

Corporate Finance: Agency models of capital structure

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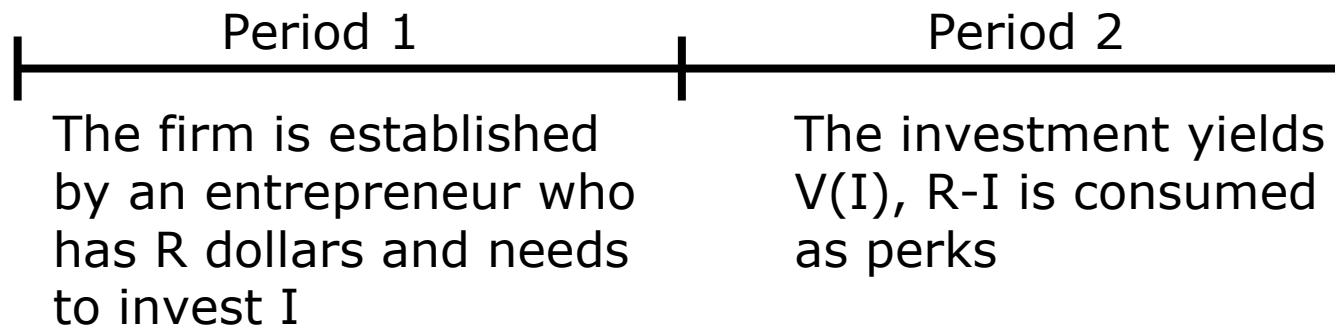
Recanati School of Business

Jensen and Meckling, JFE 1976

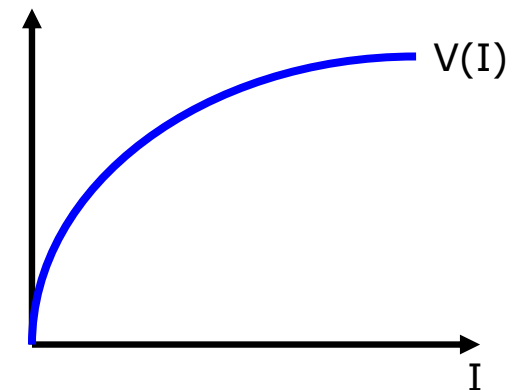
The agency cost of outside equity

The investment model

- The timing:



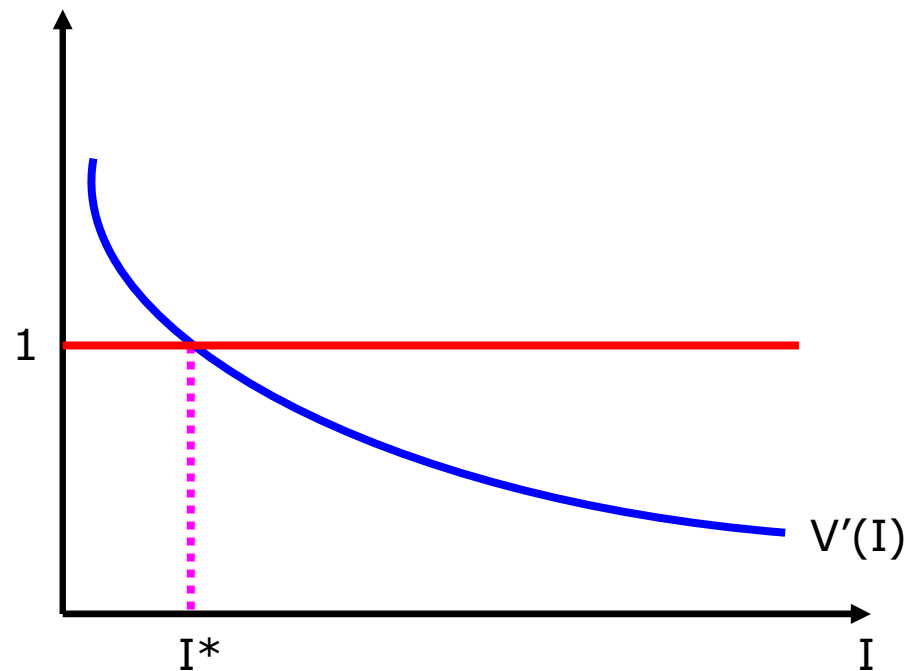
- $V'(I) > 0 > V''(I)$ and $V(0) = 0$
- $V'(0) = \infty$ and $V'(\infty) = 0$ (interior sol'n)
- The entrepreneur's payoff: $U(I) = V(I) + (R-I)$



Internal financing

□ F.O.C for the entrepreneur's problem:

$$U'(I) = V'(I) - 1 = 0 \Rightarrow V'(I^*) = 1.$$



Internal financing

- The entrepreneur's payoff at the optimum:

$$U(I^*) = V(I^*) - I^* + R$$

- But what if $I^* > R$? In that case the entrepreneur must have external financing

Debt financing

- The entrepreneur issues debt with face value D to raise $I^* - R$ upfront
- The entrepreneur's payoff:

$$U(I, D) = \underbrace{V(I) - D}_{\text{Payoff ex post}} + \underbrace{R + (I^* - R) - I}_{\substack{\text{Cash flow ex ante} \\ \text{Perks}}}$$

- $U(I, D)$ is maximized at I^* (the firm invests optimally)
- Debt is safe: ex post the firm has $V(I^*) > I^* > I^* - R \Rightarrow D^* = I^* - R$
- The entrepreneur's payoff:

$$\begin{aligned} U(I^*, D^*) &= V(I^*) - D^* + R + (I^* - R) - I^* \\ &= V(I^*) - I^* + R \end{aligned}$$

