.

For a spin-$\frac{1}{2}$ particle, a large weak value of σ_z can be obtained by selecting states $|a\rangle$ and $|b\rangle$ which have spins lying in the x–z plane and making an angle $\frac{1}{2}\pi-\phi$ with the z-axis. The situation is illustrated in fig. 1, where the states of a spin-$\frac{1}{2}$ particle are represented as points on the surface of a spatial sphere with orthogonal states at opposite ends of a diame-

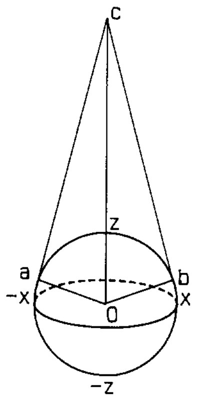

Fig. 1. Spin states involved in weak measurement of σ_z for a spin-$\frac{1}{2}$ particle. States are represented as points on the surface of a unit spatial sphere. States at opposite ends of a diameter are orthogonal. Values of the spin variables in the directions Oa and Ob are fixed to be $+\frac{1}{2}$ by preselection and postselection. Oc represents the weak value of σ_z.

ter. The components of σ satisfy the geometrical relation $\sigma_a+\sigma_b=2\sigma_z \sin\phi$. Since an operator relation of the form $A=B+C$ insures that the weak values satisfy $A_w=B_w+C_w$, and since the weak values of σ_a and σ_b are both equal to $+1$, the value selected by the filters, this relation shows that $(\sigma_z)_w$ has the value $1/\sin\phi$. This value is much greater than unity for small ϕ.

The experiment we discuss is a weak measurement of a two-valued photon polarization variable analogous to a spin component of a spin-$\frac{1}{2}$ particle [#1]. The variable Q in question can be either of the following:

$$Q_1=|x\rangle\langle x|-|y\rangle\langle y| ,$$

$$Q_c=|L\rangle\langle L|-|R\rangle\langle R| , \qquad (2)$$

where $|x\rangle$ and $|y\rangle$ are orthogonal states of linear polarization, and $|L\rangle$ and $|R\rangle$ are left- and right-circularly polarized states. We examine the conditions under which the weak value of Q is well outside its eigenvalue range.

The proposed measuring configuration, fig. 2, is an optical model of the Stern–Gerlach apparatus, employing a birefringent or optically active prism to give different deflections to states of different polarization. It is adapted for the purpose of weak mea-

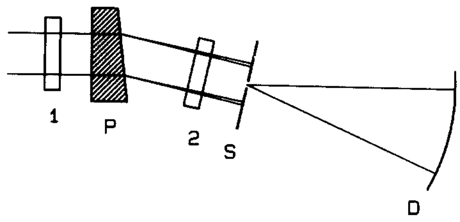

Fig. 2. Schematic diagram of the system proposed for weak measurement of the photon polarization variable $Q=|x\rangle\langle x|-|y\rangle\langle y|$. The polarization state of photons in a plane wave is selected by filter 1, and the photons are then deflected by the birefringent prism P with axes oriented in the x and y directions. The filter 2 postselects the polarization state before the photons pass through a narrow slit in the screen S to be observed at D.

surement by adding filters to preselect and postselect states $|\psi_1\rangle$ and $|\psi_2\rangle$, and by providing a small aperture in order to give the required poor resolution. After passing through the first filter, a plane wave is normally incident on the front face of the prism and is deflected at the second face, the normal to which makes an angle χ with the incident beam direction. The angle of deflection depends on the index of refraction of the material, and is different for the two orthogonal states of polarization. The two beams with slightly different directions then pass through a second filter and through a slit of width a. In a conventional Stern–Gerlach type experiment, where the second filter is not present, and where a is large enough to resolve the two peaks, the angular positions of the centers of the two patterns are correlated with the polarization state of the photon, and thus give a measure of the quantity Q. Taking $\theta=0$ at the midpoint between the two peaks, the relation is

$$\theta=cQ , \qquad (3)$$

where

$$c=\tfrac{1}{2}[\sin^{-1}(n_1\sin\chi)-\sin^{-1}(n_2\sin\chi)] . \qquad (4)$$

Here n_1 and n_2 are the indices of refraction for the two polarization states. If a is now made narrow, the peaks become broad and overlap, but their centers remain in the same location. The intensity on the screen with the second filter absent is the sum of the intensities of the peaks. If the second filter is present, however, the intensity is determined by a superposition of the two amplitudes. This pattern can take the form of a broad peak centered at the weak value

[#1] A somewhat different optical realization of weak measurements was suggested in ref. [3].

of Q, far outside the interval $(-1, +1)$ in which the expectation value of Q lies.

