Backward integration, forward integration, and vertical foreclosure*

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Abstract

I show that partial vertical integration may either alleviates or exacerbate the concern for vertical foreclosure relative to full vertical integration and I examine its implications for consumer welfare.

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Keywords: vertical integration, backward integration, forward integration, vertical foreclosure, controlling and passive integration, investment, consumer surplus

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1 Introduction

One of the main antitrust concerns that vertical mergers raise is the possibility that the merger will lead to the foreclosure of either upstream or downstream rivals. According to the European Commission, "A merger is said to result in foreclosure where actual or potential rivals' access to supplies or markets is hampered or eliminated as a result of the merger, thereby reducing these companies' ability and/or incentive to compete... Such foreclosure is regarded as anti-competitive where the merging companies — and, possibly, some of its competitors as well — are as a result able to profitably increase the price charged to consumers."[1] While most of the literature on vertical foreclosure has focused on full vertical mergers, in reality, many vertical mergers involve partial acquisitions of less than 100% of the shares of a supplier (partial backward integration) or a buyer (partial forward integration).[2] This begs the question of whether partial vertical integration alleviates, or rather exacerbates, the concern for vertical foreclosure, and what are its implications for consumer welfare.

To address this question, I consider a model with a single upstream manufacturer, U, that sells an input to two downstream firms, D_1 and D_2. The two downstream firms first invest in an attempt to boost the willingness of consumers to pay for their respective products, then they simultaneously bargain with U over the input price, and finally they produce their final products and compete by setting the prices.

[1] See "Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings," Official Journal of the European Union, (O.J. 2008/C 265/07), available at *http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:C:2008:265:0006:0025:EN:PDF*

Interestingly, according to the guidelines, foreclosure arises even if "the foreclosed rivals are not forced to exit the market: It is sufficient that the rivals are disadvantaged and consequently led to compete less effectively."

[2] See European Commission (2013) for a number of recent cases from Europe, and Gilo and Spiegel (2011) for recent cases from Israel. Partial integration is common in the U.S. cable TV industry (see Waterman and Weiss, 1997, p. 24-32). Recent prominent examples include News Corp.'s (a major owner of TV broadcast stations and programming networks) acquisition of a 34% stake in Hughes Electronics Corporation in 2003, which gave it a de facto control over DirecTV Holdings, LLC (a direct broadcast satellite service provider which is wholly-owned by Hughes), and the 2011 joint venture agreement between Comcast, GE, and NBCU, which gave Comcast (the largest cable operator and Internet service provider in the U.S.) a controlling 51% stake in a joint venture that owns broadcast TV networks and stations, and various cable programming. In the UK, BSkyB (a leading TV broadcaster) acquired in 2006 a 17.9% stake in ITV (UK's largest TV content producer). The UK competition commission found that the acquisition gave BSkyB effective control over ITN and argued that BSkyB would use it to "reduce ITV's investment in content" and "influence investment by ITV in high-definition television (HDTV) or in other services requiring additional spectrum."

The three firms impose externalities on each other. First, D_1 and D_2 impose a positive vertical externality on U because their investments boost their willingness to pay for the input. Second, D_1 and D_2 impose negative horizontal externalities on each other because the investment of D_i lowers the expected profit of D_j. These horizontal externalities also have vertical implications since they negatively affect the willingness of the rival downstream firm to pay for the input. The results in my model are driven by the effect of vertical integration on these externalities. In particular, integration between U and one of the downstream firms, say D_1, creates three effects: (i) following integration, D_1 internalizes the positive vertical externality of its investment on U and hence it invests more, (ii) following integration, D_1 internalizes the negative horizontal externality of its investment on D_2's willingness to pay for the input and hence on U's profit from selling to D_2; this effect weakens D_1's incentive to invest, and (iii) holding D_1's investment fixed, U requires a higher input price from D_2 to compensate for the negative horizontal externality that D_2 imposes on D_1; this higher input price lowers D_2's profit on the margin and weakens its incentive to invest.

Downstream foreclosure arises in my model because following integration with U, D_1 ends up investing more, while D_2 invests less, so in expectation, D_1 gains market share at D_2's expense. When D_1 controls U while holding a fraction $\alpha < 1$ of U's shares (partial backward integration), D_2 must pay an even higher price for the input to ensure that a fraction α of this price compensates D_1 for the erosion of its profit due to competition with D_2 (otherwise D_1 will use its control to induce U to refuse to sell to D_2). Hence, D_2 invests even less than under full vertical integration. D_1 in turn invests more because it now internalizes only a fraction of the negative horizontal externality of its investment on D_2's willingness to pay for the input and hence on U's profit. Consequently, D_2 is more likely to be foreclosed in the downstream market. Under partial forward integration, the opposite is true since U gets only a fraction α in D_1's profit and hence does not fully internalize the negative horizontal externality that D_2 imposes on D_1's profit. Consequently, U will charge D_2 a lower price for the input than under full vertical integration and will use its control over D_1 to cut D_1's investment in order to limit the negative externality on D_2's willingness to pay for the input. In sum, my analysis shows that partial backward integration exacerbates the concern for downstream foreclosure, while partial forward integration alleviates it.[3]

[3] Although I focus in this paper on the effect of vertical integration on the foreclosure of downstream rivals, it is also possible to examine its effects on the foreclosure of upstream rivals. For instance, one can study a model with two upstream suppliers U_1 and U_2 which sell a homogenous input to a single downstream firm, D, which uses the input to produce a final product. In such a model one can assume that the upstream firms invest in order to boosts the quality of the input they sell to D. Such a model will be a miror image of the model that I consider in this paper.

In addition, I also study the possibility of passive integration, where the acquirer acquires a stake in the target's cash flow rights, but no say in its decision making. I show that passive backward integration (D_1 gets a stake in U's profit, but no say in how the input is priced), leads to less foreclosure than controlling backward integration, while passive forward integration (U gets a stake in D_1's profit, but no say in how D_1 invests), leads to more foreclosure than controlling forward integration, though the effect on consumers depends on the size of the acquired stake as well as about the marginal benefit from investment relative to its marginal cost.

The rest of the paper proceeds as follows: Section 2 presents the model and Section 3 characterizes the non-integration benchmark. In Section 4, I solve for the equilibrium under full vertical integration and evaluate its welfare effects. In Section 5, I turn to partial backward and partial forward integration and evaluate their welfare effects. In section 6, I review the relevant literature in order to put my own contribution in context. Concluding remarks are in Section 7. All proofs are in the Appendix.

2 The Model

Two downstream firms, D_1 and D_2, purchase an input from an upstream supplier U and use it to produce a final product. The downstream firms face a unit mass of identical final consumers, each of whom is interested in buying at most one unit. The utility of a final consumer if he buys from D_i is $V_i - p_i$, where V_i is the quality of the final product and p_i is its price. If a consumer does not buy, his utility is 0.[4]

I assume that initially $V_1 = V_2 = \underline{V}$. By investing, D_i can try to increase V_i to $\overline{V}$; the probability that D_i succeeds to raise V_i to $\overline{V}$ is q_i. The cost of investment is increasing and convex. To obtain closed form solutions, I will assume that the cost of investment is $\frac{kq_i^2}{2}$, where $k > \overline{V} - \underline{V} \equiv \Delta$.[5] The total cost of each D_i is then equal to the sum of $\frac{kq_i^2}{2}$ and the price that D_i pays U for the input. The upstream supplier U incurs a constant cost c if it serves only one downstream firm and $2c$ if it serves both downstream firms. To avoid uninteresting cases, I will assume that each downstream firm also receives additional revenue $\overline{R}$ (say from selling to "captive

[4] The unit demand function implies that there is no double marginlaization in my model and it allows me to focus on other, more novel, effects of vertical integration.

[5] The assumption that the cost function is quadratic is only made for convinience. All results go through for any increasing and convex cost function that satisfies appropriate restrictions (needed to ensure that the equilibrium is well-behaved). The assumption that $k > \Delta$ ensures that in equilibrium, $q_1, q_2 < 1$ (q_1 and q_2 are probabilities).

consumers"), where $\overline{R} > c + \underline{V}$.[6]

The sequence of events is as follows:

- **Stage 1:** D_1 and D_2 simultaneously choose how much to invest in the respective qualities of their final products.

- **Stage 2:** Given q_1 and q_2, the two downstream firms buy the input from U. The price that each D_i pays U is determined by bilateral bargaining. Following Bolton and Whinston (1991) and Rey and Tirole (2007), I will assume that the bargaining between D_i and U is such that with probability 1/2, D_i makes a take-it-or-leave-it offer to U, and with probability 1/2, U makes a take-it-or-leave-it offer to D_i (a random proposer model).

- **Stage 3:** The qualities of the final products of the two downstream firms, V_1 and V_2, are realized and become common knowledge.

- **Stage 4:** D_1 and D_2 simultaneously set their prices, p_1 and p_2.

Two comments about the sequence of events are now in order. First, note that investments, which are the main strategic decisions of D_1 and D_2 are chosen before w_1 and w_2 are negotiated. Hence, there is no scope in my model for secret contract renegotiation as in Hart and Tirole (1991). Second, in the Appendix, I solve for the equilibrium in a setting where stages 2 and 3 are reversed, i.e., U contracts with D_1 and D_2 only after V_1 and V_2 are realized. It turns out that in this alternative setting, the input prices are independent of the investment levels, so vertical integration does not affect the input prices, as it does in the main text.

Before characterizing the equilibrium, it is worth noting that consumers end up buying a high quality product unless the investments of both D_1 and D_2 fail; hence, social surplus is

$$\begin{aligned} W &= \left(1-(1-q_1)(1-q_2)\right)\overline{V} + (1-q_1)(1-q_2)\underline{V} + 2\left(\overline{R}-c\right) - \frac{kq_1^2}{2} - \frac{kq_2^2}{2} \\ &= \overline{V} - (1-q_1)(1-q_2)\Delta + 2\left(\overline{R}-c\right) - \frac{kq_1^2}{2} - \frac{kq_2^2}{2}. \end{aligned}$$

The first-best levels of investment maximize W and are equal to $q^{fb} \equiv \frac{H}{H+1}$, where $H \equiv \frac{\Delta}{k} < 1$.

[6] As will become clear later, this assumption ensures that the industry surplus is higher when both D_1 and D_2 are served by U than when only one downstream firm is served. Hence, U does not wish to foreclose D_2 in order to enable D_1 to monopoloze the downstream market as in, say, Hart and Tirole (1990). Indeed, having two downstream firms increases the chance of offering a high quality product to consumers due to the fact that the realizations of V_1 and V_2 are independent of each other.

3 The non-integrated equilibrium

Since p_1 and p_2 are set simultaneously after D_1 and D_2 have already sunk their costs (the cost of investment in quality and the cost of the input), the Nash equilibrium prices are $p_1 = p_2 = 0$ if $V_1 = V_2$ and $p_i = \overline{V} - \underline{V} \equiv \Delta$ and $p_j = 0$ if $V_i = \overline{V}$ and $V_j = \underline{V}$.[7] Together with the additional revenue $\overline{R}$, the downstream revenues of D_1 and D_2 are summarized in the following table (the left entry in each cell is D_1's revenue and the right entry is D_2's revenue):

Table 1: The downstream revenues

	$V_2 = \overline{V}$	$V_2 = \underline{V}$
$V_1 = \overline{V}$	$\overline{R}, \overline{R}$	$\overline{R} + \Delta,\ \overline{R}$
$V_1 = \underline{V}$	$\overline{R},\ \overline{R} + \Delta$	$\overline{R}, \overline{R}$

Notice that D_i earns Δ only when $V_i = \overline{V}$ and $V_j = \underline{V}$ (D_i succeeds to raise V_i to $\overline{V}$ while D_j fails); the probability of this event is $q_i\,(1 - q_j)$. The variable Δ reflects the premium that D_i gets when it is the sole provider of high quality in the downstream market. Also notice that with probability $\phi_i \equiv q_j\,(1 - q_i)$, $V_i = \underline{V}$ and $V_j = \overline{V}$, in which case D_i sells only to captive customers. Hence, ϕ_i can serve as a measure of "downstream foreclosure."[8]

Next, consider stage 2 of the game, in which each D_i bargains with U over the input price. When D_i makes a take-it-or-leave-it offer to U, it offers a price c for the input, which is the minimal price that U will accept. When U makes a take-it-or-leave-it offer, it offers a price equal to the entire expected revenue of D_i, which is $q_i\,(1 - q_j)\,\Delta + \overline{R}$. The expected price that D_i pays for the input is therefore

$$w_i^* = \frac{q_i\,(1 - q_j)\,\Delta + \overline{R} + c}{2}. \tag{1}$$

[7] To simplify matters, I assume that when indifferent, consumers buy from the high quality firm. If $V_1 = V_2$, consumers randomize between D_1 and D_2.

[8] Foreclosure in my model is not due to U's "refusal to deal" with one of the downstream firms. Rather it is due to the diminished expected sales of the nonintegrated downstream firm. Indeed, after the cost of investment is sunk, total profits are $q_1\,(1 - q_2)\,\Delta + q_2\,(1 - q_1)\,\Delta + 2\left(\overline{R} - c\right)$ when both D_1 and D_2 are served, and $\underline{V} + q_1\Delta + \overline{R} - c$ when only one downstream firm, say D_1, is served. Since in equilibrium $q_1 < 1/2$, the assumption that $\overline{R} > c + \underline{V}$ is sufficient (but not necessary) to ensure that the former exceeds the latter. Hence, in equilibrium, U will deal with both D_1 and D_2.

Given w_i^* and given a pair of investments in quality, q_i and q_j, the expected profit of D_i is

$$\begin{aligned}\pi_i &= q_i(1-q_j)\Delta + \overline{R} - w_i^* - \frac{kq_i^2}{2} \\ &= \frac{q_i(1-q_j)\Delta + \overline{R} - c}{2} - \frac{kq_i^2}{2}.\end{aligned} \tag{2}$$

In stage 1 of the game, D_1 and D_2 choose q_1 and q_2 to maximize their respective profits. Recalling that $H \equiv \frac{\Delta}{k}$, the best-response function of D_i, $i = 1, 2$, is defined by the following first-order condition:

$$\pi_i' = \frac{(1-q_j)\Delta}{2} - kq_i = 0, \qquad \Rightarrow \qquad q_i = \frac{(1-q_j)H}{2}. \tag{3}$$

The equilibrium levels of investment are defined by the intersection of the two best-response functions and are given by

$$q_1^* = q_2^* = \frac{H}{H+2}. \tag{4}$$

Notice that q_1^* and q_2^* are below their first-best level $q^{fb} \equiv \frac{H}{H+1}$. Intuitively, D_1 and D_2 underinvest relative to the first best because some of the benefit from their investment accrues to U.

Figure 1 illustrates the best-response functions of D_1 and D_2 and the Nash equilibrium levels of investment.

