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The choice of exchange rate bands: balancing credibility and flexibility

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Abstract

This paper develops a framework for the optimal choice of exchange rate bands within an environment in which policymakers dislike nominal exchange rate variability, but value the flexibility to adjust the nominal exchange rate in response to shocks, in order to attain real exchange rate objectives. The paper provides an endogenous characterization of the optimal exchange rate band in terms of the underlying distribution of shocks to the current and capital accounts of the balance of payments and in terms of the commitment reputation of policymakers.

Keywords: Exchange rate bands; Pegs; Floats; Partial commitment; Credibility; Reputation; Flexibility

JEL classification: F31; F33; E5

1. Introduction

During the last decade, a large number of countries, including Brazil, Chile, Colombia, Ecuador, Finland, Hungary, Israel, Mexico, Norway, Poland, Russia, Sweden, The Czech Republic, The Slovak Republic, Venezuela and a number of emerging Asian countries have used unilateral exchange rate bands. Under this regime, there is a policy commitment to maintain exchange rates within a zone of known width around an announced reference rate. Adopting an exchange rate band forces policymakers to take a stand on key choices,

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such as the band's width, the exchange rate to be used as a central parity rate, the exit strategies such as the frequency and form of realignments, and the method of intervention to support the band. Over the years, most of the above mentioned countries have moved toward greater exchange rate flexibility (see International Monetary Fund, 1998). In some cases this has taken the form of a widening of previously existing currency bands or even a shift to a float.

Most of the existing literature on currency bands offers little guidance on the policy tradeoffs involved in the above mentioned choices. While some of the earlier work on exchange rate target zones (e.g., Williamson, 1985; Frenkel and Godstein, 1986; Williamson and Miller, 1987) partially and informally dealt with these important policy choices, the recent voluminous literature on target zones (e.g., chapters 1 and 2 in Krugman and Miller, 1992; Svensson, 1992) has not addressed the real world decision regarding the characteristics of such a band.

The purpose of this paper is to provide a precise framework for the analysis of the tradeoffs involved in the choice of unilateral exchange rate bands. We view these bands as the outcome of an optimization problem of a policymaker whose objective function trades off the option of moving the real exchange rate in a desired direction against the level and variability of the nominal exchange rate. We believe that this formulation captures an important real world aspect of exchange rate policy determination in small open economies. The authorities in such economies have shown their concern in preserving and improving the competitiveness of exports and the current account position, while at the same time avoiding the possible inflationary consequences of nominal exchange rate depreciation. This tradeoff is at the heart of a recent discussion and analysis of exchange rate policy in emerging Asian countries by Dornbusch and Park (1999).[1] Exchange rate bands are seen in this context as a simple and verifiable system for the policymaker to make a credible anti-inflation commitment while retaining some degree of flexibility in the determination of exchange rates to shield exports and the current account from the impact of adverse shocks to both the current and the capital accounts.[2]

Dornbusch and Park (1999) recommend that emerging Asian economies like Indonesia, Malaysia, Thailand and Taiwan adopt exchange rate bands. But they argue that currently there is no scientific basis to determine a good band width and mention that Williamson (1996) recommends a 7–10 percent range on either side. This paper makes a step towards putting the discussion of determinants of band width on a firmer analytical basis by providing a positive theory of exchange rate band determination. Our model differs from most existing models of exchange rate bands in two main respects. First, the width of the

[1] This tradeoff is also relevant for Chile as is transparent from a statement by the former President of the Banco Central de Chile; see Zahler (1992), who focuses on policy dilemmas that arise under high capital mobility. See also Leiderman and Bufman (1996).

[2] Different rationales for the existence of exchange rate bands are provided in recent work by Krugman and Miller (1993) and Svensson (1994). The former argue that the real world motivation for target zones is to a large extent the concern about irrational and unstable market behavior. The latter stresses the role of exchange rate bands in increasing the independence and flexibility of monetary policy compared to fixed exchange rates. Sutherland (1995) and Miller and Zhang (1996) analyze the optimal determination of band width. But, unlike this paper, they do not allow for the possibility of realignments under some circumstances.

band is determined endogenously as the solution to policymakers' maximization problem. Since the model allows for bands of any width, it admits pegs (bands of zero width) and floats (bands of infinite width) as special cases. Second, we endogenize policymakers' decisions about realignments by considering conditions under which the realization of exogenous shocks is such that it pays policymakers to exit from the band. Therefore, the imperfect credibility of the band is partly due to the public's uncertainty about the strength of the policymaker's commitment to the band.

