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Solution to Problem set 4

Problem 1

(a) If the entrepreneur can use debt financing, he will issue debt with face value I. Since the cash flow of the firm in period 2 is 1 + X > I, the debt is completely riskless, so investors will be willing to buy it in period 1 and pay for it exactly I dollars. Hence, the entrepreneur's payoff in period 2 if he uses debt financing is 1 + X - I which is more than X which is the value of the firm absent investment. Hence the entrepreneur will always invest which is the efficient thing to do.

(b) If the entrepreneur does not raise I dollars and invests, his personal wealth in period 2 is $W_0 = X$. If he issues equity that represents a fraction α of the total equity of the firm, then since the capital market is perfectly competitive, and since the expected value of X is 1/2, the following equation should hold:

$$I = \alpha\left(1/2 + 1\right) = 3\alpha/2. \tag{1}$$

This expression implies that the equity fraction that outsiders will require in order to be willing to give the entrepreneur I dollars in period 1 is α* = 2I/3. Given α*, the personal wealth of the entrepreneur if his firm is worth X, is given by:

$$W_1 = \left(1 - \alpha^*\right)\left(X + 1\right) = \left(1 - \frac{2I}{3}\right)\left(X + 1\right). \tag{2}$$

Comparing W_0 and W_1 reveals that the entrepreneur will invest in the project if and only if:

$$X \leq \frac{3 - 2I}{2I}. \tag{3}$$

Clearly this inequality is more likely to hold when X is small.

(c) If X_H is the highest value of X for which the entrepreneur invests, then the expected value of firms that invest is $X_H/2 + 1$. Using this expression, the value of α* will be determined by the following equation:

$$I = \alpha\left(X_H/2 + 1\right). \tag{4}$$

Given α*, the personal wealth of the entrepreneur if his firm is worth X, is given by:

$$W_1 = (1 - \alpha^*)(X + 1) = \left(1 - \frac{I}{\frac{X_H}{2} + 1}\right)(X + 1). \quad (5)$$

Comparing W_0 and W_1 reveals that the entrepreneur will invest in the project if and only if:

$$X \leq \frac{\frac{X_H}{2} + 1}{I} - 1. \quad (6)$$

Since the left side of the inequality decreases with X, it follows that if the inequality holds at $X = X_H$, then it surely holds for all $X < X_H$. Setting $X_H = 1$, it follows that a sufficient condition for the inequality to hold for all X is that $I \leq 3/4$.

(d) If I = 4/5, then some entrepreneur types will not invest since (6) is violated. The highest type that still invests is the one for which (6) holds with equality. That is, X_H is given by the solution to the equation,

$$X_H = \frac{\frac{X_H}{2} + 1}{I} - 1. \quad (7)$$

Solving this equation and using the fact that I = 4/5, shows that $X_H = 2/3$. Hence, the entrepreneur will invest provided that $X \leq 2/3$.

<u>Problem 2</u>

(a) If T is common knowledge then the value of the firm is equal to the expected cash flow of the firm so that V(D) = T/2. This expression represents the combined value of equity and debt and it is independent of D for the usual Modigliani-Miller argument (recall that C is a private cost to the manager).

(b) The expected payoff of the manager is given by

$$U_T(D) = \frac{T}{2} - C\frac{D}{T}. \quad (1)$$

The second term here is the expected personal loss to the manager in the event of bankruptcy. The probability of this event is D/T. Clearly U(D) is maximized when D = 0.

(c) The figure is as we drew in class.

(d) In a separating equilibrium, the identity of each firm is fully revealed. Hence, a manager of an L-type firm has no reason to issue debt so his payoff is simply 3/2 = 1.5. On the other hand, if he pretends to be a manager of an H-type firm and the market falsely believe that this is the case, then his payoff, given the capital market's beliefs is given by

$$U_L(D|H) = \frac{5}{2} - C\frac{D}{3}. \tag{2}$$

The largest D that a manager of an L type firm would be willing to issue to fool the market is given by the solution to $U_L(D|H) = 1.5$. Solving for D, we get

$$D^* = \frac{3}{C}. \tag{3}$$

e) Clearly, a manager of an L-type firm will never issue more debt than D* because he is better-off issuing no debt and being believed to be managing a bad firm. On the other hand, a manager of an H-type firm will be willing to issue more debt than D*. To see why, note that if a manager of an H-type firm does not issue any debt and if the market (falsely) believes that he is the manager of an L-type firm, his payoff is 1.5. But if he issues D* and convinces the market of his identity, his payoff is

$$U_H(D^*|H) = \frac{5}{2} - C\frac{D^*}{5} = 2.5 - C\frac{3}{5C} = 1.9 > 1.5. \tag{4}$$

Hence, if the market observe a manager issuing debt with face value D* or more they should realize that this cannot be a manager of an L-type firm.

(f) As we can see, the model predicts a positive correlation between debt and earnings - on average, firms with higher debt levels have higher earnings than firms with low debt levels. The model also predicts that the debt levels of good firms decrease with the size of the personal losses that their mangers bear, but increase with the degree of information asymmetry which is reflected in the gap between H and L (you can solve the model with L and H instead of 3 and 5 to see that). Hence we should expect firms in more volatile industries (where there is more scope for private information) to have higher debt levels than firms in traditional and predictable industries.

Problem 3

(a) When X is common knowledge, the market does learn anything from the dividend policy of the firm so $V_0 = V_1$. If V_0 and V_1 are not the same then there are opportunities for arbitrage. Hence if the market is perfect there should not be such opportunities (i.e., they should be fully exhausted) so $V_0 = V_1$. The value of the firm as a function of y is

$$V_0 = \int_0^X \frac{\tilde{X} - y t}{X} d\tilde{X} - C \int_0^y \frac{\tilde{X}}{X} d\tilde{X}$$

$$= \frac{X}{2} - t y - \frac{C y}{X}. \tag{5}$$

Clearly, V_0 decreases with t so its optimal to pay no dividends.