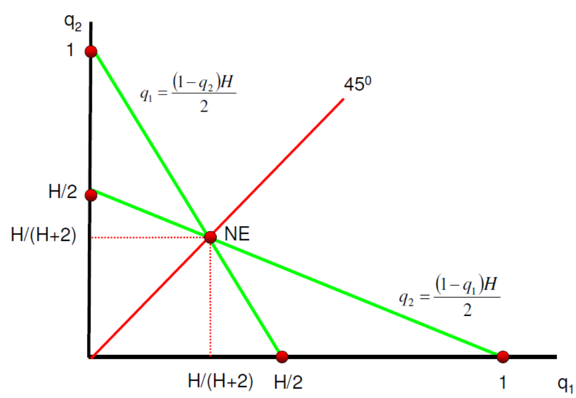

Figure 1: The Nash equilibrium investments under non integration

Using (4), the probability that D_i is foreclosed in the downstream market is

$$\phi_i^* \equiv q_j^*(1-q_i^*) = \frac{2H}{(H+2)^2}. \tag{5}$$

4 The vertically integrated equilibrium

Suppose that D_1 and U fully merge and choose the strategy of the vertically integrated entity, VI, to maximize their joint profit. The merger does not affect the outcome in stages 3 and 4 of the game; in particular, the downstream revenues are still given by Table 1.

Moving to stage 2 in which VI and D_2 bargain over the input price, note that when D_2 makes a take-it-or-leave-it offer, it offers an input price, w, that leaves VI indifferent between selling to D_2 and refusing to sell to D_2:

$$\underbrace{q_1\overline{V} + (1-q_1)\,\underline{V} + \overline{R} - c}_{VI\text{'s profit if it refuses to sell to } D_2} = \underbrace{q_1(1-q_2)\,\Delta + \overline{R} + w - 2c}_{VI\text{'s profit if it sells to } D_2} \quad \Rightarrow \quad w = q_1q_2\Delta + c + \underline{V}.$$

D_2 is willing to make this offer since its resulting expected profit is $q_2(1-q_1)\,\Delta + \overline{R} - w = q_2(1-2q_1)\,\Delta + \overline{R} - \underline{V} - c$, which is positive since, as I show later, $q_1 \leq 1/2$, and since by assumption, $\overline{R} > c + \underline{V}$. When VI makes a take-it-or-leave-it offer, it offers $q_2(1-q_1)\,\Delta + \overline{R}$, which is equal to the entire expected revenue of D_2. The expected input price that D_2 will pay U is therefore

$$w_2^{VI} = \frac{q_2(1-q_1)\,\Delta + \overline{R}}{2} + \frac{q_1q_2\Delta + \underline{V} + c}{2} = \frac{q_2\Delta + \overline{R} + \underline{V} + c}{2}.$$

Notice that w_2^{VI} increases with q_2, but is independent of q_1. The reason for this is that the input price that D_2 proposes is equal to the difference between the expected monopoly profit of VI (VI's profit when it refuses to sell to D_2) and its expected duopoly profit (VI's profit when it sells to D_2), which is $q_1q_2\Delta + \underline{V}$, plus the cost of producing for D_2. The input price that VI proposes is equal to the expected duopoly profit of D_2, which is $q_2(1-q_1)\,\Delta + \overline{R}$. The sum of the two then is $q_2\Delta + \overline{R} + \underline{V}$, which is increasing with q_2, but is independent of q_1.[9]

The fact that w_2^{VI} is increasing with q_2 reflects the fact that following integration, U internalizes the negative horizontal externality it imposes on D_1 when it deals with D_2 and hence it requires compensation for the erosion in its downstream profit due to selling the input to D_2. Consequently, holding q_1 and q_2 fixed, $w_2^{VI} > w_2^*$: following the integration of D_1 and U, D_2 ends up paying U a higher price for the input (this can be seen as another sense in which vertical integration leads to foreclosure).

[9] Note that if the bargaining between VI and D_2 was asymmetric in the sense that VI made a take-it-or-leave offer with probability $\gamma \neq 1/2$ and D_2 made a take-it-or-leave offer with probability $1-\gamma$, then w_2^{VI} would be equal to $\gamma\left(q_2\Delta + \overline{R}\right) + (1-2\gamma)\,q_1q_2\Delta + (1-\gamma)\,(c+\underline{V})$. Here, w_2^{VI} increases with q_1 if $\gamma < 1/2$ and decreases with q_1 if $\gamma > 1/2$, so in choosing q_1, VI would also take into account its effect on w_2^{VI} and would invest more if $\gamma < 1/2$ and invest less if $\gamma > 1/2$.

Given w_2^{VI}, the expected profits of VI and D_2 are

$$\pi_{VI} = \underbrace{q_1 (1 - q_2) \Delta + \overline{R} - \frac{kq_1^2}{2}}_{\text{Downstream profit}} + \underbrace{w_2^{VI} - 2c}_{\text{Upstream profit}},$$

and

$$\begin{aligned}\pi_2 &= q_2 (1 - q_1) \Delta + \overline{R} - w_2^{VI} - \frac{kq_2^2}{2} \\ &= \frac{q_2 (1 - 2q_1) \Delta + \overline{R} - c - \underline{V}}{2} - \frac{kq_2^2}{2}.\end{aligned}$$

The equilibrium investment levels under vertical integration, q_1^{VI} and q_2^{VI}, are defined by the following pair of first-order conditions:

$$\pi'_{VI} = (1 - q_2) \Delta - kq_1 = 0, \qquad \Rightarrow \qquad q_1 = (1 - q_2) H, \tag{6}$$

and

$$\pi'_2 = \frac{(1 - 2q_1) \Delta}{2} - kq_2 = 0, \qquad \Rightarrow \qquad q_2 = \left(\frac{1}{2} - q_1\right) H. \tag{7}$$

Notice from (7) that $q_2 = 0$ whenever $q_1 \geq 1/2$; hence $q_1 \leq 1/2$ in every interior equilibrium, as I have assumed above.

The best-response functions of the integrated firm VI and of D_2 are illustrated in Figures 2 and 3. Figure 2 shows the best-response functions when $H < 1/2$ (D_i gets a limited premium from being the sole provider of high quality in the downstream market). In this case, the Nash equilibrium is interior. The best-response functions in the non-integrated case are shown by the dotted lines.

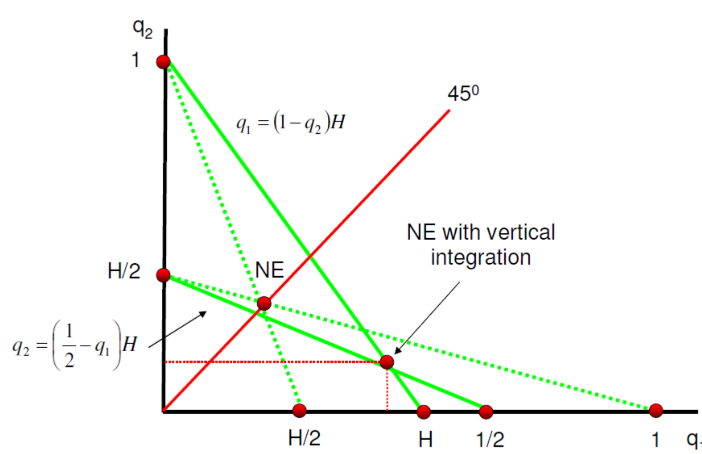

Figure 2: The Nash equilibrium investments under vertical integration - an interior equilibrium

To understand the figure, recall that under vertical integration, D_1 internalizes the positive externality of its investment on U's profit and hence it invests more than under no integration, especially if D_2's investment is low (in which case D_1's marginal benefit from investment is high since it has a high probability of earning Δ in the downstream market). Hence, D_1's best-response function rotates counterclockwise. The clockwise rotation of D_2's best-response function reflects the increase in w_2, which, as mentioned earlier, is due to the fact that under vertical integration, U internalizes the negative externality that selling the input imposes on D_1's downstream profit; D_2 invest less than under no integration, especially when q_1 is low, so that D_2's marginal benefit from investment is high.

Figure 3 shows that when $H \geq 1/2$ (D_i gets a large premium from being the sole provider of high quality in the downstream market), the best-response function of D_1 lies everywhere above the best-response function of D_2, so in equilibrium, $q_2^{VI} = 0$.

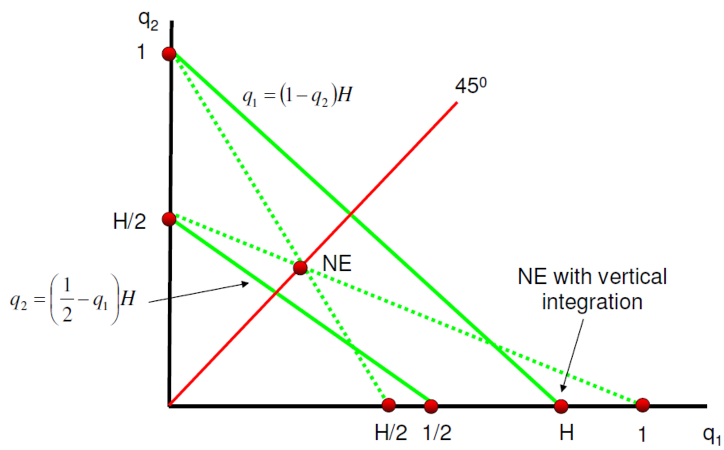

Figure 3: The Nash equilibrium investments under vertical integration - firm 2 does not invest

Solving (6) and (7), the equilibrium levels of investment are

$$q_1^{VI} = \begin{cases} \frac{H(2-H)}{2(1-H^2)} & \text{if} \quad H < \frac{1}{2}, \\ H & \text{if} \quad H \geq \frac{1}{2}, \end{cases} \tag{8}$$

and

$$q_2^{VI} = \begin{cases} \frac{H(1-2H)}{2(1-H^2)} & \text{if} \quad H < \frac{1}{2}, \\ 0 & \text{if} \quad H \geq \frac{1}{2}. \end{cases} \tag{9}$$

Notice that since $H \equiv \frac{\Delta}{k} < 1$, the denominators of q_1^{VI} and q_2^{VI} are both positive. It is easy to check that $q_1^{VI} > q_i^* > q_2^{VI}$: following vertical integration, D_1 invests more, while D_2 invests less.

This result is due to a combination of 3 effects: (i) following vertical integration, D_1 internalizes the positive externality of its investment on U and hence it invests more; (ii) investments are strategic substitutes, so the higher investment of D_1 lowers the investment of D_2; and (iii) following vertical integration, D_2 pays a higher price for the input and hence makes a smaller profit on the margin; this in turn lowers D_2's benefit from investing.[10] Since q_1 increases while q_2 decreases, the probability that D_2 is foreclosed in the downstream market, ϕ_2^{VI}, is higher than in the non-integration case.

One can also check that $q_1^{VI} > q^{fb} > q_2^{VI}$: under vertical integration, the vertically integrated firm overinvests relative to the first best level, while the non-integrated firm underinvests.

The next proposition summarizes the discussion so far and proves that D_1 and U find it optimal to vertically integrate.

Proposition 1: *Vertical integration is profitable for the upstream supplier U and downstream firm D_1. Relative to the non-integration benchmark, vertical integration leads to more investment by D_1 (above the first-best level), less investment by D_2 (below the first-best level), and a higher ϕ_2 (D_2 is more likely to be foreclosed in the downstream market).*

4.1 The welfare effects of vertical integration

To examine how vertical integration affects welfare, recall that the Nash equilibrium prices are $p_1 = p_2 = 0$ if $V_1 = V_2$ and $p_i = \overline{V} - \underline{V} \equiv \Delta$ and $p_j = 0$ if $V_i = \overline{V}$ and $V_j = \underline{V}$. Hence, consumer surplus in the downstream market is given by the following table:[11]

Table 2: Consumer surplus

	$V_2 = \overline{V}$	$V_2 = \underline{V}$
$V_1 = \overline{V}$	$\overline{V}$	$\overline{V} - \Delta = \underline{V}$
$V_1 = \underline{V}$	$\overline{V} - \Delta = \underline{V}$	$\underline{V}$

Expected consumer surplus is therefore

$$S(q_1, q_2) = q_1 q_2 \overline{V} + (1 - q_1 q_2)\underline{V} = \underline{V} + q_1 q_2 \Delta. \tag{10}$$

[10] Buehler and Schmutzler (2008) also show that following vertical integartion, D_1 invests more and D_2 invests less than under non-integration. In their model though, D_1 and D_2 engage in Cournot competition and investements are cost-reducing. They call this result the "intimidation effect" of vertical mergers.

[11] The surplus of "captive consumers" is constant and hence I will ignore it.

The next proposition compares expected consumer surplus absent integration, $S^* \equiv S(q_1^*, q_2^*)$, and under vertical integration $S^{VI} \equiv S\left(q_1^{VI}, q_2^{VI}\right)$.

Proposition 2: *Vertical integration benefits consumers when $H < 0.323$, but harms consumers otherwise.*

Equation (10) shows that vertical integration affects consumers only through its effect on $q_1 q_2$, which is the probability that both firms offer high quality; in that case (and only then), consumers enjoy high quality at a low price. Equation (4) shows that $q_1^* q_2^*$ is strictly increasing with H. Equations (8) and (9) in turn show that q_1^{VI} is strictly increasing with H, while q_2^{VI} is an inverse U-shaped function of H; hence $q_1^{VI} q_2^{VI}$ is first increasing and then decreasing with H. Not surprisingly then, vertical integration harms consumers when H is sufficiently large.

5 Partial vertical integration

So far I have assumed that under vertical integration, D_1 and U fully merge. In reality though, vertical integration is often partial: the acquiring firm (D_1 in the case of backward integration and U in the case of forward integration) buys only a partial stake in the target firm. In this section, I explore the effects of partial integration (backward and forward) on foreclosure and on welfare.

5.1 Partial backward integration by $\mathbf{D}_1$

Suppose that D_1 acquires a stake $\alpha < 1$ in U. For now, I will assume that α is a controlling stake, which de facto, allows D_1 to choose U's strategy. Towards the end of this subsection, I will examine the case where α is a passive stake, so that U's strategy is effectively chosen by other shareholders who do not own shares in D_1 or D_2.

As in the full integration case, the equilibrium prices and downstream revenues are given by Table 1. Since D_1 fully controls U, it will set w_1 unilaterally at some level (the precise value of w_1 does not matter for now). As for the bargaining between D_2 and U over w_2, note that when D_2 makes a take-it-or-leave-it offer, it will make an offer that leaves D_1 (which controls U) indifferent between selling the input to D_2 at w_2 and selling to D_2:

$$\underbrace{q_1\overline{V} + (1-q_1)\underline{V} + \overline{R} - w_1}_{\substack{D_1\text{'s profit if } U \text{ refuses} \\ \text{to sell to } D_2}} + \underbrace{\alpha(w_1 - c)}_{\substack{D_1\text{'s share} \\ \text{in } U\text{'s profit}}} = \underbrace{q_1(1-q_2)\Delta + \overline{R} - w_1}_{\substack{D_1\text{'s profit if } U \\ \text{sells to } D_2}} + \underbrace{\alpha(w_1 + w_2 - 2c)}_{\substack{D_1\text{'s share} \\ \text{in } U\text{'s profit}}},$$

$$\Rightarrow \qquad w_2 = \frac{q_1 q_2 \Delta + \alpha c + \underline{V}}{\alpha}.$$

When D_1 makes a take-it-or-leave-it offer on U's behalf, it will offer a price $w_2 = q_2(1-q_1)\Delta + \overline{R}$, which is equal to the entire expected revenue of D_2. The expected input price that D_2 pays under partial backward integration (denoted BI) is therefore

$$w_2^{BI} = \frac{q_2(1-q_1)\Delta + \overline{R}}{2} + \frac{q_1 q_2 \Delta + \alpha c + \underline{V}}{2\alpha}.$$

Notice that w_2^{BI} is decreasing in α and is equal to w_2^{VI} when $\alpha = 1$ (full integration). Hence, holding q_1 and q_2 fixed, $w_2^{BI} > w_2^{VI}$ for all $\alpha < 1$. The reason why w_2 is higher when α is small is that D_2 must compensate D_1 for the erosion in D_1's downstream profit due to competition with D_2. Since D_1 gets only a fraction α of U's profits, the input price must be high enough so that a fraction α of it will cover the entire erosion of D_1's downstream profit.