We view the choice of band width as involving a tradeoff between the flexibility to react to unanticipated exchange rate misalignments and the minimization of nominal variability. A similar policy tradeoff has been investigated by Flood and Isard (1989) and Lohmann (1992) for closed economies, and by Cukierman et al. (1992) and Obstfeld (1996, 1997) for open economies.[3] Policymakers value flexibility because it enables them to rely on the nominal exchange rate to move closer to *their* real exchange rate target when *they* believe that the real exchange rate deviates from this desired level. But since they dislike nominal exchange rate variability they prefer to achieve those objectives with less nominal variability. In many cases the preannouncement of a band has a moderating impact on the public's expectation regarding changes in the exchange rate, which leads in turn to more price stability.[4]

The preannouncement of a band commands *some* credibility since the public anticipates that, at least some, policymakers will bear a political cost if they abandon the band. This cost may be due to instability on financial markets, to an overall loss of confidence in monetary policy or to a personal loss of reputation that may undermine the future careers of policymakers in charge. In any event, absent such a cost, the public will correctly perceive the announcement of a band as an empty statement, and hence policymakers will have no incentive to make announcements in the first place. The prevalence of exchange rate bands suggests that in reality, policymakers do stand to lose if they abandon their commitment which, in turn, makes their announcement meaningful. Whether an existing band is maintained or not depends on the realization of shocks and on the political costs of abandoning the band. These costs imply the existence of a range of effective commitment that lies outside the band. In this range the preannouncement of a band deters dependable

[3] The model in Cukierman et al. (1992) can be viewed as a complement to the current model, since it assumes that the exchange rate is pegged, but allows policymakers to choose the degree of commitment to the peg by choosing the cost of breaking their commitment. Here the cost of exiting the band is specified exogenously, but policymakers can determine the tradeoff between credibility and flexibility ex ante by choosing the width of the band. Obstfeld uses a *fixed* exchange rate framework with escape clauses for periods of stress to show that such clauses lead to multiple equilibria and to self fulfilling (possibly welfare reducing) exchange rate crises. A shared feature of his framework and ours is that both allow for costly realignments when external shocks are sufficiently large. The main difference is that Obstfeld examines the destabilizing effects of fixed exchange rates with escape clauses whereas we are interested in characterizing the optimal exchange rate band. A secondary difference is that in Obstfeld, the temptation to realign is driven by employment considerations whereas in our framework it is driven by sufficiently large (positive or negative) differences between actual and desired real exchange rates that need not be due only to employment considerations.

[4] The existence of a band also has a moderating impact on the variability of expected currency depreciations. Although not identical, this mechanism is similar to the 'honeymoon' effect discussed in the target zone literature; see Krugman and Miller (1992) and Svensson (1992).

policymakers from adjusting the exchange rate. Thus, pegs and bands are limited commitments which (as in Flood and Isard, 1989 and Lohmann, 1992 for example) are maintained only for some ranges of shock realizations.

An essential aspect of the analysis is that the exchange rate regime is chosen by policymakers prior to the realization of shocks. The first and main part of the paper focuses on the case in which expectations, and therefore prices, are determined after the realization of shocks, so that price formation is subject to political, but not to economic uncertainty. In that case a peg (a zero band width) is, inter alia, optimal only if the policymaker's reputation is perfect in the sense that once a peg is announced, the public expects the policymaker to keep the exchange rate fixed *under all circumstances*. However, if the policymaker's reputation is not perfect so that the public expects him to exit the band with some positive probability, then the optimal regime is either a band of a finite width or a free float (a band of infinite width). The latter is optimal only in the (unlikely) case where large exchange rate misalignments (i.e., differences between desired and actual real exchange rates) are more likely than small ones. Otherwise, it is optimal for the policymaker to set up a two sided band. Unless the distribution of misalignments is symmetric, the optimal band is not necessarily symmetric. The model is used to characterize the dependence of the optimal band on the policymaker's reputation for being able to maintain his commitment to the band, on the cost of exiting from the band, and on the level of uncertainty about potential exchange rate misalignments.

