

# Corporate Finance: Credit rationing

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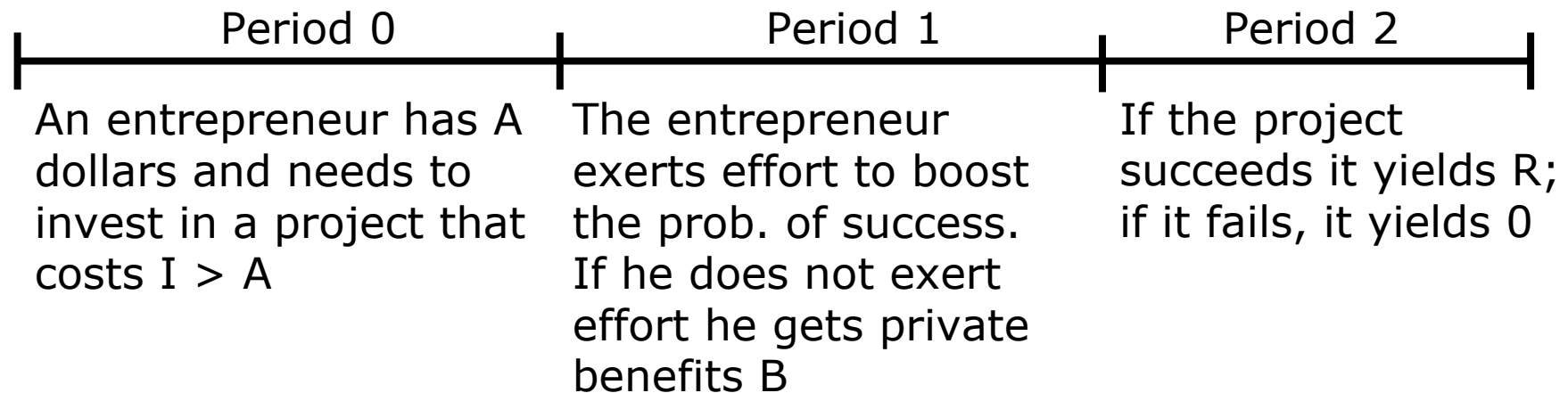
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The Theory of Corporate Finance

# The model

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- The timing:



- Effort raises the prob. of success from  $p_L$  to  $p_H$
- $\Delta p \equiv p_H - p_L$
- The project is viable only if there's effort:

$$\underbrace{p_H R - I}_{\text{NPV}} > 0 > \underbrace{p_L R - I + B}_{\text{NPV} + \text{Benefits}} \Rightarrow R > \frac{B}{\Delta p}$$

# The loan agreement

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- The loan can be debt or equity (the model cannot distinguish between them)
- Incentive compatibility (to ensure effort):

$$\underbrace{p_H R_b}_{\text{Entrepreneur's expected payoff with effort}} > \underbrace{p_L R_b + B}_{\text{Entrepreneur's expected payoff without effort}} \Rightarrow R_b > \frac{B}{\underbrace{p_H - p_L}_{\text{Cost of MH}}} \equiv \frac{B}{\Delta p}$$

- Creditor's individual rationality:

$$p_H \underbrace{(R - R_b)}_{\text{Maximal pledgeable income}} \equiv p_H \underbrace{\left( R - \frac{B}{\Delta p} \right)}_{(+)} \geq \underbrace{I - A}_{\text{Required funds}}$$

# Credit rationing

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- Creditor's individual rationality:

$$p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A \Rightarrow A \geq \bar{A} \equiv \underbrace{p_H \frac{B}{\Delta p} - \overbrace{(p_H R - I)}^{\text{NPV with effort}}}_{(+)\text{ by assumption}}$$

- An entrepreneur must have  $\bar{A}$  to get funds
- When  $A < \bar{A}$ , we get credit rationing: the creditor gets too little ex post to agree to give the entrepreneur  $I - A$
- Credit rationing is "more severe" when  $B$  is large: there's more agency problem or MH