As in the full integration case, w_2^{BI} is increasing with q_2, since an increase in q_2 implies a larger negative externality on D_1, which the vertically integrated U must be compensated for in order to agree to sell the input to D_2. But unlike the full integration case, w_2^{BI} is now also increasing with q_1. The reason for this is as follows: an increase in q_1 makes it more likely that D_1 will produce a high quality product; hence, the negative externality that D_2 imposes on D_1 becomes more significant, so a higher w_2^{BI} is needed to compensate D_1. On the other hand, the higher q_1 is, the larger is the negative externality that D_1 imposes on D_2 and hence the lower is D_2's willingness to pay for the input. When $\alpha = 1$, the two externalities just cancel each other out. But when $\alpha < 1$, D_1 takes into account the entire loss of downstream profits, but only a fraction α of the decrease in D_2's willingness to pay for the input (which accrues to U). Hence, the first effect dominates. The fact that w_2^{BI} is increasing with q_1 strengthens D_1's incentive to invest.

Given w_2^{BI}, the expected profits of D_1 and D_2 are

$$\begin{aligned} \pi_1 &= \underbrace{q_1(1-q_2)\Delta + \overline{R} - w_1 - \frac{kq_1^2}{2}}_{D_1\text{'s profit}} + \underbrace{\alpha\left(w_1 + w_2^{BI} - 2c\right)}_{U\text{'s profit}} \\ &= \frac{((2-(1+\alpha)q_2)q_1 + \alpha q_2)\Delta + (2+\alpha)\overline{R} - 3\alpha c + \underline{V}}{2} - (1-\alpha)w_1 - \frac{kq_1^2}{2}, \end{aligned}$$

and

$$\begin{aligned}\pi_2 &= q_2\left(1-q_1\right)\Delta+\overline{R}-w_2^{BI}-\frac{kq_2^2}{2}\\ &= \frac{q_2\left(\alpha-\left(1+\alpha\right)q_1\right)\Delta+\alpha\left(\overline{R}-c\right)-\underline{V}}{2\alpha}-\frac{kq_2^2}{2}.\end{aligned}$$

The equilibrium levels of investment under partial backward integration, q_1^{BI} and q_2^{BI}, are defined by the following first-order conditions:

$$\pi_1'=\left(1-\frac{(1+\alpha)}{2}q_2\right)\Delta-kq_1=0, \qquad \Rightarrow \qquad q_1=\left(1-\frac{(1+\alpha)\,q_2}{2}\right)H, \tag{11}$$

and

$$\pi_2'=\frac{\left(\alpha-\left(1+\alpha\right)q_1\right)\Delta}{2\alpha}-kq_2=0, \qquad \Rightarrow \qquad q_2=\left(\frac{1}{2}-\frac{(1+\alpha)\,q_1}{2\alpha}\right)H. \tag{12}$$

Figure 4 shows the interior Nash equilibrium which obtains when $H<\frac{\alpha}{1+\alpha}$ (equivalently, when $\alpha>\frac{H}{1-H}$). Compared with full integration, now the best-response function of D_1 rotates clockwise around its horizontal intercept, while the best-response function of D_2 rotates clockwise around its vertical intercept. Intuitively, D_1 invests more when $\alpha<1$ because it internalizes only a fraction α of the negative effect of its investment on U's revenue from selling the input to D_2. In turn, D_2 invests less because it now pays U a higher input price.

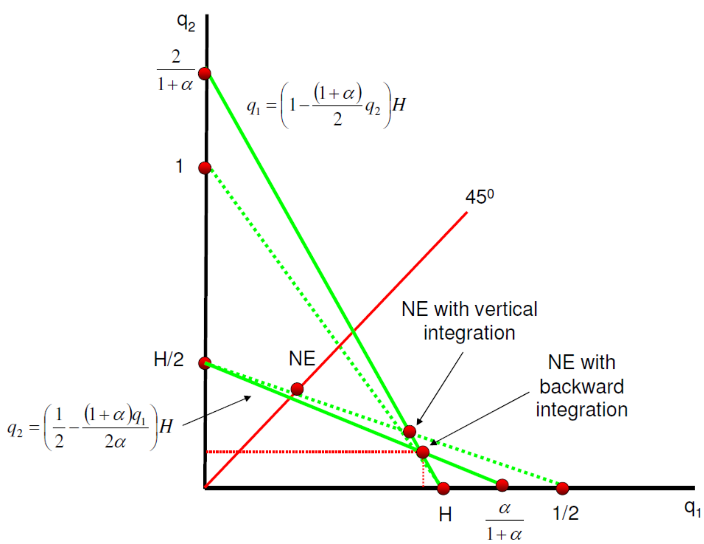

Figure 4: The interior Nash equilibrium investments under partial backward integration

Solving (11) and (12), the equilibrium levels of investment are

$$q_1^{BI}=\begin{cases}\frac{\alpha H(4-(1+\alpha)H)}{4\alpha-(1+\alpha)^2H^2} & \text{if} \quad H<\frac{\alpha}{1+\alpha},\\ H & \text{if} \quad H\geq\frac{\alpha}{1+\alpha},\end{cases} \tag{13}$$

and

$$q_2^{BI} = \begin{cases} \frac{2H(\alpha-(1+\alpha)H)}{4\alpha-(1+\alpha)^2H^2} & \text{if} \quad H < \frac{\alpha}{1+\alpha}, \\ 0 & \text{if} \quad H \geq \frac{\alpha}{1+\alpha}. \end{cases} \tag{14}$$

Note that when $\alpha = 1$, the equilibrium under backward integration coincides with the equilibrium under full integration. In the next proposition, I examine what happens when $\alpha < 1$ (backward integration becomes "more partial").

Proposition 3: *Suppose that D_1 acquires a controlling stake α in U. Then, a decrease in α below 1 leads to*

(i) *more investment by D_1, less investment by D_2, and a higher $\phi_2^{BI} \equiv q_1^{BI}\left(1-q_2^{BI}\right)$ (relative to full integration, D_2 is more likely to be foreclosed in the downstream market);*

(ii) *a lower consumer surplus in an interior Nash equilibria.*

Part (i) of Proposition 3 is obvious from Figure 4. Since q_1^{BI} increases and q_2^{BI} decreases as α gets lower, and since $\phi_2^{BI} = \phi_2^{VI}$ when $\alpha = 1$, it follows that $\phi_2^{BI} > \phi_2^{VI}$ for all $\alpha < 1$: D_2 is more likely to be foreclosed when D_1 has only a partial controlling stake in U. Recalling that $q_1^{VI} > q^{fb} > q_2^{VI}$, the fact that $q_1^{BI} > q_1^{VI}$ and $q_2^{BI} < q_2^{VI}$ implies that under partial backward integration, there is more overinvestment by D_1 and more underinvestment by D_2 relative to the first best than under full vertical integration.

Part (ii) of Proposition 3 implies that partial backward integration harms consumers more than full vertical integration. The reason is that the decrease in q_2^{BI} has a bigger effect on the probability that consumers will enjoy a high quality product at a relatively low price than the increase in q_1^{BI}.

The next step is to examine D_1's incentive to acquire a controlling stake $\alpha < 1$ in U. To address this question, I will assume that initially, U is controlled by a single shareholder, whose equity stake is γ. Suppose that D_1 offers a price T to U's controlling shareholder for an equity stake $\alpha \leq \gamma$ in U. The offer is accepted if it increases the payoff of U's controlling shareholder relative to no integration, i.e., if

$$(\gamma - \alpha)\,\pi_U^{BI} + T \geq \gamma\pi_U^*.$$

D_1's controlling shareholder would find it profitable to make this offer only if his stake, γ_1, in D_1's profit, π_1^{BI}, plus D_1's share in U's profit, π_U^{BI}, minus the payment T, exceeds his share in D_1's profit absent integration, i.e., only if

$$\gamma_1\left(\pi_1^{BI}+\alpha\pi_U^{BI}-T\right)\geq\gamma_1\pi_1^*.$$

The two inequalities can both hold only if

$$\pi_1^{BI}-\pi_1^*\geq\gamma\left(\pi_U^*-\pi_U^{BI}\right),\tag{15}$$

where $\pi_1^{BI}-\pi_1^*$ is the downstream gain from partial backward integration, and $\gamma\left(\pi_U^*-\pi_U^{BI}\right)$ is the decrease in the value of the stake that U's controlling shareholder has in U.[12] Notice that α, which is the actual acquired share, affects matters only through its effect on π_1^{BI} and π_U^{BI}, but it does not affect matters directly. This is because D_1 needs to compensate U's controlling shareholder not only for the shares it sells, but also for the drop in the value of its remaining shares.

When D_1 controls U with a partial ownership stake, it obviously wishes to set w_1 low in order to divert funds from the minority shareholders of U to itself. This incentive, however, exists even if D_1 were a monopoly in the downstream market and is independent of the main issue that I address here. I will therefore shut down this effect by assuming that under partial backward integration, w_1 remains equal to its value absent integration. This assumption ensures that backward introgression is not driven by D_1's desire to exploit the minority shareholders of U.[13]

Proposition 4: *Suppose that U is initially controlled by a single shareholder, whose equity stake is γ, and suppose that D_1 offers to acquire an equity stake $\alpha\leq\gamma$ from U's controlling shareholder for a price T. Then,*

(i) *acquiring the entire stake γ is always profitable for D_1;*

(ii) *partial backward integration always harms the minority shareholders of U if $q_2^{BI}>0$ and it also harms the minority shareholders of U if $q_2^{BI}=0$ provided that $\underline{V}$ is not too large;*

(iii) *if $\gamma>\frac{H}{1-H}$ (in which case $H<\frac{\gamma}{1+\gamma}$, so $q_2^{BI}>0$), then D_1 may prefer to acquire less than the entire controlling stake of U's initial controlling shareholder if H is sufficiently small or γ is sufficiently close to $\frac{H}{1-H}$;*

(iv) *if $\gamma\leq\frac{H}{1-H}$ (in which case $H\geq\frac{\gamma}{1+\gamma}$, so $q_2^{BI}=0$), then D_1 prefers to acquire the smallest equity stake in U, subject to gaining control over U.*

[12] The proof of Proposition 4 below establishes that $\pi_1^{BI}-\pi_1^*>0$, and provided that $\underline{V}$ is not too high, $\pi_U^*-\pi_U^{BI}>0$.

[13] The following analysis then understates the incentive to backward integrate since it abstarcts from the ability of D_1 to lower the price that it pays for the input.

So far I examined cases in which partial backward integration gives D_1 full control over U. I now consider the opposite extreme in which D_1 acquires a passive stake $\alpha < 1$ in U, which gives it cash flow rights, but no say in how U prices its input. For concreteness, I will refer to this case as "passive backward integration," and will refer to the case where D_1 gains control over U as "controlling backward integration." This analysis is important because many antitrust authorities, e.g., the European Commission (EC), do not have the tools to deal with passive acquisitions, which reflects the belief that passive acquisitions do not harm competition. Currently however, the EC considers the extension of its Merger Regulation to allow it to intervene in some acquisitions of non-controlling minority shareholdings (see European Commission, 2013).

Under passive backward integration, D_1 has no influence over U's decisions, so $w_2 = w^*$. Consequently, D_2's problem is exactly as in the non-integration case and hence its best-response function is given by (3). Given that D_1 gets a fraction α of U's profit, its expected profit under passive backward integration is given by

$$\begin{aligned}
\pi_1 &= \underbrace{q_1(1-q_2)\Delta - w_1^* + \overline{R} - c - \frac{kq_1^2}{2}}_{D_1\text{'s profit}} + \underbrace{\alpha\left(w_1^* + w_2^* - 2c\right)}_{U\text{'s profit}} \\
&= \frac{q_1(1-q_2)\Delta + \overline{R} - c}{2} + \alpha\left(\frac{q_1(1-q_2)\Delta + \overline{R} + c}{2} + \frac{q_2(1-q_1)\Delta + \overline{R} + c}{2} - 2c\right) - \frac{kq_1^2}{2}.
\end{aligned}$$

The best-response function of D_1 is now defined by the following first-order condition:

$$\begin{aligned}
\pi_1' &= \frac{(1-q_2)\Delta}{2} + \alpha\left(\frac{(1-q_2)\Delta}{2} - \frac{q_2\Delta}{2}\right) - kq_1 = 0, \qquad (16)\\
\Rightarrow &\qquad q_1 = \left(\frac{(1+\alpha) - (1+2\alpha)q_2}{2}\right)H.
\end{aligned}$$

The top line in (16) is similar to D_1's best-response function under non integration (equation (3)), except for the second term, which captures the effect of q_1 on D_1's share in U's profit. An increase in q_1 has a positive effect on w_1 and a negative effect on w_2. That is, D_1's passive stake in U allows D_1 to (partially) internalize the positive vertical externality of q_1 on U's profit and the negative horizontal externality on D_2's profit, which U (partially) internalize through w_2. The top line in equation (16) shows that the positive vertical externality on U's profit outweighs the negative horizontal externality on D_2's profit if and only if $q_2 < 1/2$. Consequently, as Figure 5 below shows, the best-response function of D_1 shifts outward relative to the non-integration case for $q_2 < 1/2$ and inward for $q_2 > 1/2$. Notice from (3) that $q_2 < \frac{H}{2} < 1/2$, where the last inequality follows since $H < 1$. This implies that in the relevant range, the best-response function of D_1 shifts outward, so

q_1 will be higher than in the non-integration case, and since investments are strategic substitutes, q_2 will be lower than in the non-integration case.

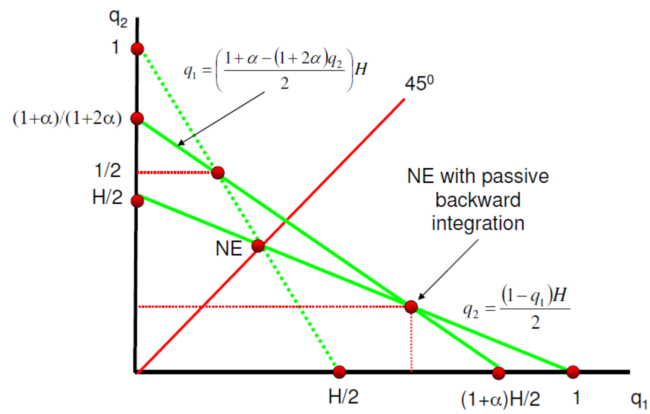

Figure 5: The Nash equilibrium investments under passive backward integration

Solving equations (16) and (3), the equilibrium levels of investment are

$$q_1^{BIpass} = \frac{H\left(2\left(1+\alpha\right)-\left(1+2\alpha\right)H\right)}{4-\left(1+2\alpha\right)H^2}, \qquad q_2^{BIpass} = \frac{H\left(2-\left(1+\alpha\right)H\right)}{4-\left(1+2\alpha\right)H^2}. \tag{17}$$

Since $\alpha \leq 1$ and $H < 1$, the equilibrium is interior, so $q_1^{BIpass} > 0$ and $q_2^{BIpass} > 0$.