The second part of the paper examines the case where price formation is subject to both economic and political uncertainty. We show that when prices are set prior to the realization of shocks, one sided bands are usually optimal for the policymaker. In particular, it is optimal to set up a one sided band that moderates expectations about the more probable policy action in the absence of exchange rate commitments. For example if devaluations are, a priori, more probable it is optimal to set a one sided band or peg that imposes limits only on devaluations.

The paper is organized as follows. Section 2 presents the basic model and develops the notion that an exchange rate band can be seen as a partially credible commitment device. Section 3 characterizes the equilibrium and shows that the band gives rise to a range of effective commitment beyond which the policymaker opts for realignment. Section 4 provides a comparative statics analysis of the determinants of the width of the exchange rate band. Section 5 examines the case where the public expectation reflects both political uncertainty and uncertainty about misalignments shocks. This is followed by concluding remarks. All proofs are in Appendix.

2. A model of exchange rate bands

2.1. The policymaker's objective

We consider a model in which exchange rate policy is driven by a fundamental tradeoff between allowing the real exchange rate to move in a desired direction and maintaining nominal exchange rate stability. This tradeoff arises under either current account shocks or capital account shocks. Due to these shocks, there is uncertainty regarding the direction and magnitude of desired changes in real exchange rates. We capture this uncertainty by postulating that the policymaker has the following objective function:

$$V(\pi, \pi^e) = x(\pi - \pi^e) - \frac{\pi^2}{2}, \tag{1}$$

where π and π^e are the actual and the expected rates of changes in the nominal exchange rate (positive values of π and π^e are associated with actual and expected depreciations while negative values are associated with appreciations), and x is a continuous random variable with positive density in the range $[\underline{x}, \bar{x}]$, where $-\infty \leq \underline{x} < 0 < \bar{x} \leq \infty$. We assume that π is chosen directly by policymakers, though in practice it is implemented via interest rate policy and/or intervention in the foreign exchange market. It is shown in the first part of the Appendix that x can be viewed as an approximate measure of the divergence between the desired level and the existing level of the real exchange rate. This divergence depends in turn on various stochastic shocks to the current account, to employment, and/or to the capital account of the balance of payments. In countries with persistent balance of payments deficits the mean value of x is positive although particular realizations may be negative.[5] The mean value of x is a measure of the average degree of dissatisfaction of policymakers with the existing real exchange rate relative to the political costs of exchange rate variability. Thus x can be viewed as a measure of the extent of real exchange rate misalignment, as perceived by policymakers.[6]

Eq. (1) has the following interpretation. Assuming that domestic wages and prices are (at least partially) sticky in the short run, expected changes in the nominal exchange rate affect their predetermined components.[7] Hence, if real trade shocks, employment shocks, or capital flow shocks are such that $x > 0$ (i.e., the real exchange rate desired by the policymaker exceeds the existing level), the policymaker wishes to produce an unexpected real depreciation by setting π above π^e. Similarly, if $x < 0$, the policymaker wishes to produce a real appreciation by setting π below π^e. Thus the first term in (1) represents the policymaker's benefit from unexpected depreciation (when $x > 0$) or unexpected appreciation (when $x < 0$). When $x > 0$, an increase in x leads to a larger misalignment and hence an *unanticipated depreciation* of a given size becomes more valuable to the policymaker. Conversely, when $x < 0$, a decrease in x leads to a larger misalignment and hence an *unanticipated appreciation* of a given size is more valuable to the policymaker. When $x = 0$ the policymaker is content with

[5] Persistent budget deficits and/or overly ambitious employment objectives may lead to such situations. Precise discussions of such cases appear in chapter 5 of Cukierman (1992) and in Horn and Persson (1988) respectively.