Equity financing with commitment

- The entrepreneur issues equity with equity participation $1-\alpha$ (the entrepreneur keeps α) and commits to invest I^*
- To raise I^*-R :

$$\underbrace{(1-\alpha)V(I^*)}_{E^*} = I^* - R \quad \Rightarrow \quad \alpha^* = 1 - \frac{I^* - R}{V(I^*)}.$$

- The entrepreneur's payoff:

$$\begin{aligned} U(I^*, \alpha^*) &= \underbrace{\alpha^* V(I^*)}_{\text{Ex post payoff}} + \underbrace{E^* + R - I^*}_{\text{Perks}} \\ &= V(I^*) - I^* + R \end{aligned}$$

Equity financing without commitment

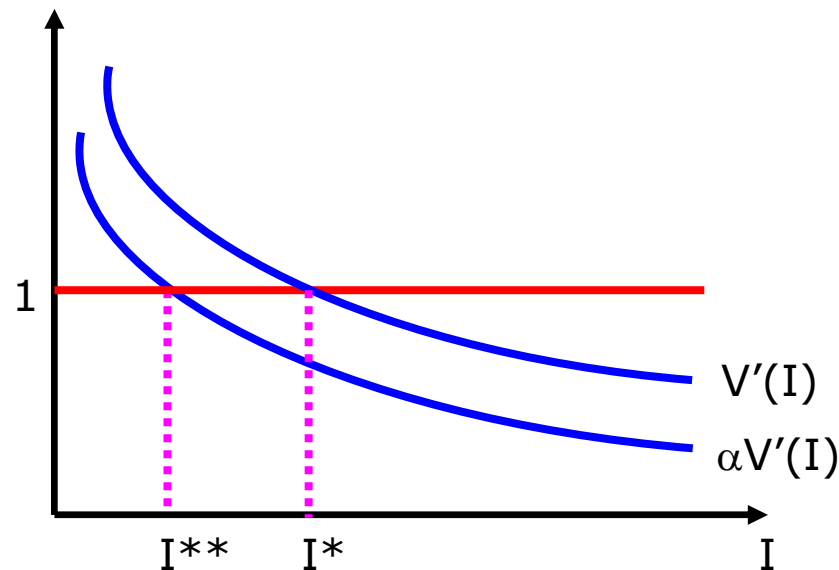
- The entrepreneur issues equity with equity participation $1-\alpha$, but cannot commit to invest I^*
- After receiving E , but before choosing I , the entrepreneur's payoff:

$$U(I, \alpha) = \underbrace{\alpha V(I)}_{\text{Ex post payoff}} + \underbrace{E + R - I}_{\text{Perks}}$$

Equity financing without commitment

- F.O.C for the entrepreneur's problem:

$$U'(I, \alpha) = \alpha V'(I) - 1 = 0 \Rightarrow \alpha V'(I^{**}) = 1.$$



- We get underinvestment
- $\alpha \downarrow$ (more outside equity) $\Rightarrow I^* \downarrow$ (more underinvestment)

The agency cost of outside equity

- The entrepreneur's payoff:

$$\begin{aligned}U(I^{**}, \alpha) &= \alpha V(I^{**}) + E^{**} - I^{**} + R \\ &= \alpha V(I^{**}) + (1 - \alpha)V(I^{**}) - I^{**} + R \\ &= V(I^{**}) - I^{**} + R\end{aligned}$$

- The entrepreneur's payoff with commitment:

$$U(I^*, \alpha) = V(I^*) - I^* + R$$

- By revealed preferences (and since $I^{**} < I^*$):

$$V(I^*) - I^* + R > V(I^{**}) - I^{**} + R$$

- The entrepreneur bears the cost of underinvestment (outside investors break even)

The optimal choice of α

- How does α affect $U(I^{**}, \alpha)$?

$$\frac{\partial U(I^{**}, \alpha)}{\partial \alpha} = \underbrace{\left(\overbrace{V'(I^{**})}^{1/\alpha} - 1 \right)}_{(+)} \underbrace{\frac{\partial I^{**}}{\partial \alpha}}_{(+)} > 0$$

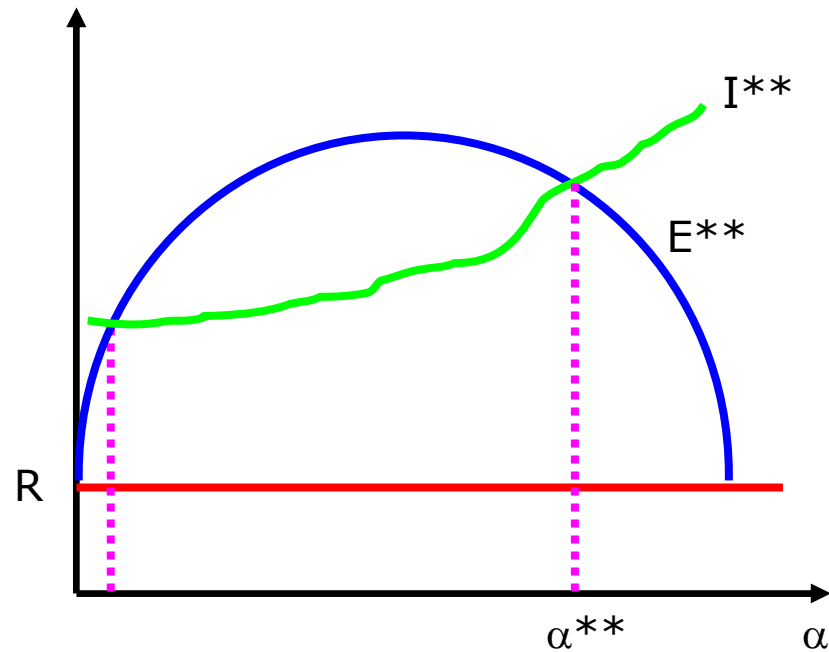
- The entrepreneur will raise α up to the point where

$$I^{**} = R + \underbrace{(1 - \alpha)V(I^{**})}_{E^{**}}$$

- $E^{**} = 0$ at $\alpha = 0$ (since then $I^{**} = 0$) and at $\alpha = 1 \Rightarrow E^{**}$ is inverse U-shaped