Proposition 5: *Suppose that D_1 acquires a passive stake α in U. Then, in an interior equilibrium:*

(i) *a decrease in α below 1 leads to less investment by D_1, more investment by D_2, and a lower $\phi_2 \equiv q_1(1-q_2)$;*

(ii) *compared to controlling backward integration, D_1 invests less, D_2 invests more, and ϕ_2 is lower (D_2 is less likely to be foreclosed in the downstream market when D_1's stake in U is passive);*

(iii) *holding α constant, consumer surplus is higher when D_1's stake in U is passive if H is sufficiently large, but lower when H is small;*

(iv) *consumer surplus is higher than in the non-integration case.*

Figure 6 illustrates the investment levels under non integration, controlling backward integration, and passive backward integration.[14] The investment levels under full integration are equal to q_1^{BI} and q_2^{BI} when $\alpha = 1$. Since $q_1^{BIpass} < q_1^{BI}$ and $q_2^{BIpass} > q_2^{BI}$ at $\alpha = 1$, D_2 is less likely to be foreclosed under passive backward integration than under full vertical integration. Figure 6 illustrates Part (i) of Proposition 5 by showing that α has an opposite effect on investment under controlling and passive ownership. The figure also illustrates part (ii) of the proposition: D_1 invests more, while D_2 invests less when D_1 has a controlling, rather than a passive, stake in U.

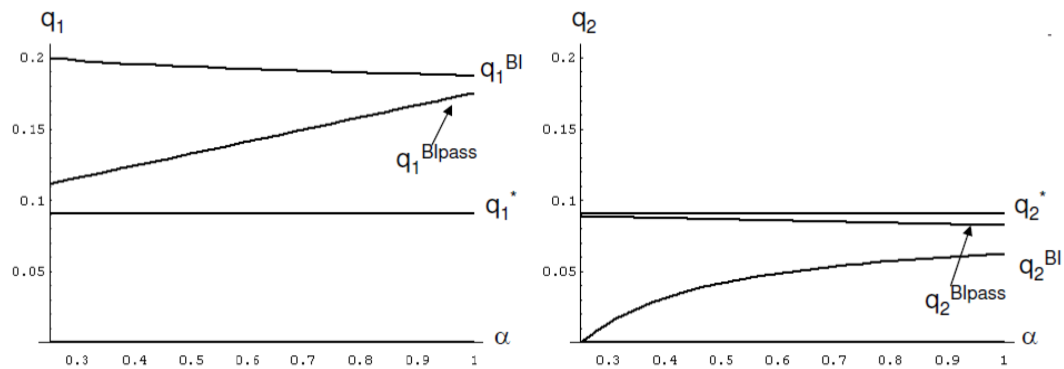

Figure 6: The investments in interior equilibria under non integration, controlling backward integration, and passive backward integartion, as functions of α

Part (iii) of Proposition 5 is illustrated in Figure 7. The figure shows that controlling backward integration is better for consumers than passive backward integration when H is low, and conversely when H is high. The figure also shows that controlling backward integration is particularly likely to be better for consumers when α is intermediate. These results suggest that there is no reason to treat passive acquisitions in vertically related firms more leniently than controlling acquisitions, as many antitrust authorities currently do.

[14] The figure is drawn under the assumtion that $H = \frac{1}{5}$. For this value of H, there are interior equilibria under controlling backward integration only when $\alpha > \frac{1}{4}$.

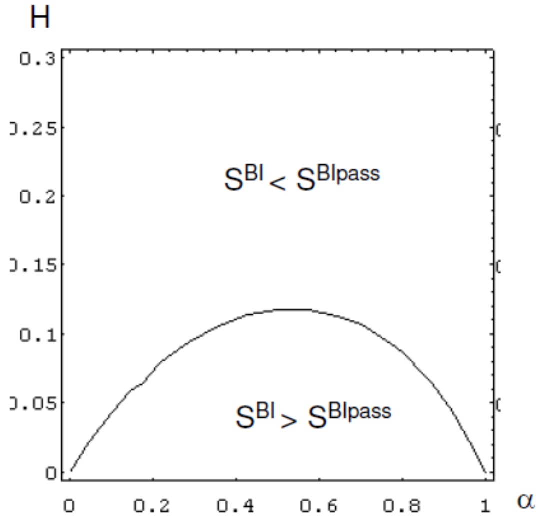

Figure 7: The difference between consumer surplus in an interior Nash equilibrium, under passive backward integration, S^{BIpass}, and under controlling backward integration, S^{BI}

Having examined the two polar cases of controlling and passive backward integration, one may now wonder what happens in intermediate cases, in which partial backward integration gives D_1 some, but not full, control over U. There is no generally agreed upon way to model partial control. One possible way to model this situation is to assume that with probability α, U's decisions are made by D_1 (as in the full control case), and with probability $1-\alpha$, they are made by U's remaining shareholders (as in the passive ownership case). Since I already assumed that w_1 is the same regardless of whether D_1 does or does not control U, partial control will only affect w_2. Given my assumption about U's decisions, the expected price that D_2 will pay for the input is

$$\begin{aligned} \alpha w_2^{BI} + (1-\alpha)\, w_2^* &= \alpha \left[\frac{q_2 (1-q_1)\, \Delta + \overline{R}}{2} + \frac{q_1 q_2 \Delta + \alpha c + \underline{V}}{2\alpha} \right] + (1-\alpha) \left[\frac{q_2 (1-q_1)\, \Delta + \overline{R} + c}{2} \right] \\ &= \frac{q_2 \Delta + \overline{R} + \underline{V} + c}{2} = w_2^{VI}, \end{aligned}$$

exactly as in the full integration case. The implication is that D_2's best-response function is given by (7). Since I assumed that under partial integration, w_1 is as in the non-integration case, D_1's best-response function is still given by (11). The equilibrium outcome is therefore a hybrid of the full integration and the partial backward integration cases. In particular, it is more favorable to D_2 than the equilibrium under partial backward integration considered above.

5.2 Partial forward integration by U

Now, assume that U_1 acquires a stake $\alpha < 1$ in D_1. As in the partial backward integration case, I will begin by assuming that α gives D_1 full control over U. Towards the end of the section, I will also consider the case where α is a passive stake. As before, the equilibrium prices and downstream revenues are given by Table 1.

Since U gets the full upstream profit, but only part of the downstream profit of D_1, it will prefer to charge D_1 a high input price and thereby divert funds from the minority shareholders of D_1 to itself. As in the partial backward integration case, I will shut down this effect by assuming that w_1 remains equal to its value absent integration. Moving to the bargaining between U and D_2 over w_2, when D_2 makes a take-it-or-leave-it offer, its offer leaves U indifferent between selling to D_2 at w_2 and refusing to sell to D_2:

$$\underbrace{w_1 - c}_{\substack{U\text{'s profit if it} \\ \text{refuses to sell to } D_2}} + \alpha \underbrace{\left(q_1 \overline{V} + (1-q_1)\underline{V} + \overline{R} - w_1\right)}_{\substack{U\text{'s share} \\ \text{in } D_1\text{'s profit}}} = \underbrace{w_1 + w_2 - 2c}_{\substack{U\text{'s profit if it} \\ \text{sells to } D_2}} + \alpha \underbrace{\left(q_1 (1-q_2)\Delta + \overline{R} - w_1\right)}_{\substack{U\text{'s share} \\ \text{in } D_1\text{'s profit}}},$$

$$\Rightarrow \qquad w_2 = \alpha \left(\underline{V} + q_1 q_2 \Delta\right) + c.$$

When U makes a take-it-or-leave-it offer, it offers $w_2 = q_2(1-q_1)\Delta + \overline{R}$, which is equal to the entire expected revenue of D_2. The expected value of w_2 under partial forward integration (denoted FI) is therefore

$$\begin{aligned} w_2^{FI} &= \frac{q_2(1-q_1)\Delta + \overline{R}}{2} + \frac{\alpha(\underline{V} + q_1 q_2 \Delta) + c}{2} \\ &= \frac{q_2(1-(1-\alpha)q_1)\Delta + \alpha\underline{V} + \overline{R} + c}{2}. \end{aligned} \tag{18}$$

Notice that w_2^{FI} increases with α and is equal to w_2^{VI} when $\alpha = 1$ (full integration). Hence, $w_2^{FI} < w_2^{VI}$ for all $\alpha < 1$. The reason for this is that when U owns only part of D_1, it requires only partial compensation for the negative externality that D_2 imposes on D_1. Given w_2^{FI}, the expected profits of U (which now chooses D_1's strategy) and D_2 are

$$\begin{aligned} \pi_1 &= \underbrace{w_1 + w_2^{FI} - 2c}_{U\text{'s profit}} + \alpha \underbrace{\left(q_1(1-q_2)\Delta + \overline{R} - w_1 - \frac{kq_1^2}{2}\right)}_{D_1\text{'s profit}} \\ &= \frac{(q_2 + q_1(2\alpha - (1+\alpha)q_2))\Delta + \alpha\underline{V} + (1+2\alpha)\overline{R} - 3c}{2} + (1-\alpha)w_1 - \alpha\frac{kq_1^2}{2}. \end{aligned}$$

and

$$\begin{aligned}\pi_2 &= q_2(1-q_1)\Delta + \overline{R} - w_2^{FI} - \frac{kq_2^2}{2} \\ &= \frac{q_2(1-q_1)\Delta - \alpha(\underline{V} + q_1 q_2 \Delta) + \overline{R} - c}{2} - \frac{kq_2^2}{2}.\end{aligned}$$

The equilibrium levels of investment under partial forward integration, q_1^{FI} and q_2^{FI}, are defined by the following first-order conditions:

$$\pi_1' = \left(\alpha - \frac{(1+\alpha)q_2}{2}\right)\Delta - \alpha k q_1 = 0, \qquad q_1 = \left(1 - \frac{(1+\alpha)q_2}{2\alpha}\right)H, \tag{19}$$

and

$$\pi_2' = \frac{(1-(1+\alpha)q_1)\Delta}{2} - kq_2, \qquad q_2 = \left(\frac{1-(1+\alpha)q_1}{2}\right)H. \tag{20}$$

Figure 8 below shows the interior Nash equilibrium, which obtains when $H < \frac{1}{1+\alpha}$ and $\frac{H}{2} < \frac{2\alpha}{1+\alpha}$. When $H > \frac{1}{1+\alpha}$, the best-response function of D_1 lies everywhere above that of D_2, so $q_2 = 0$. When $\frac{H}{2} > \frac{2\alpha}{1+\alpha}$, the opposite happens: now D_2's best-response function lies everywhere above that of D_1, so $q_1 = 0$. The latter situation cannot arise under full integration or under partial backward integration because by assumption, $H < 1$. However, under partial forward integration, when α is sufficiently small, U may prefer to set $q_1 = 0$ in order to eliminate the negative externality that D_1 imposes on D_2 and thereby maximize its profit from dealing with D_2. When this is the case, forward integration leads to a voluntary foreclosure of D_1 by its controller U. To restrict the number of different cases that can arise, I will restrict attention to cases where $\alpha > 1/4$. Then $H < \frac{1}{1+\alpha}$ also implies $\frac{H}{2} < \frac{2\alpha}{1+\alpha}$, so $H < \frac{1}{1+\alpha}$ is sufficient for an interior Nash equilibrium.

Figure 8 shows that relative to full vertical integration, the best-response function of D_1 rotates counterclockwise around its horizontal intercept, while that of D_2 rotates counterclockwise around its vertical intercept. The rotation of D_1's best-response function reflects the fact that U, who now chooses q_1, captures only a fraction of D_1's downstream profit, but bears the full negative impact of q_1 on w_2^{FI}. Hence, U has an incentive to restrict q_1. The rotation of D_2's best-response function in turn reflects the fact that under forward integration, D_2 pays a lower price for the input than it does under full vertical integration.

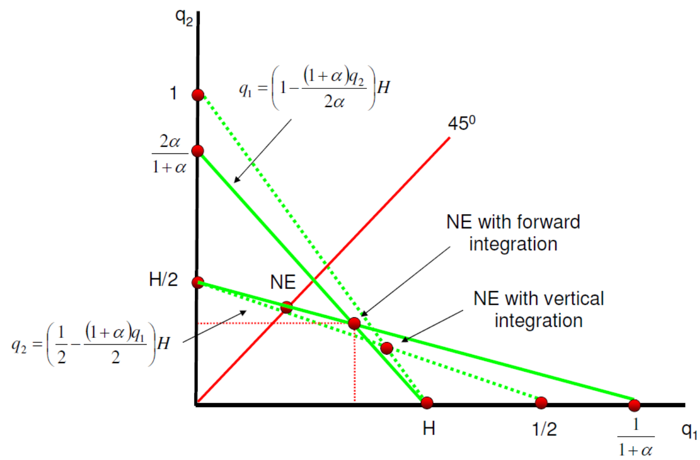

Figure 8: The interior Nash equilibrium investments under partial forward integration

Solving (19) and (20), the equilibrium levels of investment are

$$q_1^{FI} = \begin{cases} \frac{H(4\alpha-(1+\alpha)H)}{4\alpha-(1+\alpha)^2H^2} & \text{if} \quad H < \frac{1}{1+\alpha}, \\ H & \text{if} \quad H \geq \frac{!}{1+\alpha}, \end{cases} \tag{21}$$

and

$$q_2^{FI} = \begin{cases} \frac{2\alpha H(1-(1+\alpha)H)}{4\alpha-(1+\alpha)^2H^2} & \text{if} \quad H < \frac{1}{1+\alpha}, \\ 0 & \text{if} \quad H \geq \frac{1}{1+\alpha}. \end{cases} \tag{22}$$

When $\alpha = 1$, the equilibrium coincides with the equilibrium under full vertical integration. In the next proposition, I examine what happens as α drops below 1 (integration becomes "more partial").

Proposition 6: *Suppose that U acquires a controlling stake α in D_1. Then, a decrease in α below 1 leads to*

(i) *less investment by D_1, more investment by D_2, and a lower $\phi_2^{FI} \equiv q_1^{FI}\left(1 - q_2^{FI}\right)$ (relative to full integration, D_2 is less likely to be foreclosed in the downstream market);*

(ii) *assuming that $\alpha > 1/4$, a higher consumer surplus in an interior equilibrium for sufficiently high α and H.*

Intuitively, under partial forward integration, U internalizes only a fraction of the negative externality that selling the input to D_2 imposes on D_1; hence, holding q_1 fixed, w_2 is lower so $q_2^{FI} > q_2^{VI}$. Since investments are strategic substitutes, this leads to a lower q_1. This effect is compounded by the fact that U has an incentive to restrict q_1, because it captures the full profit from selling the input to D_2, but captures only a fraction of D_1's profits. By restricting q_1, U boosts its profit from selling the input to D_2. Given that $q_1^{FI} < q_1^{VI}$ while $q_2^{FI} > q_2^{VI}$, the probability that D_2 is foreclosed in the downstream market, ϕ_2^{FI}, is lower than under full vertical integration.