[6] Note that the arguments in the paper in no way rely on the presumption that the real exchange rate desired by policymakers is also the optimal one from a social welfare point of view (although we do not exclude such a possibility). For example policymakers might desire to create a real depreciation because the interests of exporters are dear to their hearts or, alternatively, because they wish to reduce persistent deficits in the current account of the balance of payments. Although we always have in mind misalignments as perceived by policymaker, we do not always carry the adjective 'perceived' for reasons of brevity.

[7] A similar wage price sluggishness appears in Beetsma and van der Ploeg (1998). But they take the width of the band to be exogenous.

the current level of the real exchange rate and does not wish to produce exchange rate surprises. The second term in (1) reflects the political costs of exchange rate variability.[8]

2.2. The exchange rate band and the policymaker's type

We consider exchange rate bands as a partial commitment device. The reason why a policymaker would willingly restrict his freedom to adjust the exchange rate optimally ex post is that by doing so he increases the credibility of his exchange rate policy. This enables the policymaker to attain the same real exchange rate objectives with less nominal exchange rate variability. The cost associated with this gain in credibility is a loss in the policymaker's flexibility to freely move the exchange rate in a desired direction. The optimal width of the band is determined by trading-off the benefits of credibility against the cost of reduced flexibility.[9]

To reflect this tradeoff, let $[\underline{e}, \overline{e}]$ be the exchange rate band set by the policymaker around the preexisting nominal exchange rate, e_{-1}; the policymaker then commits to cap the nominal exchange rate, e, from below by $\underline{e}$ and cap it from above by $\overline{e}$. Assuming, without loss of generality, that e_{-1} serves as the reference or central parity rate, the exchange rate band induces a permissible range of rates of change in the exchange rate, $[\underline{\pi}, \overline{\pi}]$, where $\underline{\pi} \equiv (\underline{e} - e_{-1})/e_{-1} < 0$ and $\overline{\pi} \equiv (\overline{e} - e_{-1})/e_{-1} > 0$. Within this range, the domestic currency is depreciated if $\pi \in (0, \overline{\pi}]$ and appreciated if $\pi \in [\underline{\pi}, 0)$; the absolute value of $\underline{\pi}$ then is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation that the exchange rate band permits.

Obviously, the announcement of a band can have *some* impact on exchange rate expectations only if it is common knowledge that the policymaker will bear a cost if he were to abandon the band. Absent such a cost, the public will correctly perceive the policymaker's announcement as an empty statement and will ignore it. This cost may be due to financial instability, loss of confidence in monetary policy, a personal loss of reputation that may undermine the future career of the policymaker, or some combination of those factors.

In practice there is typically a considerable amount of uncertainty about the commitment ability of policymakers.[10] To reflect this uncertainty we assume that there are two possible types of policymakers. The first type, to which we refer as dependable (D), incurs a fixed cost, $c > 0$, whenever he allows the nominal exchange rate to move outside the band. Following Svensson (1992, 1994), we refer to situations in which the nominal exchange rate is set outside the band as 'realignments'. A realignment involves either an

[8] Assuming that depreciations are inflationary, $\pi^2/2$ could also reflect the political cost of inflation when $\pi > 0$.

[9] This tradeoff has been investigated in Flood and Isard (1989), Lohmann (1992), Cukierman et al. (1992), and Obstfeld (1997). It is analogous although not identical to Rogoff's (1985) well known trade off between credibility and flexibility in the context of a conservative central bank. As far as we know the current paper is the first to apply this point of view to the, endogenous, choice of exchange rate bands.

[10] Frankel et al. (2001) provide empirical evidence which shows that it can be very hard for the public to verify that an announced exchange rate regime is actually in operation, especially in the case of wide bands and basket pegs.