The optimal choice of α

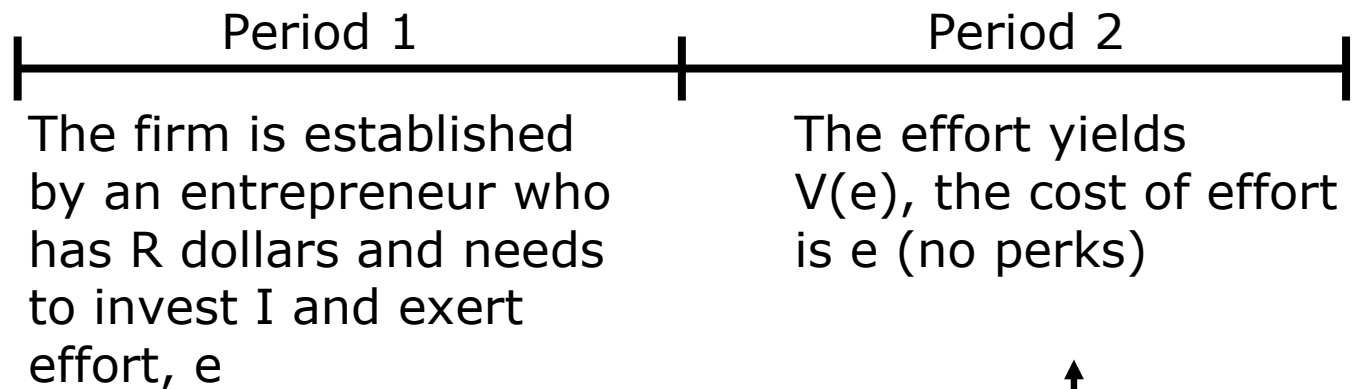
- The entrepreneur's budget constraint:



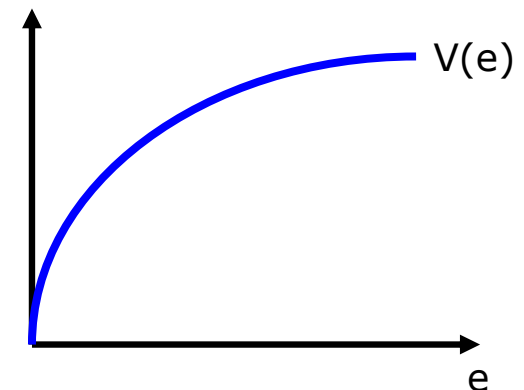
- The relevant sol'n is with the maximal α

The effort model

- The timing:



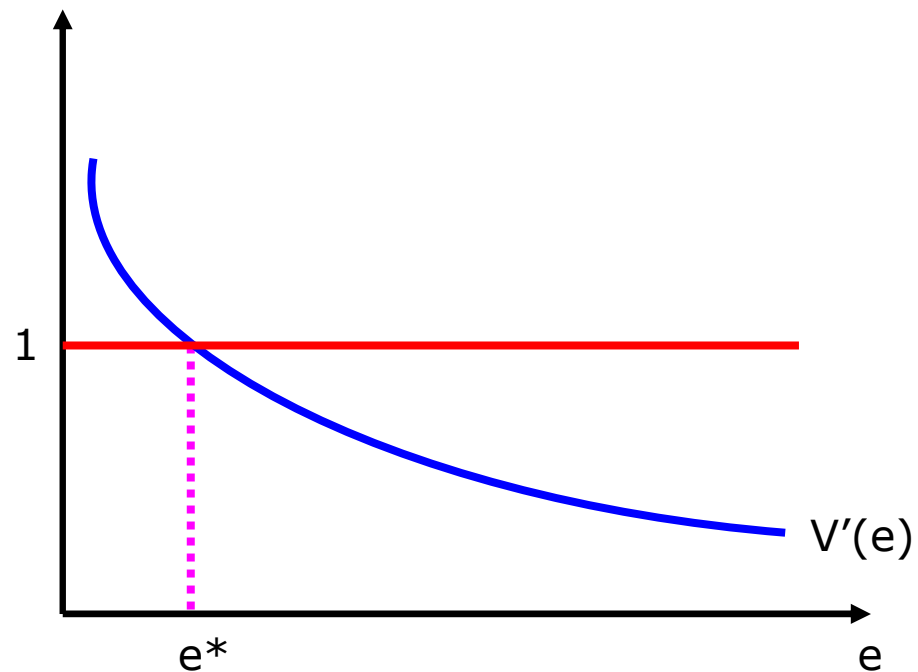
- $V'(e) > 0 > V''(e)$ and $V(0) = 0$
- $V'(0) = \infty$ and $V'(\infty) = 0$ (interior sol'n)
- The entrepreneur's payoff: $U(e) = V(e) - e$