(b) As usual, in a separating equilibium, $y_L = 0$ because the manager of an L-type firm has nothing to gain by paying dividends (he cannot fool investors anyway) and he can save the taxes of dividends.

(c)-(d) The figure looks like the one I drew in class except that the lines are straight rather than being parabolas. The lines for an H-type firm are less steep than those for an L-type firm.

(e) To find out the maximum amount of dividend, y*, that a manager of an L-type firm would be willing to pay in order to fool investors into believing that the firm is an H-type, note that the indifference curve of an L-type firm between paying a dividend y and having a period 0 value, V_0, on the one hand, and paying no dividends and having a V_0 = L/2, is given by the following equation:

$$b V_0 + (1 - b)\left(\frac{L}{2} - t y - \frac{C y}{L}\right) = \frac{L}{2}. \tag{6}$$

We can also write this equation as

$$V_0 = \frac{L}{2} + \frac{1 - b}{b}\left(t y + \frac{C y}{L}\right). \tag{7}$$

If an L-type firm convinces investors that it is an H-type, its value in period 0 will be

$$V_H = \frac{H}{2} - t y - \frac{C y}{H}. \tag{8}$$

Setting $V_0 = V_H$ and solving for y, we get:

$$y^* = \frac{b(H - L)LH}{2\left(\left(bL + (1 - b)H\right)C + t H L\right)}. \tag{9}$$

(f) When the manager of an H-type firm chooses y*, investors realize that the firm must be an L-type because it never pays an L-type firm to pay a dividend of y* as this is too costly given the relatively high probability of financial distress (an H-type firm has a smaller probability of such a distress given that its cash flow is likely to be high).

To show that paying a dividend of y* and being believed to be an H-type is better for the manager of an H-type firm than paying no dividend and being believed to be an L-type firm, note first that if an H-type firm pays no dividends, the payoff of its manager is

$$W_H(0) = b\frac{L}{2} + (1-b)\frac{H}{2}. \tag{10}$$

On the other hand, if the firm pays a dividend of y* and thereby signals its type to investors then the payoff of its manager is

$$W_H(y^*) = \frac{H}{2} - t\,y^* - \frac{C\,y^*}{H}. \tag{11}$$

Now,

$$W_H(y^*) - W_H(0) = \frac{(1-b)\,b\,C\,(H-L)^2}{2\big((bL + (1-b)H)C + tHL\big)} > 0. \tag{12}$$

Hence, it is indeed better to pay a dividend of y* than not paying any dividend.

(g) The model implies that dividends are positive correlated with high firm value and high expected cash flows. Comparative statics results on y* establish that y* increases with b and with H-L and decreases with C and t.

(h) A straightforward computation establishes that

$$A = \frac{(1-b)(C+tL)}{b\,L}. \tag{13}$$

This expression increases with t implying that the bang-for-the-buck increases with t.

Problem 4

(a) The manager's problem is to choose y to maximize his payoff given by:

$$\sqrt{X-y} + (1-t)\,y + b\sqrt{X-y}. \tag{14}$$

The first order condition for an interior solution to the manager's problem is:

$$1 - t = \frac{1}{2\sqrt{X-y}} + \frac{b}{2\sqrt{X-y}}. \tag{15}$$

The left hand side describes the marginal benefit from dividends associated with the increase in shareholders' wealth. The right hand side describes the marginal cost of dividends and it comes from two sources: first more dividends leave less money for investment in period 1, and second, more dividends leave less money for the manager and hence lower his benefits of control. Since the marginal benefit from dividends is a constant while the marginal cost is increasing with y, the second order condition for y* surely holds (the objective function is concave in y*). To ensure that y* > 0, the marginal benefit of dividends at y = 0 has to exceed the marginal cost. Hence a necessary and sufficient condition for y* > 0 is:

$$1 - t > \frac{1 + b}{2\sqrt{X}}. \tag{16}$$

Solving the first order condition for y* we get:

$$y^* = X - \left(\frac{1+b}{2(1-t)}\right)^2. \tag{17}$$

(b) It is easy to see that y* decreases with t (an increase in t lowers the marginal benefit from dividends), decreases with b (an increase in b raises the marginal cost of dividends since the manager gets a larger benefit of control from money left in the firm), and increases with X (an increase in X lowers the marginal cost of dividends since the manager's objective is concave).

(c) Substituting for y* in V(y) and rearranging yields:

$$V(y^*) = X(1-t) + \frac{(1-b)(1+b)}{4(1-t)} = X(1-t) + \frac{1-b^2}{4(1-t)}. \tag{18}$$

(d) The expected value of the firm is given by:

$$EV(y^*) = \int_0^1 \left[X(1-t) + \frac{1-b^2}{4(1-t)} \right] db = X(1-t) + \frac{1}{6(1-t)}. \tag{19}$$

(e) The change in firm value due to dividend payments is given by:

$$A(y) \equiv V(y) - EV(y^*) = \sqrt{X - y} + (1-t)y - X(1-t) - \frac{1}{6(1-t)}. \tag{20}$$

Given this expression, the marginal bang-for-the-buck, is equal to:

$$A'(y^*) = \frac{1}{2\sqrt{X - y^*}} + (1-t) = \frac{b(1-t)}{1+b}. \tag{21}$$

Clearly, the marginal bang-for-the-buck is decreasing with t. This is in contrast this result with the corresponding result in Bhattacharya (1979), where the bang-for-the-buck is rising with t. Hence we have a good way to test the validity of the two models.