Fig. 3 illustrates the choice of states for preselection and postselection. The states are represented as points on the surface of the Poincaré sphere. In fig. 3a, where Q_c is measured, states a and b are elliptically polarized states obtained by superposing L with x and iy, respectively (cf. fig. 1). If the operator Q_1 is measured (fig. 3b), then the states a and b are states of linear polarization obtained by superposing x and $x \pm y$. Measurement of linear polarization appears more attractive because of the simplicity of linear polarization filters. We therefore confine our considerations to this case for the rest of this paper.

Let the crystal be cut in such a way that the x-axis lies in the plane of fig. 2, and the y-axis is parallel to both faces of the prism. The two linear polarization states selected by the filters (fig. 3b) lie in the xy plane, perpendicular to the direction of propagation of the incident beam, and make an angle $\frac{1}{4}\pi-\phi$ with the x-axis:

$$|\psi_1\rangle = \cos(\tfrac{1}{4}\pi-\phi)|x\rangle + \sin(\tfrac{1}{4}\pi-\phi)|y\rangle\,,$$
$$|\psi_2\rangle = \cos(\tfrac{1}{4}\pi-\phi)|x\rangle - \sin(\tfrac{1}{4}\pi-\phi)|y\rangle\,. \qquad (5)$$

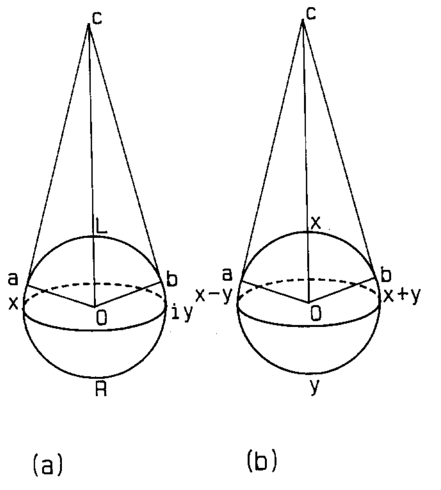

Fig. 3. Polarization states involved in weak measurement of (a) $Q_c = |L\rangle\langle L| - |R\rangle\langle R|$, and (b) $Q_1 = |x\rangle\langle x| - |y\rangle\langle y|$, represented on the surface of the Poincaré sphere. States at opposite ends of a diameter are orthogonal. Values of the polarization variables in the directions Oa and Ob are fixed to be +1 by preselection and postselection. Oc represents the weak value of the variable measured.

Then the weak value of the polarization operator is

$$(Q_1)_w = \frac{\langle\psi_2|(|x\rangle\langle x| - |y\rangle\langle y|)|\psi_1\rangle}{\langle\psi_2|\psi_1\rangle} = \frac{1}{\sin 2\phi}\,. \qquad (6)$$

The amplitude at a distance L from the slit arising from a single coherent plane wave passing through a conducting slit of width a is [8]

$$u(\phi) = \frac{e^{i(kL+\pi/4)}}{\sqrt{2\pi kL}} \times \left(\frac{\sin(\frac{1}{2}ka\sin\theta)}{\sin(\theta/2)} \pm i\frac{\cos(\frac{1}{2}ka\sin\theta)}{\cos(\theta/2)}\right), \qquad (7)$$

where $\pm$ corresponds to polarizations parallel and perpendicular to the slit. This amplitude yields the following intensity distribution:

$$dI = \frac{ka}{2\pi}\left[\left(\frac{2\sin(\frac{1}{2}ka\sin\theta)}{ka\sin\theta}\right)^2\cos\theta + [ka\cos(\theta/2)]^{-2}\right]d\theta\,. \qquad (8)$$

Each polarization separately yields the pattern (8) shifted through an angle $\theta_i = \pm c$ (eq. (3)). In our arrangement, the amplitude will be a superposition of two amplitudes of the form (7), shifted by $\pm c$:

$$\Psi(\theta) = \alpha_1 u(\theta - c) + \alpha_2 u(\theta + c)\,, \qquad (9)$$

in which α_1 and α_2 are given by

$$\alpha_1 = \langle\psi_2|x\rangle\langle x|\psi_1\rangle = \cos^2(\tfrac{1}{4}\pi-\phi)\,,$$
$$\alpha_2 = \langle\psi_2|y\rangle\langle y|\psi_1\rangle = -\sin^2(\tfrac{1}{4}\pi-\phi)\,. \qquad (10)$$

The second term in the amplitude (7) gives rise to a small θ-dependent elliptical polarization, produced by diffraction at the slit. Placement of the second filter before the slit, as in fig. 2, minimizes the effect of this term by causing the postselection to occur when the polarization is purely linear. Otherwise, if the quantity $1/ka$ were appreciable, this term could mask the effect we are seeking. With the choice of parameters described below, with $ka \approx 100$, this term is negligible in the relevant region of the diffraction pattern, and the slit could be placed before the final filter.