Part (ii) of Proposition 6 is illustrated in Figure 9. When $\alpha > 1/4$, an interior solution obtains when $H < \frac{1}{1+\alpha}$. As the figure shows, consumer surplus under partial forward integration, S^{FI}, increases as α decreases when α and H are relatively large. In particular, when $\alpha > 1/2$, S^{FI} increases as α decreases for all values of H for which there exists an interior solution. When $1/4 < \alpha < 1/2$, S^{FI} increases as α decreases only when H is sufficiently large.

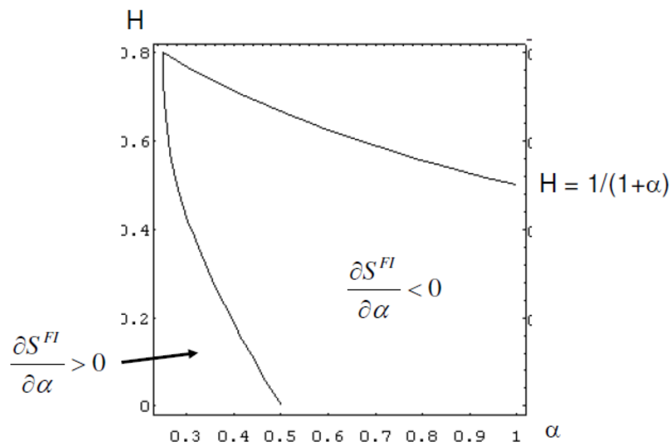

Figure 9: The effect of α on consumer surplus under partial backward integration

The next step is to examine U's incentive to acquire a controlling stake in D_1. To address this question, I will assume that initially, D_1 is controlled by a single shareholder, whose equity stakes is γ_1. Analogously to the backward integration case, U can make an acceptable offer to the initial controlling shareholder of D_1 in return for a controlling equity stake of $\alpha \le \gamma_1$ provided that

$$\gamma_1 \left(\pi_1^{FI} - \pi_1^*\right) \ge \pi_U^* - \pi_U^{FI}, \tag{23}$$

where the right-hand side is the increase in the value of the initial stake that D_1's initial controlling shareholder holds, and the right-hand side is the decrease in U's value. The next proposition is based on (23).

Proposition 7: *Suppose that D_1 is initially controlled by a single shareholder, whose equity stake* *is γ_1 and suppose that U offers to acquire an equity stake $\alpha \leq \gamma_1$ from D_1's controlling shareholder for a price T. Then,*

(i) *acquiring the entire stake γ_1 is profitable for U;*

(ii) *partial forward integration benefits the minority shareholders of D_1.*

(iii) *if $\gamma_1 < \frac{1-H}{H}$ (in which case $H < \frac{1}{1+\gamma_1}$, so $q_2^{BI} > 0$), then U will prefer to acquire a controlling stake $\alpha < \gamma_1$ provided that $\underline{V}$ is not too large;*

(iv) *if $\gamma_1 > \frac{1-H}{H}$ (in which case $H > \frac{1}{1+\gamma_1}$, so $q_2^{BI} = 0$), then acquiring the entire stake γ_1 is profitable.*

I conclude this section by considering the case where U acquires a passive stake, α, in D_1, rather than a controlling stake. This stake does not affect D_1's behavior; hence D_1's best-response function is given by (3), as in the non-integration case. The passive stake of U in D_1, however, does affect the price at which the input is sold to D_2, since now U internalizes the negative externality that D_2 imposes on D_1. The resulting input price is as in the controlling forward integration case. Hence, D_2's best-response function is given by (20). To simplify matters, I will focus on interior equilibria. Solving equations (16) and (3), the (interior) equilibrium levels of investment are

$$q_1^{FIpass} = \frac{H(2-H)}{2-(1+\alpha)H^2}, \qquad q_2^{FIpass} = \frac{H(1-(1+\alpha)H)}{2-(1+\alpha)H^2}. \tag{24}$$

Proposition 8: *Suppose that U acquires a passive stake α in D_1. Then, in an interior equilibrium:*

(i) *a decrease in α below 1 leads to less investment by D_1, more investment by D_2, and a lower $\phi_2 \equiv q_1(1-q_2)$;*

(ii) *compared to controlling forward integration, D_1 invests more, D_2 invest less, and ϕ_2 is higher (D_2 is more likely to be foreclosed in the downstream market);*

(iii) *holding α constant, consumer surplus is higher when D_1's stake in U is passive if H is sufficiently large, but lower when H is small;*

(iv) *consumer surplus is higher than in the non-integtation case when H is sufficiently small but is lower when H is large.*

Part (i) of Proposition 8 shows that α (U's stake in D_1) has the same effect on investments as in the controlling forward integration case. Part (ii) of the proposition shows that D_1 invests more, while D_2 invests less when D_1 has a passive rather than controlling stake in U. Part (iii) of Proposition 8 is illustrated in Figure 10, which shows that passive forward integration is better for consumers than controlling forward integration when H is low, and conversely when H is high.

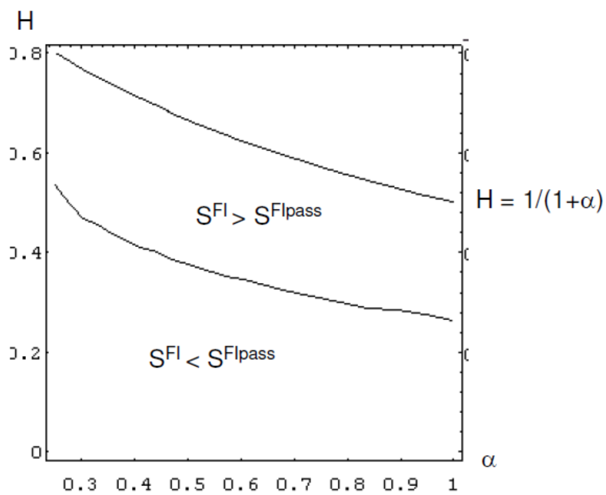

Figure 10: The difference between consumer surplus in an interior Nash equilibrium, under passive forward integration, S^{FIpass}, and controlling forward integration, S^{FI}

6 Related literature

There is a sizeable literature on vertical foreclosure.[15] In this section, I review this literature in order to put my own contribution in context. Admittedly, the literature review is on the long side, but I believe that it is important to understand the different effects of vertical integration that were identified earlier in order to evaluate the contribution of the current paper.

Roughly speaking, there are three main strands of the literature. One strand, pioneered by Ordover, Saloner and Salop (1990) and Salinger (1988), considers models in which the vertically integrated firm deliberately forecloses downstream rivals in order to raise their costs and thereby boost the profits of its own downstream unit. Ordover, Saloner, and Salop (1990) consider a model with two identical upstream firms U_1 and U_2 and two downstream firms D_1 and D_2. Following vertical integration between U_1 and D_1, the merged entity commits not to sell to D_2. As a result,

[15] See Rey and Tirole (2007) and Riordan (2008) for literature surveys.

U_2 becomes the exclusive supplier of D_2, and hence it charges D_2 a higher wholesale price. This makes D_2 softer in the downstream market and boosts D_1's profit.[16] Salinger (1988) obtains a similar result in a successive Cournot oligopoly model, but in his model, vertical integration is also beneficial because it eliminates double marginaliization within the integrated entity.[17] My model differs from these papers in several important respects: first, I consider a model with a single upstream firm. Second, in my model there is a unit demand function for the final product, so there is no double marginalization problem (this allows me to focus on less familiar effects of vertical integration). Third, foreclosure in my model is a by-product of the effect of vertical integration on the incentives of D_1 and D_2 to invest, rather than an outright refusal to sell to non-integrated rivals. In fact, in my model D_2 continues to buy from U even when the latter integrates with D_1.[18]

Building on the logic of the raising rivals' costs argument, Baumol and Ordover (1994) show that partial backward integration can lead to foreclosure even when full vertical integration does not. Specifically, they show that under full integration between a bottleneck owner, B, and one

[16] The assumption that U_1 can commit not to supply D_2 was criticized as being problematic: see Hart and Tirole (1990) and Reiffen (1992), and see Ordover, Salop, and Saloner (1992) for a response. Several papers have proposed models that are immune to this criticism. Ma (1997) shows that when U_1 and U_2 offer differentiated inputs, it is in U_1's interest, once it integrates with D_1, to foreclose D_2. This allows D_1 to monopolize the downstream market. Chen (2001) shows that when D_1 and D_2 can choose which upstream firm to buy from, then once U_1 and D_1 integrate, D_2 will choose to buy from U_1 (even if it charges a higher wholesale price than U_2) because this choice induces D_1 to be softer in the downstream market in order to protect D_2's sales and hence U_1's profits from selling to D_2. This results in a de facto foreclosure of U_2. Choi and Yi (2001) assume that U_1 and U_2 need to choose which input to produce. Absent integration, they choose to produce a generalized input that fits both D_1 and D_2, but once U_1 integrates with D_1, it produces a specialized input that fits only D_1. This de facto foreclosure of D_2 allows U_2 to charge D_2 a higher wholesale price and confers a strategic advantage on D_1 in the downstream market. Church and Gandal (2000) show that vertical integration between a hardware and a software firm may induce the integrated firm make its software incompatible with the hardware of the nonintegrated hardware firm.

[17] This effect does not arise in Ordover, Saloner, and Salop (1990) since they assume that U_1 and U_2 initially engage in Bertrand competition and hence sell the input at marginal cost. Gaudet and Van Long (1996) show that the intergrated firm may in fact wish to buy inputs from nonintegrated upstream suppliers in order to further inflate the wholesale price that nonintegrated downstream rivals pay. Riordan (1998) shows that backward integration by a dominant firm into an upstream competitive industry reduces its monopsonistic power in the upstream market and hence leads to a higher input price. This hurts downstream rivals and leads to a higher retail price in the downstream market. Loertscher and Reisinger (2010) consider a similar model and show that if the downstream firms are Cournot competitors, then, under fairly general conditions, vertical integration is procompetitive because efficiency effects tend to dominate foreclosure effects.

[18] Since U always deals with D_2, my model does not feature a "committment problem."

of several competing downstream firms, V, B will continue to deal with V's downstream rivals, so long as this is efficient. But when V controls B with a partial ownership stake, then V has an incentive to divert business to itself, even if downstream rivals are more efficient. The reason is that while V fully captures the benefits from the diversion in the downstream market, it internalizes only part of the associated loss to B in the upstream market.[19]

A second strand of the literature, due to Hart and Tirole (1990), views foreclosure as an instrument that allows U to extract monopoly profits from the downstream market. Specifically, Hart and Tirole (1990) consider a setting where U faces two competing downstream firms, D_1 and D_2. Ideally, U would like to supply only one downstream firms, say D_1, in order to eliminate competition downstream; U can then use a non linear tariff (say a two-part tariff) to fully extract D_1's resulting monopoly profit. However, D_1 fears that after it accepts the non-linear tariff, U will secretly sell to D_2 and thereby make even more money at D_1's expense. Hart and Tirole show that due to this fear, U cannot make more than the duopoly profit in a non-integrated equilibrium. But if U integrates with D_1, then it can credibly commit not sell with D_2 as such sales erode its downstream profit. Hence, integration leads to a foreclosure of D_2 and to a higher retail price.[20] This theory differs from mine because, as in the first strand of the literature, it also views foreclosure as a deliberate refusal to sell to D_2 in order to boost the downstream profit of D_1.

My paper is closely related to the third strand of the literature, due to Bolton and Whinston (1991, 1993). In this strand, foreclosure is a by-product of the effect of vertical integration on the incentives of downstream firms to invest, rather than a deliberate refusal to supply downstream rivals. Bolton and Whinston consider a setting with one upstream firm, U, and two downstream

[19] Reiffen (1998) builds on this logic and examines the stock market reaction to Union Pacific (UP) Railroad's attempt in 1995 to convert a 30% nonvoting stake in Chicago Northwestern (CNW) Railroad to voting shares. A group of competing railroads argued that since the remaining 70% of CNW's shares were held by dispersed shareholders, UP would gain effective control over CNW and would use it to foreclose them from some of CNW's transportation routes. Reiffen finds however that CNW's stock price reacted positively, rather than negatively, to events that made the merger more likely to take place. This is inconsistent with the idea that UP would have diverted profits from CNW to itself by foreclosing competing railroads.

[20] Baake, Kamecke, and Normann (2003), consider a related model in which U faces $n \geq 2$ downstream rivals and needs to make a cost-reducing investment before offering contracts to the downstream firms. They show that vertical integration between U and one of the downstream firms leads to downstream foreclosure, which is ex post inefficient, but it induces U to invest efficiently ex ante. Vertical integration is welfare enhancing in their model when n is suficiently large. White (2007) shows that when U's cost is private information, U has a strong incentive to signal to D_1 and D_2 that its cost is high (and consequently that sales to the rival is limited) by cutting its output below the monopoly level. Vertical integratation restores the monopoly output and hence is welfare enhancing.

firms, D_1 and D_2, which do not compete with each other downstream. Rather, with some probability, there is excess demand for the input, so D_1 and D_2 compete for a limited input supply. The two firms invest ex ante in order to boost their profits from using the upstream input. Following integration between U and D_1, D_1 internalizes the externality of its investment on U's profit and hence it invests more. Since investments are strategic substitutes, D_2 invests less. In equilibrium then, D_2 is less likely to buy the input whenever there is supply shortage. My model builds on Bolton and Whinston, but unlike in their model , there is no supply shortage in my model, and the strategic interaction between D_1 and D_2 arises because the two firms compete in the downstream market. Moreover, integration in my model affects the wholesale price that D_2 pays and hence creates a new effects that are not present in Bolton and Whinston.

Similarly to my model, Allain, Chambolle, and Rey (2010) also consider two competing downstream firms which first make value-enhancing investments and then buy an input. However, unlike in my model, there are two upstream suppliers in their model and moreover, each downstream firm must share technical information with its upstream supplier. As a result, D_2 may be reluctant to deal with U_1 when the latter is integrated with D_1, because U_1 may leak some of D_2's technical information to D_1 and thereby diminish D_2's potential advantage in the downstream market. The result is a de facto foreclosure of D_2, which weakens its incentive to invest; consequently, vertical integration harms consumers. In my model by contrast, the associated increase in D_1's investment may more than compensate consumers for the decrease in D_2's investment.

There is some empirical evidence for the foreclosure effect of vertical mergers. Waterman and Weiss (1996) find that relative to average non-integrated cable TV systems, cable systems owned by Viacom and ATC (the two major cable networks that had majority ownership ties in the four major pay networks, Showtime and the Movie Channel (Viacom) and HBO and Cinemax (ATC)) tend to (i) carry their affiliated networks more frequently and their rival networks less frequently, (ii) offer fewer pay networks in total, (iii) "favor" their affiliated networks in terms of pricing or other marketing behavior. Chipty (2001) finds that integrated cable TV system operators tend to exclude rival program services, although vertical integration does not seem to harm, and may actually benefit, consumers because of the associated efficiency gains.[21] Hastings and Gilbert

[21] Chen and Waterman (2007) use a 2004 cross-sectional database of digital cable systems in the U.S. and show that the foreclosure effect of vertical ownership ties between systems and programming suppliers persists in spite of extensive channel capacity expansion and new competition from direct broadcast satellites. In particular, they find that integrated cable systems tend to carry their affiliated networks more frequently and carry unaffiliated rival networks less frequently. They also find that integrated systems that do carry rival networks often position them on

(2005) find evidence for vertical foreclosure in the U.S. gasoline distribution industry by showing that a vertically integrated refiner (Tosco) charges higher wholesale prices in cities where it competes more with independent gas stations.