# Entrepreneur's payoff

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- When  $A < \bar{A}$ , the project is not funded so  $U = 0$
- When  $A \geq \bar{A}$ , the project is funded; if the entrepreneur has all the bargaining power, the creditor simply breaks even:

$$\underbrace{p_H R_l}_{\text{Creditor's expected payoff}} = I - A \quad \Rightarrow \quad R_l = \frac{I - A}{\underbrace{p_H}_{\text{Min payment to creditor given effort}}}$$

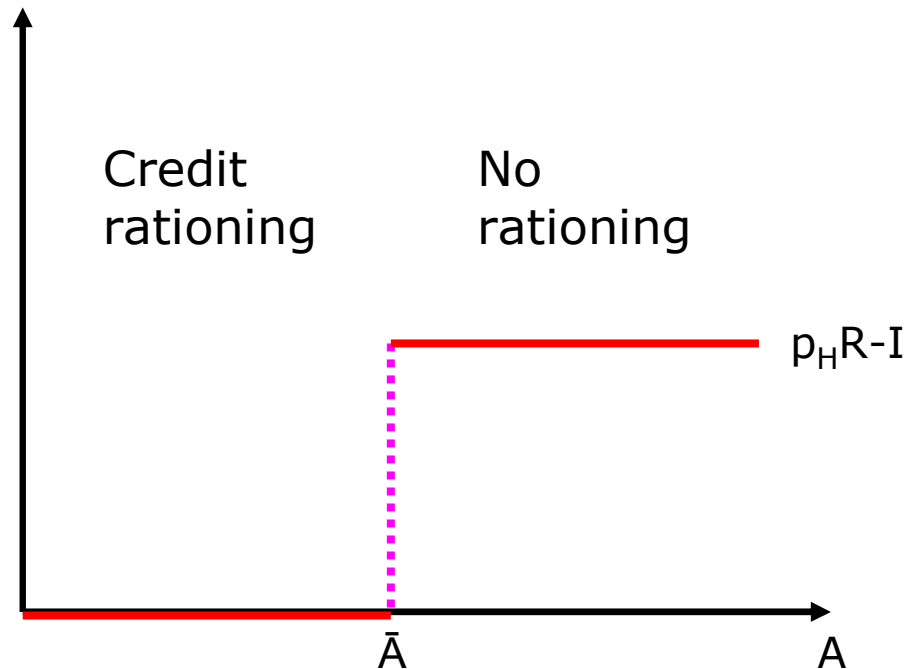
- The entrepreneur's net payoff (above and beyond  $A$  which he can consume anyway by not investing):

$$U = p_H (R - R_l) - A = p_H \left( R - \frac{I - A}{p_H} \right) - A = \underbrace{p_H R - I}_{\text{NPV with effort}}$$

- Since the creditor breaks even, the entrepreneur captures the entire NPV

# The entrepreneur's net payoff (above and beyond $A$ ) - illustration

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- The entrepreneur either gets all the NPV or nothing  $\Rightarrow$  the entrepreneur is indifferent to  $A$  above  $\bar{A}$

# Overborrowing

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- Suppose the firm can  $\uparrow$  the prob. of success by  $\tau$  by investing  $J$  which it borrows from a new creditor
- Assumption: the investment is inefficient:  $J > \tau R$
- $\Rightarrow$  No point in investing if the effort stays the same (the investment  $\downarrow$  NPV and hence  $\downarrow$  the entrepreneur's payoff); the investment's role is to transfer value from the original creditor
- The entrepreneur invests  $J$  only if it induces him to exert no effort (the alternative is to forgo  $J$  and exert effort):

$$\underbrace{(p_L + \tau)R_b - J + B}_{\text{The entrepreneur's payoff w/o effort when the new creditor breaks even}} > \underbrace{p_H R_b}_{\text{No overinvestment and effort}} \Rightarrow \underbrace{(p_L + \tau)(R - R_l) - J + B}_{\text{The entrepreneur's payoff w/o effort when the new creditor breaks even}} > p_H (R - R_l)$$

