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Weak Value and Weak Measurements

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The weak value of a variable O is a description of an effective interaction with that variable in the limit of weak coupling. For a pre- and post-selected system described at time t by the two-state vector $\langle\Phi| \ |\Psi\rangle$ [1], the weak value is [2]:

$$O_{\rm w} \equiv \frac{\langle\Phi|O|\Psi\rangle}{\langle\Phi|\Psi\rangle}. \tag{1}$$

Contrary to classical physics, variables in quantum mechanics might not have definite values at a given time. In the complete description of a usual (pre-selected) quantum system, the state $|\Psi\rangle$ yields probabilities p_i for various outcomes o_i of (an ideal) measurement of the variable O. Numerous measurements on an ► ensemble of identical systems yield an average – expectation value of O: $\sum p_i o_i$. Since $p_i = |\langle O = o_i \ |\Psi\rangle|^2$, the expectation value can be expressed as $\langle\Psi|O|\Psi\rangle$. If the coupling to the measuring device is very small, this expression is related directly to the response of the measuring device, and the measurement does not reveal the eigenvalues o_i and their probabilities p_i. Specifically, $\langle\Psi|O|\Psi\rangle$ is the shift of the quantum state of the pointer variable of the measuring device, which, otherwise, is not distorted significantly due to the measurement interaction.

For pre- and post-selected quantum system, the response of the measuring device or any other system coupled weakly to the variable O, is the shift of the quantum state by the weak value (1). The coupling can be modeled by the von Neumann measurement interaction

$$H = g(t) P O, \tag{2}$$

where $g(t)$ defines the time of the interaction, $\int g(t) = 1$, and P is conjugate to the pointer variable Q. The weakness of the interaction is achieved by choosing the ► wave function of the measuring device so that P is small. Small value of P requires also a small uncertainty in P, and thus a large uncertainty of the pointer variable Q in the initial state and consequently, a large uncertainty in the measurement. Therefore, usually, we need a large ensemble of identical pre- and post-selected quantum systems in order to measure the weak value.

For rare post-selection, when $|\langle\Phi|\Psi\rangle| \ll 1$, the weak value (1) might be far away from the range of the eigenvalues of O, so it clearly has no statistical meaning as an "average" of o_i. If we model the initial state of the pointer by a Gaussian $\Psi_{\rm in}^{\rm MD}(Q) = (\varDelta^2\pi)^{-1/4} e^{-Q^2/2\varDelta^2}$ with large $\varDelta$ ensuring small P, the final state, to a good approximation, is the shifted Gaussian $\Psi_{\rm fin}^{\rm MD}(Q) = (\varDelta^2\pi)^{-1/4} e^{-(Q-O_{\rm w})^2/2\varDelta^2}$. The standard measurement procedure with weak coupling reveals only the real part of the weak value, which is, in general, a complex number. Its imaginary part can be measured by observing the shift in P, the conjugate to the pointer variable [3,4].

The real part of the weak value is the outcome of the standard measurement procedure at the limit of weak coupling. Unusually large outcomes, such as ► spin 100 for a spin$-\frac{1}{2}$ particle [2], appear from peculiar interference effect (sometimes called Aharonov–Albert–Vaidman (AAV) effect) according to which, the superposition of the pointer wave functions shifted by small amounts yields similar wave function shifted by a large amount. The coefficients of the superposition are universal for a large class of functions for which the Fourier transforms is well localized around zero.

In the usual cases, the shift is much smaller than the spread $\varDelta$ of the initial state of the measurement pointer. But for some variables, e.g., averages of variables of a large ensemble, for very rare event in which all members of the ensemble happened to be in the appropriate post-selected states, the shift is of the order, and might be even larger than the spread of the quantum state of the pointer [5]. In such cases the weak value is obtained in a single measurement which is not really "weak".

One can get an intuitive understanding of the AAV effect, noting that the coupling of the weak measurement procedure does not change significantly the forward and the backward evolving quantum states. Thus, during the interaction, the measuring device "feels" both forward and backward evolving quantum states. The tolerance of the weak measurement procedure to the distortion due to the measurement depends on the value of the scalar product $\langle\Phi|\Psi\rangle$.

Since the quantum states remain effectively unchanged during the measurement, several weak measurements can be performed one after another and even simultaneously. "Weak-measurement elements of reality" [6], i.e., the weak values, provide self consistent but sometimes very unusual picture for pre- and post-selected quantum systems. Consider a three-box paradox in which a single particle in three boxes is described by the two-state vector

$$\frac{1}{3}\left(\langle A| + \langle B| - \langle C|\right) \quad \left(|A\rangle + |B\rangle + |C\rangle\right), \tag{3}$$

where $|A\rangle$ is a quantum state of the particle located in box A, etc. Then, there are the following weak-measurements elements of reality regarding projections on various boxes: $(\mathbf{P}_A)_{\rm w} = 1$, $(\mathbf{P}_B)_{\rm w} = 1$, $(\mathbf{P}_C)_{\rm w} = -1$. Any weak coupling to the particle in box A behaves as if there is a particle there and the same is true for box B. Finally, a weak measuring device coupled to the particle in box C is shifted by the same value, but in the opposite direction. The coupling to the projection onto all three boxes, $\mathbf{P}_{A,B,C} = \mathbf{P}_A + \mathbf{P}_B + \mathbf{P}_C$ "feels" one particle: $(\mathbf{P}_A + \mathbf{P}_B + \mathbf{P}_C)_{\rm w} = (\mathbf{P}_A)_{\rm w} + (\mathbf{P}_B)_{\rm w} + (\mathbf{P}_C)_{\rm w} = 1$.

There have been numerous experiments showing weak values [7–11], mostly of photon polarization and the AAV effect has been well confirmed. Unusual weak values were used for explanation peculiar quantum phenomena, e.g., superluminal velocity of tunneling particles [12,13]. (▶ Superluminal communication; tunneling).

When the AAV effect was discovered, it was suggested that the type of an amplification effect which takes place for unusually large weak values might lead to practical applications. Twenty years later, the first useful application has been made: Hosten and Kwiat [14] applied weak measurement procedure for measuring spin Hall effect in light. This effect is so tiny that it cannot be observed without the amplification.

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