To the best of my knowledge, apart from Baumol and Ordover (1994) and Reiffen (1998), only Greenlee and Raskovitch (2006), Hunold, Röller, and Stahl (2012), and Gilo, Levy, and Spiegel (2013) consider the competitive effects of partial vertical integration. Greenlee and Raskovitch (2006) consider n downstream firms, which hold partial passive ownership stakes in a single upstream supplier, U (the downstream firms share U's profit, but cannot affect its decisions). An increase in the ownership stake of downstream firm D_i in U, means that D_i pays a larger share of the input price to "itself" and hence demands more input. U responds to the increased demand by raising the input's price. In a broad class of homogeneous Cournot and symmetrically differentiated Bertrand settings, the two effects cancel each other out, so aggregate output and consumer surplus remain unaffected. In my model by contrast, partial backward integration affects consumers in general because it changes the incentives of the downstream firms to invest and therefore the likelihood that consumers will be able to buy a high quality product at a low prices. Hunold, Röller, and Stahl (2012) study a related model but in their setting, U competes with a less efficient supplier, V, whose cost constrains U's wholesale price. As a result, the two effects that Greenlee and Raskovitch identify do not cancel each other anymore. In equilibrium, the passive ownership stakes soften downstream price competition, because each downstream firm internalizes the negative externality of its low price on U's sales to its rival. Gilo, Levy, and Spiegel (2013) examine how the incentive to partially integrate and then foreclose rivals depends on the ownership structure of the target firm. They show that partial backward integration can arise even when full vertical integration is not profitable, especially if the upstream firm is either held by dispersed shareholders or if its controlling shareholder holds a sufficiently small stake in the firm.

Finally, in late 2003, News Corp. (a major owner of TV broadcast stations and programming networks) acquired a 34% stake in Hughes Electronics Corporation, which gave it a de facto control over Hughes's wholly-owned subsidiary DirecTV Holdings, LLC (a direct broadcast satellite service provider). The FCC (2004) argued that News Corp.'s ability to gain programming revenues via its ownership stake in DirecTV would allow it to credibly threaten to temporarily withdraw its content from competing cable TV operators during carriage negotiations and thereby raise the price for its programming. The FCC was concerned that this would harm consumers by leading to higher

digital tiers having more limited subscriber access.

prices for cable TV services.[22]

7 Conclusion

I considered the effect of partial vertical integration on the foreclosure of downstream rivals and on consumer welfare. The analysis shows that partial vertical integration affects both vertical externalities that arise because downstream investments boost the willingness of downstream firms to pay for the input, as well as horizontal externalities that downstream firms impose on each other. Partial vertical integration allows firms to (partially) internalize these externalities and this in turn affects the incentive of the downstream firms to invest. Moreover, when the partial backward integration gives the integrated downstream firm control over the upstream supplier, the price at which the input is sold to the non-integrated downstream firm increases. This is because the integrated downstream firm's share in the upstream supplier's profit must compensate it fully for the negative horizontal externality that the rival imposes on it (otherwise it will use its control over the upstream supplier to refuse to sell to the rival). And, when the partial forward integration gives the upstream supplier control over the integrated downstream firm, the upstream supplier will use its control to curb the investment of the integrated downstream firm to limit the negative horizontal externality that it imposes on the non-integrated downstream firm and hence on its willingness to pay for the input.

From an antitrust perspective, the analysis shows that partial backward integration leads to more foreclosure of non-integrated rivals than full vertical integration, while partial forward integration leads to less foreclosure. Moreover, passive vertical integration (i.e., the acquisition of a passive stake in a supplier or a downstream buyer) always leads to less foreclosure than controlling vertical integration, though it is not necessarily better for consumers than controlling vertica integration.

In general, my analysis suggests that vertical integration may either boost consumers surplus or harm it depending on whether integration is backward or forward, depending on the acquired stake and whether it is controlling or partial, and depending on the benefit of consumers from

[22] The FCC reiterated these concerns in January 2011 when it approved, subject to some conditions, an agreement between Comcast, GE, and NBCU that gives Comcast (the largest cable operator and Internet service provider in the U.S.) a controlling 51% stake (and the right to nominate 3 out of 5 directors) in a joint venture that owns two broadcast television networks (NBC and Telemundo), 26 broadcast television stations, and various cable programming like CNBC, MSNBC, Bravo, and USA Network (see Paragraph 29 in the decision).

investment relative to the cost of investment. These results suggest that antitrust authorities should examine vertical integration cases carefully whether or not integration involves control and examine how integration affects the various externalities that firms impose on each other.

8 Appendix

Following are the analysis of the case where the qualities of the final products of D_1 and D_2 are realized before the two downstream firms buy the input from U, and the proofs of Propositions 1-7.

The case where the two downstream firms buy the input from U after the qualities of their final products are realized. Once V_1 and V_2 are realized, U will sell the input to D_i if $V_i > V_j$, and if $V_1 = V_2$, then U will pick one of the downstream firms at random, say D_i, and will sell it the input. In both cases, the downstream market is monopolized by D_i. When D_i makes a take-it-or-leave-it offer to U, it offers a price equal to $V_j + \overline{R}$, which is U's profit if the bargaining with D_i fails and U makes a take-it-or-leave-it offer to D_j. When U makes a take-it-or-leave-it offer, it offers a price equal to the entire revenue of D_i, which is $V_i + \overline{R}$. The expected price that D_i pays for the input is therefore

$$w_i^*(V_i, V_j) = \frac{V_i + V_j}{2} + \overline{R}.$$

Noting that $w_i^*\left(\overline{V}, \underline{V}\right) = \frac{\overline{V}+\underline{V}}{2} + \overline{R}$, $w_i^*\left(\overline{V}, \overline{V}\right) = \overline{V} + \overline{R}$, and $w_i^*\left(\underline{V}, \underline{V}\right) = \underline{V} + \overline{R}$, the expected profit of D_i is

$$\begin{aligned}\pi_i &= q_i(1-q_j)\left(\overline{V} - w_i^*\left(\overline{V}, \underline{V}\right) + \overline{R}\right) + \frac{q_i q_j}{2}\left(\overline{V} - w_i^*\left(\overline{V}, \overline{V}\right) + \overline{R}\right) \\ &\quad + \frac{(1-q_i)(1-q_j)}{2}\left(\underline{V} - w_i^*\left(\underline{V}, \underline{V}\right) + \overline{R}\right) - \frac{kq_i^2}{2} \\ &= \frac{q_i(1-q_j)\Delta}{2} - \frac{kq_i^2}{2}.\end{aligned}$$

The resulting best-response functions of D_1 and D_2 are defined by (3) and the equilibrium levels of investment are given by (4), exactly as in the main text.

Now, suppose that D_1 and U fully merge. The merged entity, VI, will deal with D_2 only if $V_2 = \overline{V}$ and $V_1 = \underline{V}$. When D_2 makes a take-it-or-leave-it offer, it offers an input price, $w = \underline{V} + \overline{R}$, that leaves VI indifferent between selling to D_2 and foreclosing it. When VI makes a take-it-or-leave-it offer, it offers $\overline{V} + \overline{R}$, which is equal to the entire revenue of D_2. The expected input price that D_2 will pay U is therefore $w_2^{VI} = w_i^*\left(\overline{V}, \underline{V}\right) = \frac{\overline{V}+\underline{V}}{2} + \overline{R}$. Consequently, the expected profits of VI and D_2 are

$$\begin{aligned}\pi_{VI} &= \underbrace{q_1\left(\overline{V}+\overline{R}\right)+(1-q_1)(1-q_2)\left(\underline{V}+\overline{R}\right)-\frac{kq_1^2}{2}}_{\text{Downstream profit}}+\underbrace{q_2(1-q_1)w_2^{VI}-c}_{\text{Upstream profit}}\\ &= q_1\overline{V}+(1-q_1)(1-q_2)\underline{V}+q_2(1-q_1)\left(\frac{\overline{V}+\underline{V}}{2}\right)+\overline{R}-c-\frac{kq_1^2}{2},\end{aligned}$$

and

$$\begin{aligned}\pi_2 &= q_2(1-q_1)\left(\overline{V}-w_2^{VI}+\overline{R}\right)-\frac{kq_2^2}{2}\\ &= \frac{q_2(1-q_1)\Delta}{2}-\frac{kq_2^2}{2}.\end{aligned}$$

Recalling that $H \equiv \frac{\Delta}{k}$, the equilibrium investment levels under vertical integration, q_1^{VI} and q_2^{VI}, are defined by the following pair of first-order conditions:

$$\pi_{VI}' = (1-q_2)\Delta - kq_1 = 0, \qquad \Rightarrow \qquad q_1 = (1-q_2)H$$

and

$$\pi_2' = \frac{(1-q_1)\Delta}{2} - kq_2 = 0, \qquad \Rightarrow \qquad q_2 = \frac{(1-q_1)H}{2}.$$

Solving (6) and (7), the equilibrium levels of investment are

$$q_1^{VI} = \frac{H(2-H)}{2-H^2}, \qquad q_2^{VI} = \frac{H(1-H)}{2-H^2}.$$

Relative to the equilibrium in the main text, given by equation (8) and (9), now q_1^{VI} is smaller and q_2^{VI} is higher. The reason is that in the main text, an increase in q_2 leads to an increase in w_2^{VI}, because D_2 needs to compensate VI for the negative externality of D_2's investment on D_1 profit. In the current setup, where w_2^{VI} is determined after investment was realized, VI and D_2 contract only after q_2 is sunk and hence w_2^{VI} is independent of q_2, which implies that D_2 invests more. Since investments are strategic substitutes, D_1 invests less. Still, vertical integration allows D_1 to fully internalize the positive externality of its investment on U's profit, so q_1 is higher than under non integration; since investments are strategic substitutes, q_2 is lower than under non integration.

In sum, reversing the order of stages 2 and 3 in my model preserves the effect of integration on the positive externality of D_1's investment on U's profit, but it eliminates the effect of the negative externality of D_2's investment on D_1's downstream profit. ■

Proof of Proposition 1: Absent vertical integration, the joint profit of D_1 and U is

$$\pi_1^* + \pi_U^* = \underbrace{\frac{q_1^*\,(1-q_2^*)\,\Delta + \overline{R} - c}{2} - \frac{k\,(q_1^*)^2}{2}}_{\pi_1} + \underbrace{\frac{q_1^*\,(1-q_2^*)\,\Delta + \overline{R} + c}{2} + \frac{q_2^*\,(1-q_1^*)\,\Delta + \overline{R} + c}{2} - 2c}_{\pi_U}. \tag{25}$$

Substituting for q_1^* and q_2^* from (4) into (25) and simplifying,

$$\pi_1^* + \pi_U^* = \frac{5kH^2}{2\,(H+2)^2} + \frac{3\left(\overline{R}-c\right)}{2}. \tag{26}$$

On the other hand, substituting q_1^{VI} and q_2^{VI} into π_{VI} and rearranging, the profit of the vertically integrated firm is

$$\pi_{VI} = \begin{cases} \frac{kH^2\left(6-8H-H^2+4H^3\right)}{8(1-H^2)^2} + \frac{3\left(\overline{R}-c\right)+\underline{V}}{2} & \text{if} \quad H < \frac{1}{2}, \\ \frac{kH^2}{2} + \frac{3\left(\overline{R}-c\right)+\underline{V}}{2} & \text{if} \quad H \geq \frac{1}{2}. \end{cases}$$

Comparing the two expressions reveals that,

$$\pi_{VI} - (\pi_1 + \pi_U) = \begin{cases} \frac{kH^2\left[4(1-2H)+5H^2\left(2-H^2\right)+4H^3\left(1+H^2\right)\right]}{8(1-H^2)^2(H+2)^2} + \frac{\underline{V}}{2} & \text{if} \quad H < \frac{1}{2}, \\ \frac{kH^2\left(H^2+4H-1\right)}{2(H+2)^2} + \frac{\underline{V}}{2} & \text{if} \quad H \geq \frac{1}{2}. \end{cases}$$

The sign of the expression in the top line of the equation is positive given that $H < 1/2$. The expression in the bottom line is also positive since $H \geq 1/2$. Altogether then, vertical integration is profitable for U and for D_1. ■

Proof of Proposition 2: Substituting q_1^* and q_2^* from (4) into (10) yields

$$S^* = \underline{V} + \frac{kH^3}{(H+2)^2}.$$

Substituting q_1^{VI} and q_2^{VI} from (8) and (9) into (10) yields

$$S^{VI} = \begin{cases} \underline{V} + \frac{kH^3(2-H)(1-2H)}{4(1-H^2)^2} & \text{if} \quad H < \frac{1}{2}, \\ \underline{V} & \text{if} \quad H \geq \frac{1}{2}. \end{cases}$$

Now,

$$S^{VI} - S^* = \begin{cases} \frac{kH^3\left(4-12H-2H^2+3H^3-2H^4\right)}{4(1-H^2)^2(H+2)^2} & \text{if} \quad H < \frac{1}{2}, \\ -\frac{kH^3}{(H+2)^2} & \text{if} \quad H \geq \frac{1}{2}. \end{cases} \tag{27}$$

The numerator of the top line in (27) is decreasing with H, and is positive when $H < 0.323$ and negative otherwise. Hence, $S^{VI} > S^*$ for all $H < 0.323$ and $S^{VI} < S^*$ for all $H > 0.323$. ■

Proof of Proposition 3: Recalling that $H < 1$ and $\alpha < 1$,

$$\begin{aligned}
\frac{\partial q_1^{BI}}{\partial \alpha} &= \frac{H^2\left((1+\alpha)^2 H^2 - 4\left(\alpha^2 + \left(1-\alpha^2\right) H\right)\right)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} \\
&< \frac{H^2\left((1+\alpha)^2 H^2 - 4H\left(\alpha^2 H + \left(1-\alpha^2\right) H\right)\right)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} \\
&= \frac{H^4\left((1+\alpha)^2 - 4\right)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} < 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial q_2^{BI}}{\partial \alpha} &= \frac{2H^2\left(\left(4 - \left(1-\alpha^2\right) H\right) - (1+\alpha)^2 H^2\right)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} \\
&> \frac{2H^2\left(H\left(4H - \left(1-\alpha^2\right) H\right) - (1+\alpha)^2 H^2\right)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} \\
&= \frac{4H^4 (1-\alpha)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} > 0.
\end{aligned}$$

Hence, when α falls below 1, q_1^{BI} increases, q_2^{BI} decreases, and $\phi_2^{BI} \equiv q_1^{BI}\left(1 - q_2^{BI}\right)$ increases. Since $\phi_2^{BI} = \phi_2^{VI}$ when $\alpha = 1$, it follows that $\phi_2^{BI} > \phi_2^{VI}$ for all $\alpha < 1$: D_2 is foreclosed more often when D_1 and U only partially integrate.