# Overborrowing

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- The condition for overborrowing:

$$(P_L + \tau)(R - R_l) - J + B > p_H(R - R_l) \Rightarrow \underbrace{(p_H - (p_L + \tau))R_l}_{\substack{\text{Benefit} \\ \downarrow \text{in the expected payoff} \\ \text{of the initial creditor}}} > \underbrace{\Delta p R - B}_{\substack{\text{Cost} \\ \text{Net loss due to} \\ \text{lack of effort}}} + \underbrace{J - \tau R}_{\substack{\text{Net cost} \\ \text{of new} \\ \text{investment}}}$$

- Overborrowing is worthwhile only if it transfers enough value from the initial creditor to compensate for the resulting inefficiencies
- If the condition holds, the initial creditor must impose a no-extra investment/loan covenant
- $R_l \uparrow \Rightarrow$  overborrowing is more tempting
- But  $R_l = (I - A)/p_H$ ; hence,  $A \downarrow \Rightarrow R_l \uparrow \Rightarrow$  overborrowing is more likely when  $A$  is low and hence covenants are needed more

# Debt overhang

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- Suppose the firm has initial secured debt with face value  $D \leq A$
- The creditor's IR constraint:

$$\underbrace{p_H \left( R - \frac{B}{\Delta p} \right) - D}_{\text{Net pledgeable income}} \geq \underbrace{I - A}_{\text{Size of loan}} \Rightarrow A \geq \underbrace{p_H \frac{B}{\Delta p} - \overbrace{(p_H R - I)}^{\text{NPV with effort}} + D}_A$$

- $D$  raises the threshold for financing: if  $A > \bar{A}$  but  $A < \bar{A} - D$ , the project will not be financed with  $D$  but will be financed otherwise

# Debt restructuring

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- Suppose that  $D > 0$ ,  $A = 0$ , and  $\bar{A} < 0$  ( $R$  is large enough so the entrepreneur can get a loan even though he has no funds)
- Since  $A = 0$ ,  $D$  can be paid only if the project is financed and succeeds
- Absent restructuring, the creditor obtains:

$$\underbrace{p_H \left( R - \frac{B}{\Delta p} - D \right)}_{\text{Net pledgeable income}} - I \equiv \underbrace{-\bar{A}}_{(+)} - p_H D$$

- Suppose that  $-\bar{A} - p_H D < 0 \Rightarrow D$  must be restructured to ensure financing!

# Debt restructuring

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- Suppose that  $D$  is lowered to  $d$
- The creditor obtains:

$$\underbrace{p_H \left( R - \frac{B}{\Delta p} - d \right)}_{\text{Net pledgeable income}} - I \equiv \underbrace{-\bar{A}}_{(+)} - p_H d = p_H \left( -\frac{\bar{A}}{p_H} - d \right)$$

- $d$  must be set low enough to ensure that  $-\bar{A}/p_H - d \geq 0 \Rightarrow$  If the entrepreneur can make TIOLI offer, then  $d = -\bar{A}/p_H$

# Multiple projects

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- 2 identical projects
- Suppose that the entrepreneur gets  $R_2$  if both projects succeed and gets 0 otherwise (can also pay  $R_1$  if one project succeeds and  $R_0$  if none succeeds but  $R_2$  is sufficient since the entrepreneur is risk neutral)
- Incentive compatibility:

$$\underbrace{p_H^2 R_2}_{\text{Entrepreneur's payoff with effort on both projects}} > \underbrace{p_L^2 R_2 + 2B}_{\text{Entrepreneur's payoff without effort}} \Rightarrow \left( \frac{p_H + p_L}{2} \right) \underbrace{(p_H - p_L)}_{\Delta p} R_2 > B$$

$$\underbrace{p_H^2 R_2}_{\text{Entrepreneur's payoff with effort on both projects}} > \underbrace{p_H p_L R_2 + B}_{\text{Entrepreneur's payoff with effort on a single project}} \Rightarrow p_H \Delta p R_2 > B$$

- The first IC constraint implies the second

# The creditor's IR

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- Creditor's individual rationality (IR):

$$\underbrace{p_H^2 2R + 2p_H(1-p_H)R}_{\text{Expected return}} - \underbrace{p_H^2 R_2}_{\text{Entrepreneur's payoff}} = 2p_H R - p_H^2 R_2 \geq 2(I - A)$$