Internal financing

□ F.O.C for the entrepreneur's problem:

$$U'(e) = V'(e) - 1 = 0 \Rightarrow V'(e^*) = 1.$$



Internal financing

- The entrepreneur's payoff at the optimum:

$$U(e^*) = V(e^*) - e^*$$

- But what if $I > R$?
- Debt financing: debt is safe so $D^* = I - R$

Equity financing without commitment

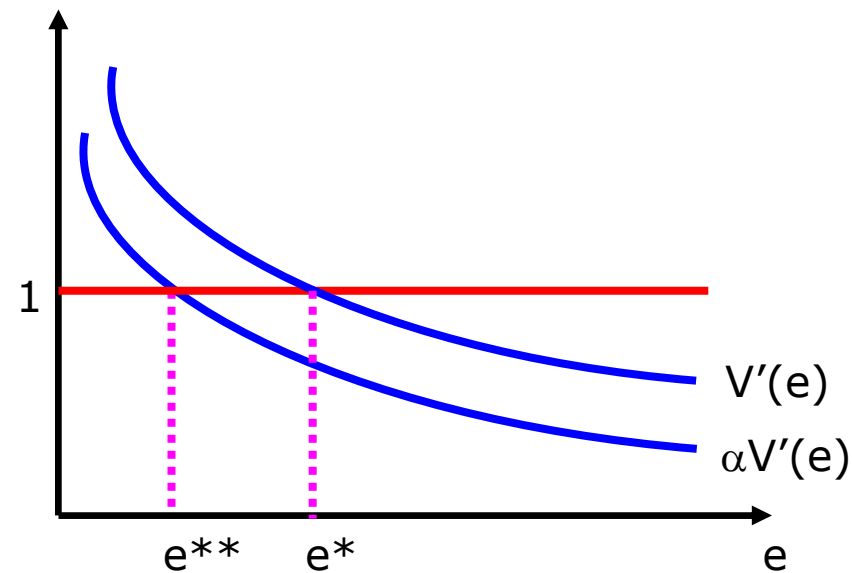
- The entrepreneur issues equity with equity participation $1-\alpha$
- After receiving E , but before choosing I , the entrepreneur's payoff is:

$$U(e, \alpha) = \alpha V(e) - e$$

Equity financing without commitment

- F.O.C for the entrepreneur's problem:

$$U'(e, \alpha) = \alpha V'(e) - 1 = 0 \Rightarrow \alpha V'(e^{**}) = 1.$$



- We get underinvestment

The agency cost of outside equity

- The entrepreneur's payoff:

$$U(e^{**}, \alpha) = \alpha V(e^{**}) - e^{**}$$

- The entrepreneur's payoff with commitment:

$$U(e^*, \alpha) = \alpha V(e^*) - e^*$$

- By revealed preferences (and since $e^{**} < e^*$):

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The optimal choice of α

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$$\frac{\partial U(e^{**}, \alpha)}{\partial \alpha} = \underbrace{\left(\overbrace{V'(e^{**})}^{1/\alpha} - 1 \right)}_{(+)} \underbrace{\frac{\partial e^{**}}{\partial \alpha}}_{(+)} > 0$$

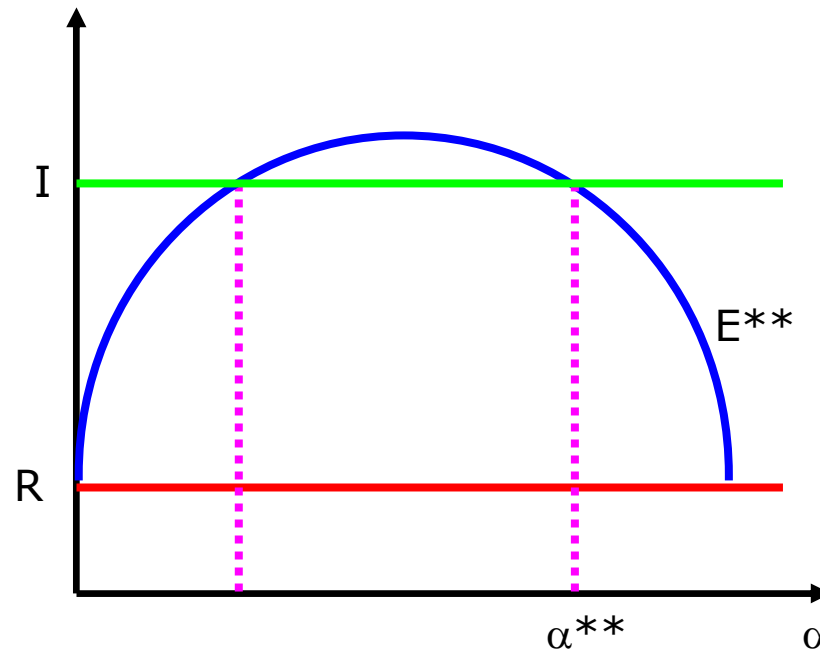
- The entrepreneur will raise α up to the point where

$$I = R + \underbrace{(1 - \alpha)V(e^{**})}_{E^{**}}$$

- $E^{**} = 0$ at $\alpha = 0$ (since then $I^{**} = 0$) and at $\alpha = 1 \Rightarrow E^{**}$ is inverse U-shaped

The optimal choice of α

- The entrepreneur's budget constraint:



- The relevant sol'n is with the maximal α

Jensen and Meckling, JFE 1976

The asset substitution problem

A simple example

- Consider a box with two sealed envelopes: one with \$100 and the other with \$0
- You can either pick one envelope from the box or receive \$70 for sure – what would you do?
- Now suppose you owe someone \$50 out of your gains
- If you take \$70, your payoff is $\$70 - \$50 = \$20$
- If you pick one of the sealed envelopes then you either have $\$100 - \$50 = \$50$, or you pick the empty envelope and your payoff is 0 because you cannot pay the \$50 \Rightarrow your expected payoff is $(\$0 + \$50)/2 = \$25$
- The lottery is better even though its NPV is only \$50