Substituting q_1^{BI} and q_2^{BI} in (10), consumer surplus under partial backward integration is

$$S^{BI} = \begin{cases} \underline{V} + \frac{2\alpha k H^3 (4-(1+\alpha)H)(\alpha-(1+\alpha)H)}{\left(4\alpha-(1+\alpha)^2 H^2\right)^2} & \text{if} \quad H < \frac{\alpha}{1+\alpha}, \\ \underline{V} & \text{if} \quad H \geq \frac{\alpha}{1+\alpha}. \end{cases}$$

Now, when $H < \frac{\alpha}{1+\alpha}$

$$\frac{\partial S^{BI}}{\partial \alpha} = \frac{2kH^4\left[(1+\alpha)^2 H^2\left(4 - 6\alpha - \alpha^2 - \left(1-\alpha^2\right) H\right) + 4\alpha\left(4 - \alpha^2 - 3\left(1-\alpha^2\right) H\right)\right]}{\left(4\alpha - (1+\alpha)^2 H^2\right)^3}.$$

The denominator of $\frac{\partial S^{BI}}{\partial \alpha}$ is positive since $H < \frac{\alpha}{1+\alpha}$ implies

$$4\alpha - (1+\alpha)^2 H^2 > 4\alpha - \alpha^2 = \alpha(4-\alpha) > 0.$$

As for the numerator of $\frac{\partial S^{BI}}{\partial \alpha}$, note that since $H < \frac{\alpha}{1+\alpha}$,

$$\begin{aligned}
& (1+\alpha)^2 H^2 \left(4 - 6\alpha - \alpha^2 - \left(1-\alpha^2\right) H\right) + 4\alpha \left(4 - \alpha^2 - 3\left(1-\alpha^2\right) H\right) \\
> \; & (1+\alpha)^2 H^2 \left(4 - 6\alpha - \alpha^2 - (1-\alpha)\alpha\right) + 4\alpha \left(4 - \alpha^2 - 3(1-\alpha)\alpha\right) \\
= \; & (1+\alpha)^2 H^2 (4 - 7\alpha) + 4\alpha \left(4 - 3\alpha + 2\alpha^2\right) \\
> \; & (1+\alpha)^2 H^2 (4 - 7\alpha) + 4\alpha \left(\frac{(1+\alpha) H}{\alpha}\right)^2 \left(4 - 3\alpha + 2\alpha^2\right) \\
= \; & \frac{(1+\alpha)^2 (4-\alpha)^2 H^2}{\alpha} > 0.
\end{aligned}$$

Hence, $\frac{\partial S^{BI}}{\partial \alpha} > 0$, implying that consumer surplus is lower when α falls below 1. ■

Proof of Proposition 4: To prove part (i) of the proposition, let $\alpha = \gamma$. Then, a straightforward (though tedious) calculations show that when $H < \frac{\alpha}{1+\alpha}$,

$$\begin{aligned}
\pi_1^{BI} - \pi_1^* - \alpha\left(\pi_U^* - \pi_U^{BI}\right) = \; & \frac{kH^2}{2(2+H)^2 \left(4\alpha - (1+\alpha)^2 H^2\right)^2} \times \\
& \left[16\alpha^2 (1 - 2\alpha H) + 4\alpha \left(6 + 3\alpha - 2\alpha^2 + 2\alpha^3\right) H^2 + 4\alpha\left(2 + \alpha + \alpha^3\right) H^3 \right. \\
& \left. - (1+\alpha)^2 \left(3 + 2\alpha^3\right) H^4 + 2\alpha (1+\alpha)^3 H^5\right] + \frac{V}{2}.
\end{aligned}$$

Since $H < 1$,

$$\begin{aligned}
& 16\alpha^2 (1 - 2\alpha H) + 4\alpha \left(6 + 3\alpha - 2\alpha^2 + 2\alpha^3\right) H^2 + 4\alpha\left(2 + \alpha + \alpha^3\right) H^3 \\
& - (1+\alpha)^2 \left(3 + 2\alpha^3\right) H^4 + 2\alpha (1-\alpha)(1+\alpha)^3 H^5 \\
> \; & 16\alpha^2 (1 - 2\alpha H) + 4\alpha \left(6 + 3\alpha - 2\alpha^2 + 2\alpha^3\right) H^3 + 4\alpha\left(2 + \alpha + \alpha^3\right) H^3 \\
& - (1+\alpha)^2 \left(3 + 2\alpha^3\right) H^3 + 2\alpha (1+\alpha)^3 H^5 \\
= \; & 16\alpha^2 (1 - 2\alpha H) - \left(3 - 26\alpha - 13\alpha^2 + 10\alpha^3 - 12\alpha^4 + 2\alpha^5\right) H^3 + 2\alpha (1+\alpha)^3 H^5.
\end{aligned} \tag{28}$$

Moreover, using the fact that $H < \frac{\alpha}{1+\alpha}$, it follows that $1 - 2\alpha H > 1 - 2\alpha\left(\frac{\alpha}{1+\alpha}\right) = \frac{16\alpha^2(1-\alpha)(1+2\alpha)}{1+\alpha} > 0$. Hence, if the coefficient of H^3 in the last line of (28) is positive, then $\pi_1^{BI} - \pi_1^* > \alpha\left(\pi_U^* - \pi_U^{BI}\right)$. If the coefficient of H^3 in (28) is negative, then given that $H < \frac{\alpha}{1+\alpha}$,

$$\begin{aligned}
& 16\alpha^2 (1 - 2\alpha H) - \left(3 - 26\alpha - 13\alpha^2 + 10\alpha^3 - 12\alpha^4 + 2\alpha^5\right) H^3 + 2\alpha (1+\alpha)^3 H^5 \\
> \; & 16\alpha^2 \left(1 - 2\alpha\left(\frac{\alpha}{1+\alpha}\right)\right) - \left(3 - 26\alpha - 13\alpha^2 + 10\alpha^3 - 12\alpha^4 + 2\alpha^5\right)\left(\frac{\alpha}{1+\alpha}\right)^3 + 2\alpha(1+\alpha)^3 H^5 \\
= \; & \frac{\alpha^2 \left(16 + 45\alpha + 42\alpha^2 - 35\alpha^3 - 42\alpha^4 + 12\alpha^5 - 2\alpha^6\right)}{(1+\alpha)^3} + 2\alpha (1+\alpha)^3 H^5 > 0.
\end{aligned}$$

Hence, once again, $\pi_1^{BI} - \pi_1^* > \alpha\left(\pi_U^* - \pi_U^{BI}\right)$.

Next, suppose that $H > \frac{\alpha}{1+\alpha}$. Then, $q_2^{BI} = H$ and $q_2^{BI} = 0$, and hence

$$\begin{aligned}
\pi_1^{BI} - \pi_1^* - \alpha\left(\pi_U^* - \pi_U^{BI}\right) &= \frac{H^2\left(1 - 2\alpha + 4H + H^2\right)}{2\left(2+H\right)^2} + \frac{\underline{V}}{2} \\
&> \frac{H^2\left(1 - 2\alpha + 4\left(\frac{\alpha}{1+\alpha}\right) + \left(\frac{\alpha}{1+\alpha}\right)^2\right)}{2\left(2+H\right)^2} + \frac{\underline{V}}{2} \\
&= \frac{H^2\left(1 + 4\alpha + 2\alpha^2 - 2\alpha^3\right)}{2\left(1+\alpha\right)^2\left(2+H\right)^2} + \frac{\underline{V}}{2} > 0.
\end{aligned}$$

This completes part (i) of the proof.

As for the minority shareholders of U, if $H < \frac{\alpha}{1+\alpha}$, then the change in their payoff depends on

$$\pi_U^* - \pi_U^{BI} = \frac{kH^3}{\left(2+H\right)^2\left(4\alpha - \left(1+\alpha\right)^2 H^2\right)^2} \times M + \frac{\underline{V}}{2\alpha},$$

where

$$M \equiv 16\alpha^2 + 4\left(4 + \alpha^2 - 2\alpha^3\right)H + 8\left(1 - \alpha - \alpha^2\right)H^2 - \left(3 + 10\alpha + 4\alpha^2 - 4\alpha^3 - \alpha^4\right)H^3 - 2\left(1 + \alpha^2\right)H^4.$$

Since the coefficients of H^3 and H^4 are negative and $H < 1$,

$$\begin{aligned}
M &> 16\alpha^2 + 4\left(4 + \alpha^2 - 2\alpha^3\right)H + 8\left(1 - \alpha - \alpha^2\right)H^2 - \left(3 + 10\alpha + 4\alpha^2 - 4\alpha^3 - \alpha^4\right)H^2 - 2\left(1 + \alpha^2\right)H^2 \\
&= 16\alpha^2 + 4\left(4 + \alpha^2 - 2\alpha^3\right)H + \left(3 - 22\alpha - 14\alpha^2 + 4\alpha^3 + \alpha^4\right)H^2.
\end{aligned}$$

If the coefficient of H^2 in the last line is positive, then the entire expression is positive, so $\pi_U^* - \pi_U^{BI} > 0$. If the coefficient of H^2 is negative, then since $H < \frac{\alpha}{1+\alpha}$,

$$\begin{aligned}
& 16\alpha^2 + 4\left(4 + \alpha^2 - 2\alpha^3\right)H + \left(3 - 22\alpha - 14\alpha^2 + 4\alpha^3 + \alpha^4\right)H^2 \\
> \; & 16\alpha^2 + 4\left(4 + \alpha^2 - 2\alpha^3\right)H + \left(3 - 22\alpha - 14\alpha^2 + 4\alpha^3 + \alpha^4\right)\left(\frac{\alpha}{1+\alpha}\right)^2 \\
= \; & \frac{\alpha}{\left(1+\alpha\right)^2}\left[16 + 35\alpha + 14\alpha^2 - 2\alpha^3 - 4\alpha^3 + \alpha^5\right] > 0.
\end{aligned}$$

Hence, again, $\pi_U^* - \pi_U^{BI} > 0$.

If $H > \frac{\alpha}{1+\alpha}$, then $q_2^{BI} = H$ and $q_2^{BI} = 0$, so

$$\begin{aligned}
\pi_U^* - \pi_U^{BI} &= \left(w_1 + \frac{\frac{2kH^2}{\left(H+2\right)^2} + \overline{R} + c}{2} - 2c\right) - \left(w_1 + \frac{\overline{R}}{2} + \frac{\alpha c + \underline{V}}{2\alpha} - 2c\right) \\
&= \frac{kH^2}{\left(H+2\right)^2} - \frac{\underline{V}}{2\alpha}.
\end{aligned}$$

This expression is positive provided that $\underline{V}$ is not too large.

Since $\pi_1^{BI} - \pi_1^* - \alpha\left(\pi_U^* - \pi_U^{BI}\right) > 0$ and since $\pi_U^* - \pi_U^{BI} > 0$, it follows that $\pi_1^{BI} - \pi_1^* > 0$: backward integration boosts the value of D_1.

To prove parts (iii) and (iv) of the proposition, note that the minimally acceptable offer for control in U is

$$T = \gamma\pi_U^* - (\gamma - \alpha)\,\pi_U^{BI}.$$

Hence, the post-acquisition payoff of D_1 is

$$\gamma_1\left(\pi_1^{BI} + \alpha\pi_U^{BI} - T\right) = \gamma_1\left(\pi_1^{BI} + \gamma\left(\pi_U^{BI} - \pi_U^*\right)\right).$$

Now assume that $\gamma > \frac{H}{1-H}$, so $H < \frac{\gamma}{1+\gamma}$. Differentiating $\pi_1^{BI} + \gamma\left(\pi_U^{BI} - \pi_U^*\right)$ with respect to α and evaluating the derivative at $\alpha = \gamma$ yields

$$\frac{\partial}{\partial\alpha}\left(\pi_1^{BI} + \gamma\left(\pi_U^{BI} - \pi_U^*\right)\right)\bigg|_{\alpha=\gamma} = \frac{z^4(1-z)\gamma^3 k\left(\gamma\left(4-z^2\right) + 4z(1-\gamma)\right)}{(1+z)^3\left(4 - z^2\gamma\right)^3} - \frac{\underline{V}}{2\gamma},$$

where $z \equiv \frac{(1+\gamma)H}{\gamma}$; since $H < \frac{\alpha H}{1+\alpha}$, $z < 1$. The first term in the derivative is positive, but goes to 0 when z goes to 0, or when z goes to 1. Since the second term is negative, the derivative is negative when z goes to 0 or goes to 1. Part (iii) of the proposition follows by noting that z goes to 0 when H is small and goes to 1 when γ approaches $\frac{H}{1-H}$.

Finally, to prove part (iv) of the proposition, note that when $\gamma \leq \frac{H}{1-H}$, the post-acquisition payoff of D_1 is

$$\pi_1^{BI} + \gamma\left(\pi_U^{BI} - \pi_U^*\right) = \frac{kH^2}{2} + \overline{R} - w_1 + \gamma\left(\frac{\underline{V}}{2\alpha} - \frac{kH^2}{(H+2)^2}\right).$$

This expression is decreasing with α, implying that D_1 would wish to obtain a minimal controlling stake in U, subject to being able to obtain control over U. ■

Proof of Proposition 5: Using (17),

$$\frac{\partial q_1^{BIpass}}{\partial\alpha} = \frac{2H(2-H)^2}{\left(4 - (1+2\alpha)H^2\right)^2} > 0, \qquad \frac{\partial q_2^{BIpass}}{\partial\alpha} = -\frac{H^2(2-H)^2}{\left(4 - (1+2\alpha)H^2\right)^2} < 0.$$

Since q_1^{BIpass} decreases and q_2^{BIpass} is increases when α falls below 1, $\phi \equiv q_1(1-q_2)$ becomes smaller.

To prove part (ii), note that $\alpha \in (0,1)$ implies that $4 > 2(1+\alpha)$, $(1+\alpha)H < (1+2\alpha)H$, and $\frac{(1+\alpha)^2}{\alpha}H^2 > (1+2\alpha)H^2$. Hence,

$$\begin{aligned} q_1^{BI} &= \frac{\alpha H(4 - (1+\alpha)H)}{4\alpha - (1+\alpha)^2 H^2} = \frac{H(4-(1+\alpha)H)}{4 - \frac{(1+\alpha)^2}{\alpha}H^2} \\ &> \frac{H(2(1+\alpha) - (1+2\alpha)H)}{4 - (1+2\alpha)H^2} = q_1^{BIpass}. \end{aligned}$$

Likewise, $\alpha \in (0,1)$ implies that $\frac{2(1+\alpha)}{\alpha} H > (1+\alpha) H$ and $\frac{(1+\alpha)^2}{\alpha} H^2 > (1+2\alpha) H^2$, so

$$\begin{aligned} q_2^{BI} &= \frac{2H(\alpha - (1+\alpha)H)}{4\alpha - (1+\alpha)^2 H^2} = \frac{H\left(2 - \frac{2(1+\alpha)}{\alpha} H\right)}{4 - \frac{(1+\alpha)^2}{\alpha} H^2} \\ &< \frac{H(2-(1+\alpha)H)}{4-(1+2\alpha)H^2} = q_2^{BIpass}. \end{aligned}$$

Since $q_1^{BI} > q_1^{BIpass}$ and $q_2^{BI} < q_2^{BIpass}$, $\phi^{BI} > \phi^{BIpass}$.

