The Smoothed Analysis of Algorithms

Shang-Hua Teng
Boston University/Akamai Technologies

Theorists have long been challenged by the existence of remarkable algorithms that are known by scientists and engineers to work well in practice, but whose theoretical analyses have been are negative or unconvincing. The root of the problem is that algorithms are usually analyzed in one of two ways: by worst-case or average-case analysis. The former can improperly suggest that an algorithm will perform poorly, while the latter can be unconvincing because the random inputs it considers may fail to resemble those encountered in practice.

We introduce smoothed analysis to help explain the success of some of these algorithms and heuristics. Smoothed analysis is a hybrid of worst-case and average-case analyses that inherits advantages of both. The smoothed complexity of an algorithm is the maximum over its inputs of the expected running time of the algorithm under slight random perturbations of that input, measured as a function of both the input length and the magnitude of the perturbations. If an algorithm has low smoothed complexity, then it should perform well on most inputs in every neighborhood of inputs.

In this talk, we will explain how smoothed analysis can help explain the excellent observed behavior of the simplex method, Gaussian elimination, and interior point methods. In particular, we show that the simplex algorithm has polynomial smoothed complexity.

This is joint work with Dan Spielman (MIT), John Dunagan (Microsoft Research), Arvind Sankar (MIT)