The model

- Two projects:
 - Safe project with return Z
 - Risky project with return $X \sim [0, \infty)$

- The firm has debt with face value D

- The management is perfect agent for equityholders – the agency problem is between equityholders and debtholders

Payoffs under the two projects

- Equityholders' payoff with the risky project:

$$Y_R = \int_D^{\infty} (X - D) dF(X)$$

- Equityholders' payoff with the safe project:

$$Y_S = \text{Max}\{Z - D, 0\}$$

- The firm surely chooses the risky project if $Z \leq D \Rightarrow$ assume that $Z > D$. Hence

$$Y_S = Z - D$$

Comparing the two projects

□ The safe project is better iff:

$$Y_S > Y_R \iff Z > X^C(D) \equiv D + \int_D^{\infty} (X - D) dF(X)$$

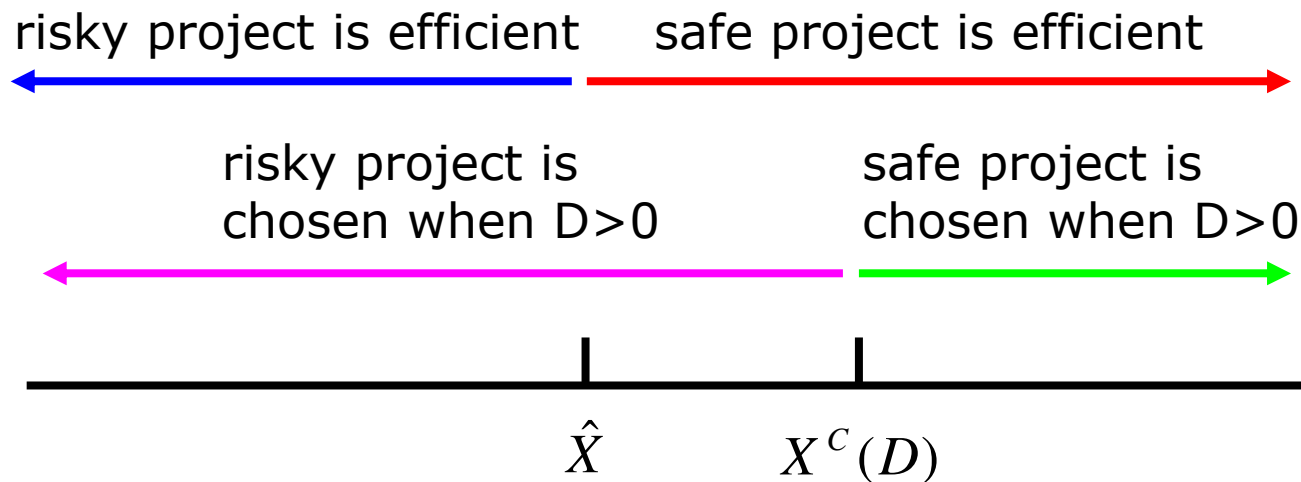
□ Properties of $X^C(D)$:

$$\frac{\partial X^C(D)}{\partial D} = 1 - \int_D^{\infty} dF(X) = 1 - (1 - F(D)) = F(D)$$

$$X^C(0) = \int_0^{\infty} X dF(X) = \hat{X}$$

Choice of projects with leverage

□ $X^C(D) > \hat{X}$ for all $D > 0$



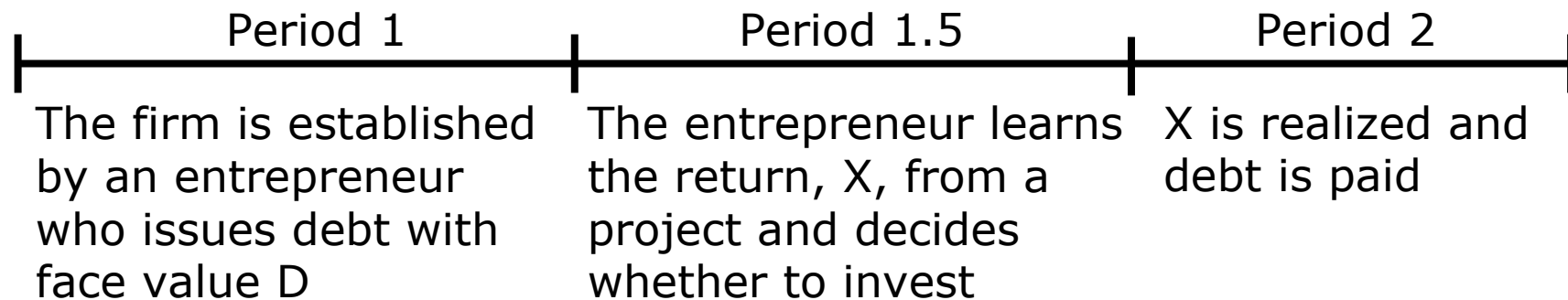
□ A leveraged firm prefers the risky project even when it is inefficient

Myers, JFE 1977

The debt overhang problem

The model

□ The timing:



□ In period 1 it is common knowledge that $X \sim [0, \infty)$

□ Absent debt, the firm invests iff $X \geq I$. The value of the firm:

$$V(0) = \int_I^{\infty} (X - I) dF(X)$$

Short-term debt (due at period 1.5)

- Suppose debt has to be paid **before** it is time to invest
- If $X - D \geq I$, the firm will invest
- If $X - D < I$, the firm will not invest and will go bankrupt. The debtholders will invest provided that $X \geq I$
- The firm invests iff $X \geq I \Rightarrow$ investment is efficient

