COMBINATORIAL ASPECTS OF DUAL-BASED HEXAHEDRAL MESH MODIFICATION

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Hexahedral meshes are used widely in finite element and finite volume analysis. While many analysts simply demand hexahedral meshes, there have also been studies which show in some cases that hexahedra actually perform better on a theoretical basis (see e.g. [1]). Fully automatic generation of hexahedral meshes suitable for FEA remains an unsolved problem.

Automatic meshing algorithms exist for generating quadrilateral and tetrahedral meshes. In most cases, these algorithms work by first generating a mesh of insufficient quality, then improving the mesh using local topology modifications which improve the geometric mesh quality. Local topology modification operations have been described extensively in the literature for both tetrahedral and quadrilateral meshes. The research described in this paper is focused toward the development of similar techniques for hexahedral meshes.

Previous research in the area of local hexahedral mesh modification is sparse. Knupp and Mitchell[2] explored an ad-hoc set of operations with the goal of converting a hexahedral mesh from subdivided tetrahedra to one of better quality. Bern et. al describe a set of flipping operations based on lifting a hexahedron to 4 dimensions[3]. Both of these efforts describe some of the same operations, for example the 1-7 and 2-6 transitions (see Figure 1). Bern et. al's work has the added benefit of a completeness proof, based on all possible combinations of 3d hexahedral meshes which can be resolved within a 4d cuboid. However, in both works, there is little unification amongst the various operations; that is, each operation is considered a separate one distinct from the others. Applying these to a mesh is assumed to involve searching for a configuration which matches the start or end of one of the stated transitions.

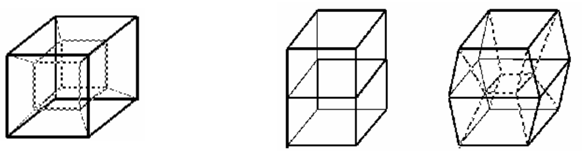

Figure 1: 1-7 transition (left); 2-6 transition (right) [3].

The research described here formulates the hexahedral mesh modification problem as an operation on the dual of a hexahedral mesh. It has been shown that the hexahedral dual is a simple arrangement of pseudo-surfaces[4]; formulating the problem in the dual therefore allows us to take advantage of the Dehn-Sommerville relations, which apply to arrangements of this type. We describe modifications to a hexahedral mesh in terms of modifications to the dual arrangement. We seek to define a set of operations which are "atomic", that is, cannot be reproduced using a combination of other atomic operations, and "complete", that is, can transition between any two meshes with the same topological (all-quadrilateral) boundary.

The proposed set consists of the two operations shown in Figure 2. The first operation inserts a new dual surface such that it forms two new vertices in the arrangement; we refer to this operation as an "atomic pillow", since it is related to the more traditional pillow operation described by Mitchell. We show that the atomic pillow is the smallest operation which inserts a new dual surface but which also begins and ends with a conformal hexahedral mesh.

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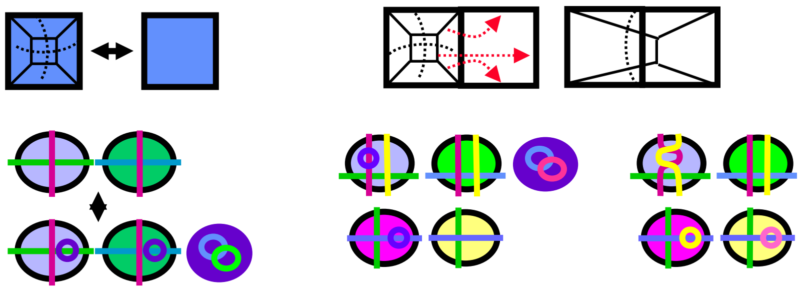

Figure 2: Atomic pillow; in the primal, a quadrilateral is "pulled apart" into two quads which share all their edges, with two new hexes between them (top left); in the dual, a new dual surface is formed (bottom left). Face open-collapse; in the primal, an edge and two quadrilaterals are split, then quads are folded . Operations shown in primal (top) and dual (bottom). See [4] for description of dual representation.

The second operation is defined as the splitting and merging of neighboring dual surfaces, which modifies the cell complex locally without changing the boundary of the arrangement. We refer to this as a "face open-collapse" operation, in reference to how this operation modifies the primal (the mesh).

We show how the atomic pillow and face open-collapse operations can be used in sequence to reproduce Bern et. al's operations. As such, we propose that this forms a smaller, more atomic set than the set described in that work. Furthermore, describing our operations as more fundamental modifications to the dual arrangement allows them to be applied to a wider context of initial conditions. Finally, describing these operations in terms of the dual surfaces gives more of a non-local flavor to the mesh modification problem, without making it fully global. We suggest that this may lend the technique towards mesh improvement techniques based on optimization, such as those described by Knupp et. al. Fitting our hexahedral mesh modification techniques into such a framework could provide a means for guiding these modifications towards a specific end, be it general mesh improvement or, more interesting, local adaptive refinement of hexahedral meshes.

We will conclude this talk with a discussion of the various interesting combinatorial aspects of this problem which are the basis of continuing research. Foremost among these is the question of whether combinatorial results on simple arrangements can be used to prove that the two operations are both necessary and complete for transitioning between two hexahedral meshes with the same quadrilateral boundary.

References

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