As for consumer surplus, I need to compare $S\left(q_1^{BI}, q_2^{BI}\right) = \underline{V} + q_1^{BI} q_2^{BI} \Delta$ with $S\left(q_1^{BI}, q_2^{BI}\right) = \underline{V} + q_1^{BIpass} q_2^{BIpass} \Delta$. Of course, when $q_2 = 0$, consumers get a fixed surplus of $\underline{V}$. In an interior Nash equilibrium, the sign of $q_1^{BI} q_2^{BI} - q_1^{BIpass} q_2^{BIpass}$ depends only on H and on α, though the difference is a polynomial of degree six. Using Mathematica, I obtain Figure 7 in the text that shows that range of values of H and α for which the difference is positive and the range for which it is negative. The statement in the proposition is based on this figure.

To prove part (iv) of the proposition, I will use (10), (4), (17), to obtain

$$S\left(q_1^{BIpass}, q_2^{BIpass}\right) - S\left(q_1^*, q_2^*\right) = \frac{\alpha k H^3 (2-H)^2 \left(4 - 2\alpha H - (1+2\alpha) H^2\right)}{(2+H)^2 \left(4\alpha - (1+\alpha)^2 H^2\right)}.$$

Since in an interior equilibrium, $H < \frac{\alpha}{1+\alpha}$,

$$\begin{aligned} 4 - 2\alpha H - (1+2\alpha) H^2 &> 4 - 2\alpha \left(\frac{\alpha}{1+\alpha}\right) - (1+2\alpha)\left(\frac{\alpha}{1+\alpha}\right)^2 \\ &= \frac{4 + 8\alpha + \alpha^2 - 4\alpha^3}{(1+\alpha)^2} > 0. \end{aligned}$$

Hence, $S\left(q_1^{BIpass}, q_2^{BIpass}\right) > S\left(q_1^*, q_2^*\right)$. ■

Proof of Proposition 6: First, assume that $\alpha > 1/4$. Then, since in a interior equilibrium, $H < \frac{1}{1+\alpha}$,

$$\begin{aligned} \frac{\partial q_1^{FI}}{\partial \alpha} &= \frac{H^2 \left(4 - 4\left(1-\alpha^2\right) H - (1+\alpha)^2 H^2\right)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} \\ &> \frac{H^2 (4 - 4(1-\alpha) - 1)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} \\ &> \frac{H^2 (4\alpha - 1)}{\left(4\alpha - (1+\alpha)^2 H^2\right)^2} > 0, \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial q_2^{FI}}{\partial \alpha} &= -\frac{2H^2\left(4\alpha^2+(1+\alpha)H(1-\alpha-(1+\alpha)H)\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&< -\frac{2H^2\left(4\alpha^2-(1+\alpha)H\alpha\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&< -\frac{2H^2\alpha(4\alpha-1)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} < 0.
\end{aligned}$$

If $\alpha < 1/4$, then in an interior equilibrium, $\frac{H}{2} < \frac{2\alpha}{1+\alpha}$,

$$\begin{aligned}
\frac{\partial q_1^{FI}}{\partial \alpha} &= \frac{H^2\left(4-4\left(1-\alpha^2\right)H-(1+\alpha)^2H^2\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&> \frac{H^2\left(4-4\left(1-\alpha^2\right)\frac{4\alpha}{1+\alpha}-(1+\alpha)^2\left(\frac{4\alpha}{1+\alpha}\right)^2\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&= \frac{4H^2(1-4\alpha)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} > 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial q_2^{FI}}{\partial \alpha} &= -\frac{2H^2\left(4\alpha^2+(1+\alpha)H(1-\alpha-(1+\alpha)H)\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&< -\frac{2H^2\left(4\alpha^2+(1+\alpha)H\left(1-\alpha-(1+\alpha)\frac{4\alpha}{1+\alpha}\right)\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&< -\frac{2H^2\left(4\alpha^2+(1+\alpha)H(1-5\alpha)\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2}.
\end{aligned}$$

If $\alpha < \frac{1}{5}$ then $\frac{\partial q_2^{FI}}{\partial \alpha} < 0$. If $1/5 < \alpha < 1/4$, then

$$\begin{aligned}
\frac{\partial q_2^{FI}}{\partial \alpha} &< -\frac{2H^2\left(4\alpha^2+(1+\alpha)\frac{4\alpha}{1+\alpha}(1-5\alpha)\right)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} \\
&< -\frac{8\alpha H^2(1-4\alpha)}{\left(4\alpha-(1+\alpha)^2H^2\right)^2} < 0.
\end{aligned}$$

Since q_1^{FI} increases and q_2^{FI} decreases with α, $\phi_2^{FI} \equiv q_1^{FI}\left(1-q_2^{FI}\right)$ increases with α. Since $\phi_2^{FI} = \phi_2^{VI}$ when $\alpha = 1$, it follows that $\phi_2^{FI} > \phi_2^{VI}$.

Substituting q_1^{FI} and q_2^{FI} in (10), consumer surplus under partial forward integration is

$$S^{FI} = \begin{cases} \underline{V} + \frac{2\alpha k H^3 (1-(1+\alpha)H)(4\alpha-(1+\alpha)H)}{\left(4\alpha-(1+\alpha)^2\right)^2} & \text{if} \quad H < \frac{1}{1+\alpha}, \\ \underline{V} & \text{if} \quad H \geq \frac{1}{1+\alpha}. \end{cases}$$

Now,

$$\frac{\partial S^{FI}}{\partial \alpha} = \frac{2kH^4 \left[4\alpha\left(1-4\alpha^2\right) - 12\alpha\left(1-\alpha^2\right)H + (1+\alpha)^2\left(1+6\alpha-4\alpha^2\right)H^2 - \left(1+2\alpha-2\alpha^3-\alpha^4\right)H^3\right]}{\left(4\alpha-(1+\alpha)^2\right)^3}.$$

The sign of $\frac{\partial S^{FI}}{\partial \alpha}$ depends on the sign of the bracketed term in the numerator. Using Mathematica, I obtain Figure 9 in the text which shows the combinations of α and H for which the bracketed term is positive or negative. The statement in the proposition is based on Figure 9. ■

Proof of Proposition 7: Note that $\pi_U^{FI} \equiv w_1 + w_2^{FI} - 2c$ and $\pi_1^{FI} \equiv q_1^{FI}\left(1-q_2^{FI}\right)\Delta + \overline{R} - w_1 - \frac{k\left(q_1^{FI}\right)^2}{2}$, and recall that I assume that w_1 is equal to its value absent integration, in order to eliminate the incentive to forward integrate in order to exploit the minority shareholders of D_1. Using (4), (21) and (22), it follows that when $H < \frac{1}{1+\alpha}$ (so $q_2^{FI} > 0$),

$$\begin{aligned} \pi_1^{FI} - \pi_1^* &= \left(q_1^{FI}\left(1-q_2^{FI}\right) - q_1^*\left(1-q_2^*\right)\right)\Delta - \frac{k\left(\left(q_1^{FI}\right)^2 - \left(q_1^*\right)^2\right)}{2} \\ &= \frac{kH^2}{2(2+H)^2\left(4\alpha-(1+\alpha)^2H^2\right)^2} \times \\ &\quad \left[16\alpha^2 - \left(4-12\alpha^2-56\alpha^3\right)H^2 + 4\left(1-4\alpha-3\alpha^2+6\alpha^3\right)H^3\right. \\ &\quad \left. + \left(4-10\alpha-23\alpha^2-12\alpha^3-3\alpha^4\right)H^4 + 2\left(1+\alpha-\alpha^2-\alpha^3\right)H^5\right]. \end{aligned}$$

The sign of this expression depends on the sign of the bracketed term. Computations with Mathematica reveal that this term is positive for all $\alpha \in [0,1]$ and all $H \in [0,1]$. Hence, the minority shareholders of D_1 benefit as stated in part (iv) of the proposition.

Likewise, using (4), (1), (18), (21), and (22), it follows that

$$\begin{aligned} \pi_U^* - \pi_U^{FI} &= w_2^* - w_2^{FI} = \frac{H^3}{(2+H)^2\left(4\alpha-(1+\alpha)^2H^2\right)^2} \times \\ &\quad \left[16\alpha^2 - 4\alpha\left(2-\alpha-4\alpha^3\right)H - 8\alpha^2\left(1+\alpha-\alpha^2\right)H^2\right. \\ &\quad \left. + \left(1+4\alpha-4\alpha^2-10\alpha^3-3\alpha^4\right)H^3 - 2\alpha^2(1+\alpha)^2H^4\right] - \frac{\alpha\underline{V}}{2}. \end{aligned}$$

Again, computations with Mathematica establish that the first term is positive for all $\alpha \in [0,1]$ and all $H \in [0,1]$, so, provided that $\underline{V}$ is not too large, $\pi_U^* - \pi_U^{FI} > 0$.

Assuming that $\underline{V}$ is not too large, both sides of (23) are positive, implying that the condition surely fails when γ_1 is small. To check if the condition hold for large enough γ_1, let's assume that $\alpha = \gamma_1$, that is, U acquires the entire stake of D_1's controlling shareholder. Then,

$$\begin{aligned}\alpha\left(\pi_1^{FI} - \pi_1^*\right) - \left(\pi_U^* - \pi_U^{FI}\right) &= \frac{H^2}{2\left(2+H\right)^2\left(4\alpha - \left(1+\alpha\right)^2 H^2\right)^2} \times \\ &\quad \left[16\alpha^2\left(\alpha - 2H\right) + 4\alpha\left(3 - 2\alpha + 3\alpha^2 + 6\alpha^3\right)H^2 + 4\alpha\left(1+\alpha^2+2\alpha^3\right)H^3 \right. \\ &\quad \left. - \left(1+\alpha\right)^2\left(2+3\alpha^3\right)H^4 + 2\alpha\left(1+\alpha\right)^3 H^5\right].\end{aligned}$$

Computations with Mathematica establish that whenever $\alpha \geq 1/4$, this expression is positive when α is sufficiently large, so the acquisition of the entire stake γ_1 is profitable if γ_1 is sufficiently large.

Next, suppose that $H > \frac{1}{1+\alpha}$. Then, $q_1^{FI} = H$ and $q_2^{FI} = 0$, and hence

$$\pi_1^{FI} - \pi_1^* = \frac{kH^2\left(1+4H+H^2\right)}{2\left(2+H\right)^2} > 0,$$

so again the minority shareholders of D_1 benefit from forward integration. Moreover,

$$\alpha\left(\pi_1^{FI} - \pi_1^*\right) - \left(\pi_U^* - \pi_U^{FI}\right) = \frac{H^2\left(-2+\alpha\left(1+4H+H^2\right)\right)}{2\left(2+H\right)^2} + \frac{\underline{V}}{2}.$$

This expression is positive when α and H are sufficiently large

When U makes a take-it-or-leave-it-offer for a stake in D_1, it needs to offer D_1 a payment T such that

$$\left(\gamma_1 - \alpha\right)\pi_1^{FI} + T \geq \gamma_1\pi_1^*, \qquad \Rightarrow \qquad T \geq \gamma_1\pi_1^* - \left(\gamma_1 - \alpha\right)\pi_1^{FI}.$$

Clearly, U will make the minimal acceptable offer, so his post-acquisition payoff is

$$\begin{aligned}\gamma\left(\pi_U^{FI} + \alpha\pi_1^{FI} - T\right) &= \gamma\left(\pi_U^{FI} + \alpha\pi_1^{FI} - \gamma_1\pi_1^* + \left(\gamma_1 - \alpha\right)\pi_1^{FI}\right) \\ &= \gamma\left(\pi_U^{FI} + \gamma_1\left(\pi_1^{FI} - \pi_1^*\right)\right).\end{aligned}$$

Now, when $H < \frac{1}{1+\alpha}$ (so $q_2^{FI} > 0$),

$$\frac{\partial}{\partial\alpha}\left(\pi_U^{FI} + \gamma_1\left(\pi_1^{FI} - \pi_1^*\right)\right) = \frac{-\alpha\left(1+\alpha\right)kH^4\left(1-t\right)\left(4\left(1-t\right) + \left(4\alpha - t\right)t\right)}{\left(4\alpha - \left(1+\alpha\right)^2 H^2\right)^3} + \frac{\underline{V}}{2},$$

where $t \equiv \left(1+\alpha\right)H < 1$, since $H < \frac{1}{1+\alpha}$. The first term is negative since $\alpha > 1/4$, implying that $4\alpha > t$. Hence, if $\frac{\underline{V}}{2}$ is not too large, U will prefer to acquire a stake $\alpha < \gamma_1$.

If $H > \frac{1}{1+\alpha}$, so $q_1^{FI} = H$ and $q_2^{FI} = 0$, then

$$\pi_U^{FI} + \gamma_1\left(\pi_1^{FI} - \pi_1^*\right) = \frac{\gamma_1 kH^2 + \alpha\underline{V} + \left(1+2\gamma_1\right)\overline{R} - 3c}{2} + \left(1-\gamma_1\right)w_1,$$

which is clearly increasing with α. $\blacksquare$

Proof of Proposition 8: Using (21),

$$\frac{\partial q_1^{FIpass}}{\partial \alpha} = \frac{H^3(2-H)}{\left(2-(1+\alpha)H^2\right)^2} > 0, \qquad \frac{\partial q_2^{FIpass}}{\partial \alpha} = -\frac{H^2(2-H)}{\left(2-(1+\alpha)H^2\right)^2} < 0.$$

Since q_1^{FIpass} decreases and q_2^{FIpass} is increases when α falls below 1, $\phi \equiv q_1(1-q_2)$ becomes smaller.

To prove part (ii), note that since $\alpha \in (0,1)$ and $H < \frac{1}{1+\alpha}$,

$$\begin{aligned} q_1^{FI} &= \frac{H(4\alpha-(1+\alpha)H)}{4\alpha-(1+\alpha)^2H^2} = \frac{H\left(\frac{4\alpha}{1+\alpha}-H\right)}{\frac{4\alpha}{1+\alpha}-(1+\alpha)H^2} \\ &< \frac{H(2-H)}{2-(1+\alpha)H^2} = q_1^{FIpass}. \end{aligned}$$

Likewise, $\alpha \in (0,1)$ implies that $\frac{2(1+\alpha)}{\alpha}H > (1+\alpha)H$ and $\frac{(1+\alpha)^2}{\alpha}H^2 > (1+2\alpha)H^2$, so

$$\begin{aligned} q_2^{FI} &= \frac{2\alpha H(1-(1+\alpha)H)}{4\alpha-(1+\alpha)^2H^2} = \frac{H(1-(1+\alpha)H)}{2-\frac{(1+\alpha)^2}{2\alpha}H^2} \\ &> \frac{H(1-(1+\alpha)H)}{2-(1+\alpha)H^2} = q_2^{FIpass}. \end{aligned}$$

Since $q_1^{FI} < q_1^{FIpass}$ and $q_2^{FI} > q_2^{FIpass}$, $\phi^{FI} < \phi^{FIpass}$.

As for consumer surplus, I need to compare $S^{FI} \equiv \underline{V} + q_1^{FI}q_2^{FI}\Delta$ with $S^{FIpass} \equiv \underline{V} + q_1^{FIpass}q_2^{FIpass}\Delta$. Using Mathematica, I obtain Figure 10 in the text on which part (iii) of the proposition is based. usng mathematica again, establishes that $S^{FIpass} > S^* \equiv \underline{V} + q_1^*q_2^*\Delta$ when H is not too high and conversely when H is close to its upper limit in interior equilibria which is $\frac{1}{1+\alpha}$. $\blacksquare$

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