The value of the firm with short-term debt

- The value of debt:

$$B(D) = \int_I^{D+I} (X - I) dF(X) + \int_{D+I}^{\infty} D dF(X)$$

- The value of equity:

$$E(D) = \int_{D+I}^{\infty} (X - D - I) dF(X)$$

- The total value of the firm:

$$\begin{aligned} V(D) &= \int_I^{D+I} (X - I) dF(X) + \int_{D+I}^{\infty} (X - I) dF(X) \\ &= \int_I^{\infty} (X - I) dF(X) \end{aligned}$$

Long-term debt

- Suppose debt has to be paid **after** it is time to invest
- If $X - D \geq I$, the firm will invest
- If $X - D < I$, the firm will not invest and will go bankrupt. The debtholders get a firm with no investment opportunities

The value of the firm with long-term debt

- The value of debt:

$$B(D) = \int_{D+I}^{\infty} D dF(X)$$

- The value of equity:

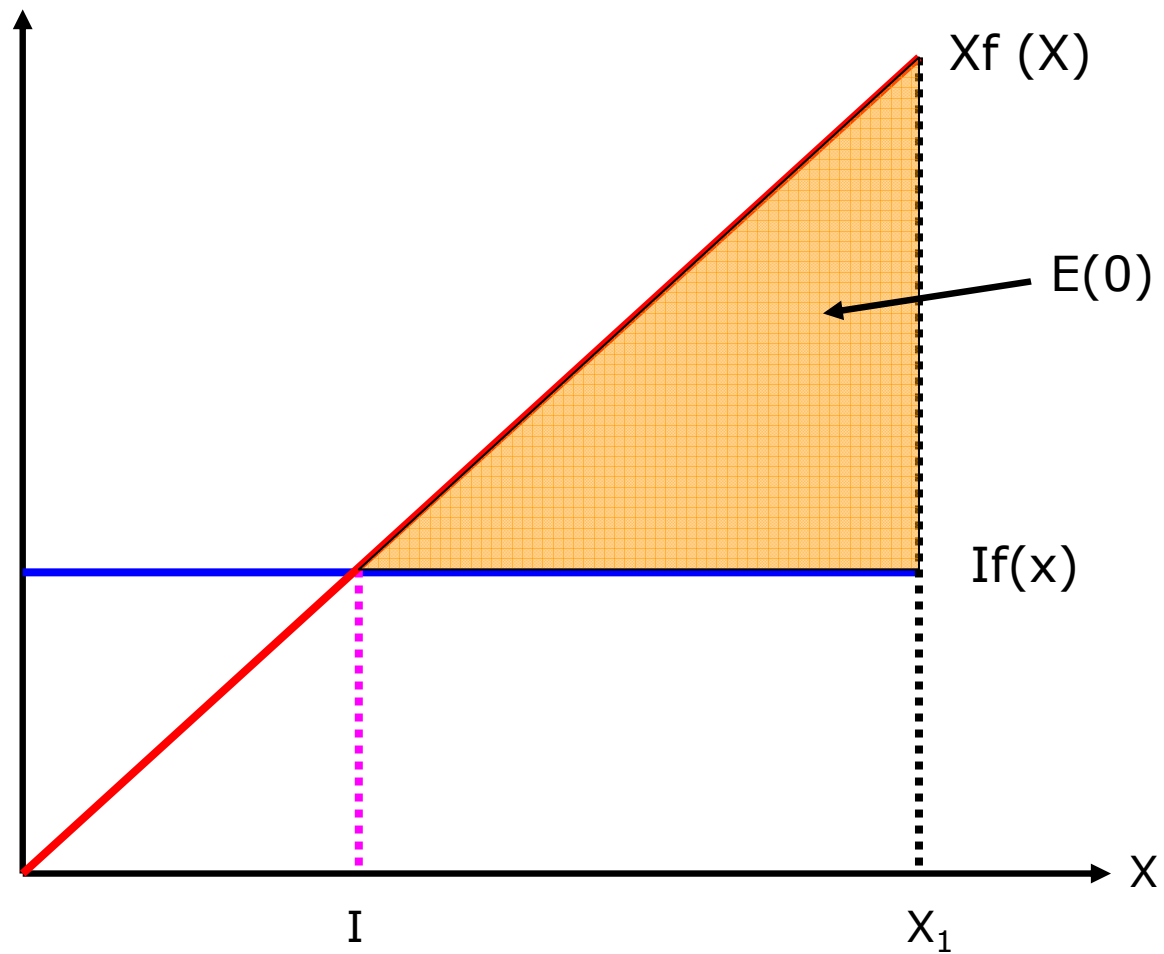
$$E(D) = \int_{D+I}^{\infty} (X - D - I) dF(X)$$

- The total value of the firm:

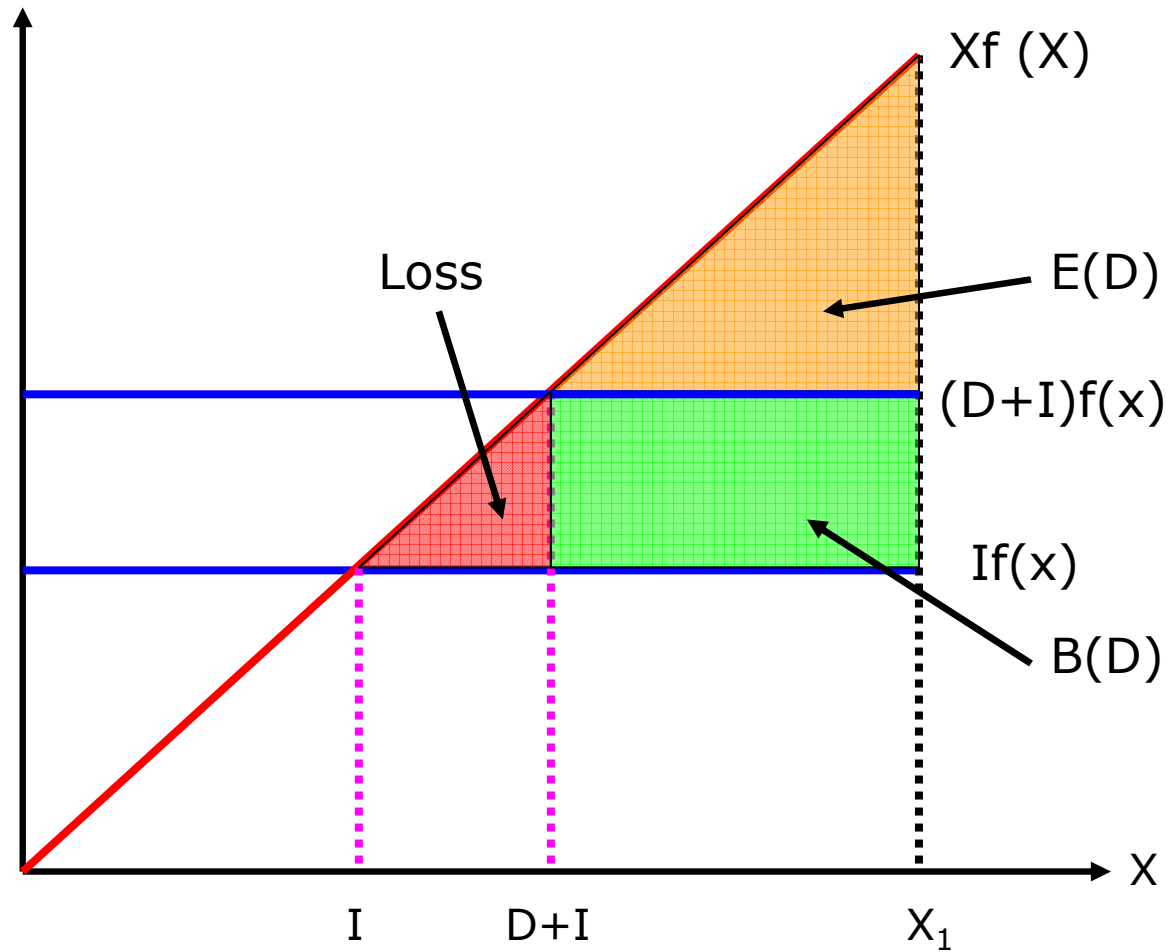
$$V(D) = \int_{D+I}^{\infty} (X - I) dF(X)$$

- The value is lower than under short-term debt. This is the debt overhang problem

Illustrating - All equity firm



Illustrating the debt overhang problem

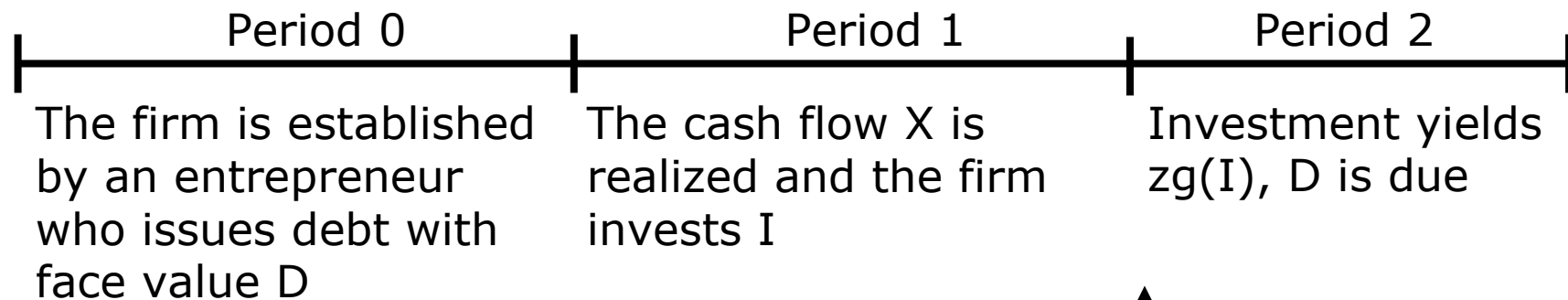


Berkovitch and Kim, JF 1990

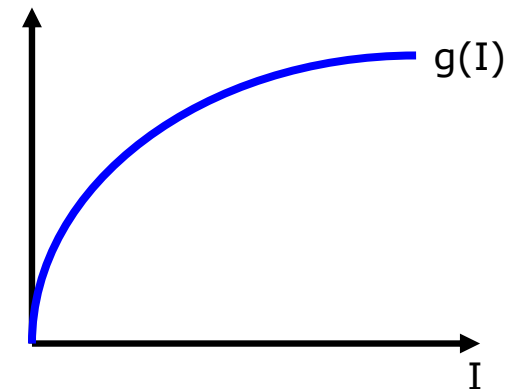
Overinvestment and underinvestment

The model

- The timing:



- $g'(I) > 0 > g''(I)$ and $g(0) = 0$
- $g'(0) = \infty$ and $g'(\infty) = 0$ (interior sol'n)
- $z \sim [0, \infty)$



Excess funds: $X > I$

- Cash flow at the end of period 2:

$$zg(I) + X - I$$

- The firm is solvent iff

$$zg(I) + X - I \geq D \quad \Rightarrow \quad z \geq z_1 \equiv \frac{D + I - X}{g(I)}$$