- From entrepreneur's IC:

$$R_2 \geq \frac{1}{p_H + p_L} \frac{2B}{\Delta p}$$

- Substituting from IC into creditor's IR:

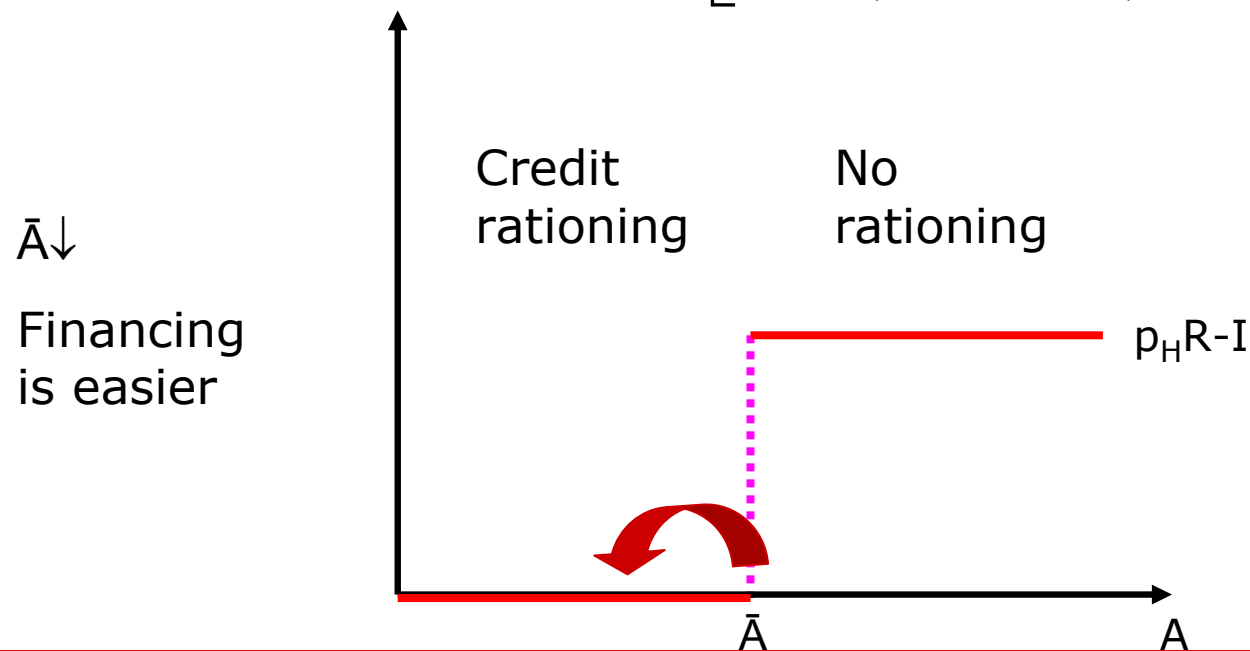
$$2p_H R - p_H^2 \frac{2B}{(p_H + p_L)\Delta p} \geq 2(I - A) \Rightarrow p_H \left[ R - \left( \frac{p_H}{p_H + p_L} \right) \frac{B}{\Delta p} \right] \geq I - A$$

# The effect of multiple projects on financing

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□ The condition for financing:

$$A \geq \bar{A} \equiv I - p_H \left[ R - \left( \frac{p_H}{p_H + p_L} \right) \frac{B}{\Delta p} \right]$$



# Multiple projects with perfect correlation

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## □ Entrepreneur's IC:

$$\underbrace{p_H R_2}_{\text{Entrepreneur's payoff with effort on both projects}} > \underbrace{p_L R_2 + 2B}_{\text{Entrepreneur's payoff without effort}} \Rightarrow R_2 > \frac{2B}{\Delta p}$$

$$\underbrace{p_H R_2}_{\text{Entrepreneur's payoff with effort on both projects}} > \underbrace{p_H p_L R_2 + B}_{\text{Entrepreneur's payoff with effort on a single projects}} \Rightarrow R_2 > \frac{B}{p_H (1 - p_L)}$$