We performed numerical calculations of the intensity distribution at the detector for a calcite prism of angle $\chi=0.01$ rad. Light of wavelength 632.8 nm was assumed, and the slit width was taken to be 0.01 mm. The angle ϕ defining the orientation of the two filters had a value of 0.05 radians. If the light intensity incident on the slit is $1.0\ \mathrm{kW/m^2}$, the maximum intensity 1 m from the slit will be $1.5\times10^{-4}\ \mathrm{kW/m^2}$. This corresponds to about 10^3 photons per second passing through a small observation slit of width 0.1 mm and height 2.0 mm.

The results are shown as the solid curve in fig. 4. The dashed curves in this figure represent the patterns produced when pure x and pure y polarizations are incident on the slit. These take the form of broad peaks centered at $\theta=\pm c$. The pattern arising from the superposition is, indeed, of nearly the same form as the pure polarization patterns. However, it is shifted from the center by an amount which is larger by a factor $1/\sin 2\phi\approx 10$, as expected from eq. (1). The calculated position of the peak agrees with the prediction of eqs. (1) and (3) to within 3%.

Care must be taken in setting up the experiment to prevent transmission and propagation effects from altering the theoretical amplitudes given by eq. (10). Propagation through the birefringent crystal produces a phase difference between x and y polarizations, and a corresponding relative phase between the amplitudes α_1 and α_2 in eq. (9). The resulting state of elliptical polarization will not produce the desired shifted diffraction pattern. This phase difference can be eliminated by careful lateral placement of the slit so that rays corresponding to the two different polarization states traverse the same optical path length. The correct position can be found by moving the slit so as to obtain a local minimum in the overall intensity of the pattern.

The absolute values of α_1 and α_2 will also be affected by the different transmission coefficients at the faces of the prism, but a rotation of the initial polarization filter can compensate for this effect. If f_x and f_y denote the products of the transmission amplitudes at the two faces, then the appropriate angle of rotation γ is determined by the equation

$$\tan(\tfrac{1}{4}\pi-\phi-\gamma)=(f_x/f_y)\tan(\tfrac{1}{4}\pi-\phi)\ . \qquad (11)$$

This rotation leaves the relation (10) unchanged except for a small modification of the total amplitude.

We have presented a simple experiment which demonstrates that the weak value of a quantum mechanical variable can greatly exceed its largest eigenvalue. Although the motivation for performing this experiment stems from the quantum mechanics of photon polarization, the collective action of the photons is to produce a new classical interference effect, the shift of the pattern by passage through a filter. This effect might have practical applications. For example, the deflection of the beam when it passes through both filters is proportional to c, eq. (4), which is proportional to n_1-n_2 when that quantity is small. The shift is large compared to the shift obtained by varying the polarization of the beam passing through the prism without the filters. The deflection could therefore act as a sensitive measure of small differences in the indices of refraction.

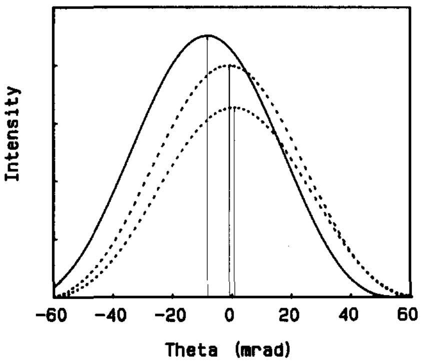

Fig. 4. The solid curve shows the intensity distribution, in arbitrary units, predicted in weak measurement of the photon polarization variable $Q_1=|x\rangle\langle x|-|y\rangle\langle y|$. The dashed curves show the intensities, on a different scale, when the postselection filter is set to pass $|x\rangle$ and $|y\rangle$, giving $Q_1=+1$ and $Q_1=-1$, respectively. The scale of the dashed curves is approximately 50 times greater than that of the solid curve. The peak of the solid curve is clearly shifted outside the interval $(-1,+1)$. The curves were calculated for values of the parameters appropriate to calcite: $n_1=1.658$, $n_2=1.486$. Other parameters (see text for definition) were taken to be: $\chi=0.01$, $\phi=0.05$, and $a=0.01$ mm. The dashed curves are split by 1.72 mrad, and the solid curve is shifted from the center of the pattern by 8.29 mrad. This corresponds to a weak value $(Q_1)_w=9.7$.

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