- The expected value of equity:

$$E_1(D) = \int_{z_1}^{\infty} (zg(I) + X - D - I) dF(z)$$

Investment with excess funds

□ F.O.C for investment:

$$\begin{aligned}\frac{\partial E_1(D)}{\partial I} &= \int_{z_1}^{\infty} (zg'(I) - 1) dF(z) \\ &= g'(I) \int_{z_1}^{\infty} z dF(z) - (1 - F(z_1)) = 0\end{aligned}$$

□ Rewriting:

$$g'(I) = \frac{1}{\int_{z_1}^{\infty} z dF(z)} \equiv \frac{1}{E(z | z \geq z_1)}$$
$$\frac{1}{1 - F(z_1)}$$

Deficit: $X < I$

- The firm covers the deficit by issuing extra equity
- Cash flow at the end of period 2: $zg(I)$
- The firm is solvent iff

$$zg(I) \geq D \Rightarrow z \geq z_2 \equiv \frac{D}{g(I)}$$

- The expected value of equity:

$$E_2(D) = \int_{z_2}^{\infty} (zg(I) - D) dF(z) - \underbrace{(I - X)}_{\text{Extra funds to cover the deficit}}$$

Investment with deficit

□ F.O.C for investment:

$$\begin{aligned}\frac{\partial E_2(D)}{\partial I} &= \int_{z_2}^{\infty} z g'(I) dF(z) - 1 \\ &= g'(I) \int_{z_2}^{\infty} z dF(z) - 1 = 0\end{aligned}$$

□ Rewriting:

$$g'(I) = \frac{1}{\int_{z_2}^{\infty} z dF(z)}$$

The value of the firm with excess funds

$$\begin{aligned} V_1(D) &= \underbrace{\int_{z_1}^{\infty} (zg(I) + X - D - I)dF(z)}_{\text{Equity}} \\ &+ \underbrace{\int_0^{z_1} (zg(I) + X - I)dF(z) + \int_{z_1}^{\infty} DdF(z)}_{\text{Debt}} \\ &= \int_0^{\infty} (zg(I) + X - I)dF(z) \\ &= \hat{z}g(I) + X - I \end{aligned}$$

The value of the firm with deficit

$$\begin{aligned} V_2(D) &= \underbrace{\int_{z_2}^{\infty} (zg(I) - D)dF(z) - (I - X)}_{\text{Equity}} + \underbrace{\int_0^{z_2} zg(I)dF(z) + \int_{z_2}^{\infty} DdF(z)}_{\text{Debt}} \\ &= \int_0^{\infty} zg(I)dF(z) - (I - X) \\ &= \hat{z}g(I) + X - I \end{aligned}$$

- The value of the firm is exactly as in the case of excess funds

Efficient investment

- F.O.C for investment:

$$\frac{\partial V(D)}{\partial I} = \hat{z}g'(I) - 1 = 0 \Rightarrow g'(I) = \frac{1}{\hat{z}}$$

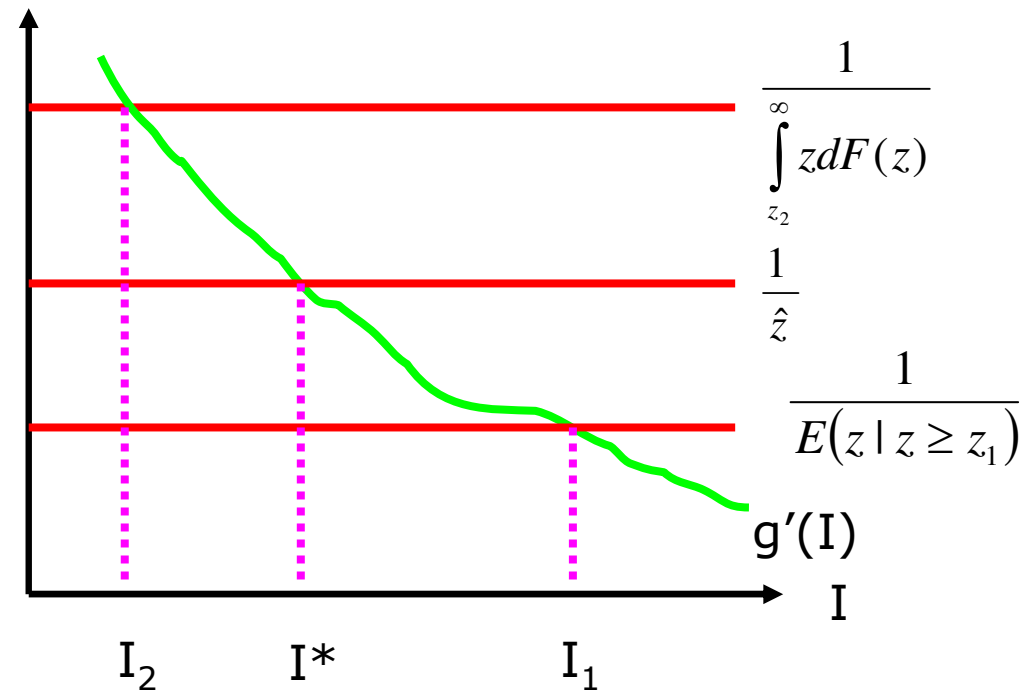
- Excess funds:

$$g'(I) = \frac{1}{\int_{z_1}^{\infty} z dF(z)} \equiv \frac{1}{E(z | z \geq z_1)} < \frac{1}{\hat{z}}$$

- Deficit:

$$g'(I) = \frac{1}{\int_{z_2}^{\infty} z dF(z)} > \frac{1}{\hat{z}}$$

Comparison



- Excess funds \Rightarrow overinvestment
- Deficit \Rightarrow underinvestment