## □ The first IC constraint implies the second:

$$p_L R_2 + 2B > p_H p_L R_2 + B$$

# The creditor's IR under perfect correlation

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- Creditor's individual rationality (IR):

$$\underbrace{p_H 2R}_{\text{Expected return}} - \underbrace{p_H R_2}_{\text{Entrepreneur's payoff}} = p_H [2R - R_2] \geq 2(I - A)$$

- From entrepreneur's IC:

$$p_H \left[ 2R - \frac{2B}{\Delta p} \right] \geq 2(I - A) \Rightarrow A \geq \bar{A} \equiv p_H \frac{B}{\Delta p} - (p_H R - I)$$

- Under perfect corr. we are back to the single project case
- Diversification helps because the projects are not perfectly correlated
- Imperfect correlation effectively lowers B to  $p_H B / (p_L + p_H)$

# Correlation or independence?

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- Suppose the entrepreneur can choose whether projects will be correlated or independent but his choice is hidden from the creditor
  
- Given  $R_2$ , the entrepreneur's payoff:
  - Correlation:  $p_H R_2$
  - Independence:  $p_H^2 R_2$
  
- ⇒ The entrepreneur will choose perfect correlation. Why is that?
  
- Asset substitution: correlation is riskier than independence. The entrepreneur is the residual claimant and likes risk

# Continuous investment

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- $I \in [0, \infty)$  is a choice variable; the entrepreneur chooses  $I$  and whether to exert effort
- Return is  $RI$  and private benefit is  $BI$
- IC for the entrepreneur:

$$p_H R_b > p_L R_b + BI \Rightarrow R_b > \frac{BI}{\Delta p}$$

- IR for the creditor:

$$p_H (RI - R_b) \geq I - A \Rightarrow p_H \left( RI - \frac{BI}{\Delta p} \right) \geq I - A$$

- Rewriting:

$$I \leq \kappa A \Rightarrow \kappa \equiv \frac{1}{\underbrace{1 - p_H R + \frac{p_H B}{\Delta p}}_{\text{multiplier}}}$$

# Continuous investment – optimal investment

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- In a competitive capital market, the lenders must break even given their anticipation that the entrepreneur will exert effort:  $p_H R_l = I - A$

- The entrepreneur's utility above and beyond A:

$$U = p_H (RI - R_l) - A = p_H \left( RI - \frac{I - A}{p_H} \right) - A = (p_H R - 1)I$$

- Assumption 1:  $p_H RI > I$  – investment has a positive NPV with effort
- Implication: the entrepreneur would like to invest as much as he can
- But if I is high, the IC constraint is violated
- Optimal investment is determined by the multiplier equation:  $I = \kappa A$
- “Invest up to  $\kappa$  times your wealth” or “Borrow  $\kappa - 1$  times your wealth”

# Continuous investment - multiplier

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- Assumption 1:  $p_H RI > I$  – investment has a positive NPV with effort
- Assumption 2:  $p_L RI + BI < I$  – investment has a negative NPV w/o effort
- Assumption 1 + 2 imply:  $p_H R > 1 > p_L R + B \Rightarrow \Delta p R > B \Rightarrow R > B/\Delta p$
- Assumption 3:  $p_H RI - I < p_H BI/\Delta p$  – NPV is lower than the cost of MH
- Since  $R > B/\Delta p$  and given Assumption 3,  $\kappa > 1$
- Implication:  $\kappa$  is a “multiplier” – each dollar of equity leads to  $\kappa$  dollars of investment
- $\kappa$  is smaller if  $B$  is large

# Continuous investment - leverage

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- The optimal investment is  $\kappa A$
- The entrepreneur needs to borrow  $(\kappa-1)A$ , where

$$\kappa - 1 = \frac{1}{1 - p_H R + \frac{p_H B}{\Delta p}} - 1 = \frac{p_H \left( R - \frac{B}{\Delta p} \right)}{1 - p_H \left( R - \frac{B}{\Delta p} \right)}$$