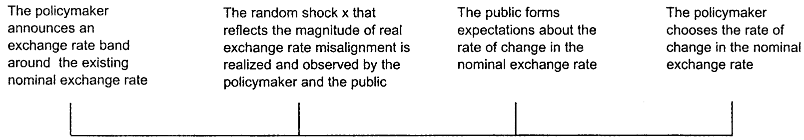

Fig. 1. The sequence of events.

upward adjustment of the center rate if e is set above $\overline{e}$, or a downward adjustment if e is set below $\underline{e}$. Since realignments are costly for a dependable policymaker, he realigns the band only if this raises his payoff by at least c. By contrast, the second type of policymaker, referred to as opportunistic (O), does not incur any cost if he sets the nominal exchange rate outside the band, and therefore the band does not constrain his actions at all.[11] We assume that the public assigns a probability $\alpha > 0$ to the policymaker being dependable. We will sometimes refer to the public's uncertainty about the policymaker type as 'political uncertainty' and to α as the policymaker's 'commitment reputation' or, following Barro (1986), just as 'reputation'.

2.3. The sequence of events and the equilibrium concept

The strategic interaction between the policymaker and the public is modeled as a three-stage game (see Fig. 1). In stage 1, the policymaker announces a band around the existing nominal exchange rate, e_{-1}.[12] The band induces a permissible range of rates of change in the exchange rate, $[\underline{\pi}, \overline{\pi}]$. In stage 2, the random shock x is realized, and given its realization and the policymaker's announcement, the public forms expectations about the rate of change in the exchange rate in stage 3, and wages and prices are set accordingly. Finally in stage 3, the policymaker chooses the rate of change in the nominal exchange rate to maximize his objective function. We believe this is a realistic formulation for policymakers who often set the interest rate and/or foreign exchange market intervention with a short run real exchange rate objective, and given price stickiness, a corresponding nominal exchange rate objective in mind.

To characterize the rational expectations equilibrium of the model, we solve the model backwards. First, given the exchange rate band, the realization of x, and the public's expectations, we solve for the rate of change in the nominal exchange rate that each type of policymaker would choose in stage 3. Second, given the policymakers' choices, and given the public's beliefs about the policymaker's type, we characterize the public's (rational) expectation that is formed in stage 2 regarding the rate of change in the nominal exchange

[11] This difference in costs may be thought of as reflecting a difference in rates of time preference between the two types in a framework in which the cost of reneging arises from a reduction in future credibility. Readers who are familiar with the literature on strategic monetary policy will recognize that the behavior of the opportunistic type corresponds to the discretionary policy of that literature.

[12] e_{-1} can be viewed as the center rate of the band, or more precisely, the rate that will be maintained absent preceived misalignments. In Proposition 1 below we show that in general, the band is not symmetric around e_{-1}. Hence, this 'center rate' is in general not literally in the middle of the band.

rate that would occur in stage 3. Third, we solve for the exchange rate band that each type of policymaker would set in stage 1 of the game.

3. The equilibrium

3.1. The choice of nominal exchange rate change

In stage 3, the policymaker chooses the rate of change in the nominal exchange rate, π, given the public's expectations, π^e, and given the realization of the random shock, x. Obviously, the policymaker's choice depends on his type. If the policymaker is opportunistic, he bears no cost when the band is realigned. Therefore, an opportunistic policymaker ignores the band that was announced in stage 1 altogether and simply chooses π with the objective of maximizing $V(\pi, \pi^e)$. The solution for this maximization problem is $\pi^o = x$.[13]

Now suppose that the policymaker is dependable and let $\pi^D(x)$ be the rate of change in the nominal exchange rate chosen by such a policymaker given the realization of the shock. Like his opportunistic counterpart a dependable policymaker also wants to choose π so as to maximize $V(\pi, \pi^e)$. Absent the band, his optimal policy is again $\pi^D(x) = x$. As long as $x \in [\underline{\pi}, \overline{\pi}]$, the exchange rate band does not constrain the policymaker, so $\pi^D(x) = x$. But if the resulting π falls outside the permissible range, $[\underline{\pi}, \overline{\pi}]$, and the dependable policymaker follows the discretionary policy, x, of his opportunistic counterpart he bears a cost c. He therefore needs to decide whether to stick to the band or realign it. It is shown in the Appendix that the dependable policymaker's choice in stage 3, as a function of the random shock x, is given by:

$$\pi^D(x) = \begin{cases} x, & \text{if } x < \underline{\pi} - \delta, \\ \underline{\pi}, & \text{if } \underline{\pi} - \delta \leq x \leq \underline{\pi}, \\ x, & \text{if } \underline{\pi} < x < \overline{\pi}, \\ \overline{\pi}, & \text{if } \overline{\pi} \leq x \leq \overline{\pi} + \delta, \\ x, & \text{if } x > \overline{\pi} + \delta, \end{cases} \tag{2}$$

where $\delta \equiv \sqrt{2c}$. The cost c, and therefore δ, reflect the present value of the future political costs of currently breaking the commitment to the band. Before continuing it is worth noting that the optimal decision of a dependable policymaker, $\pi^D(x)$, is independent of the public's expectations, π^e. This is due to the fact that the policymaker's objective function is linear in π and π^e. Consequently, our framework does not give rise to multiple equilibria as in Obstfeld (1996, 1997).[14]

[13] This solution is analogous to the well known Barro-Gordon (1983) discretionary solution for inflation. The difference is that here x may be either positive or negative, while in their model, x is always positive, producing a one sided bias.

[14] Obstfeld uses a standard Barro and Gordon (1983) objective function which is quadratic in both π and π^e. This leads to multiple equilibria that are absent in our linear specification.

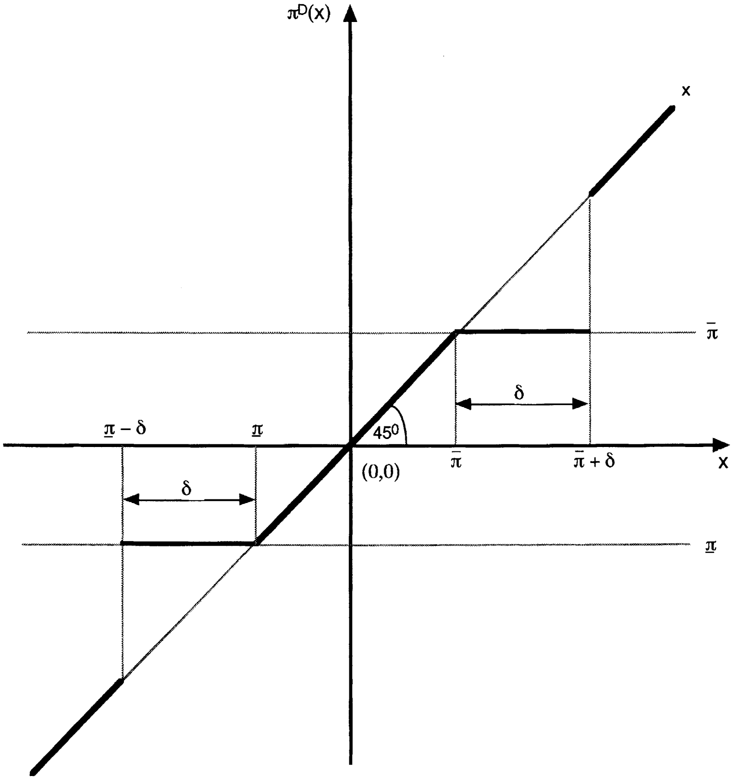

Fig. 2. The change in the exchange rate chosen by a dependable policymaker.

Unless stated otherwise, we shall assume that the political cost borne by a dependable policymaker when he realigns the exchange rate band is positive but not too large:

Assumption 1. $0 < \delta < \mathrm{Min}[-\underline{x}, \bar{x}]$.

The first inequality in Assumption 1 ensures that there is a real difference between the two policymakers' types. The second inequality implies that there exist sufficiently large realizations of x for which a dependable policymaker will realign the band. That is, even a dependable policymaker will not maintain the band *under all circumstances*.[15] Absent this assumption, the first and last lines in Eq. (2) disappear—$\pi^D(x)$ is then equal to x only inside the band and is equal to $\underline{\pi}$ if $x \leq \underline{\pi}$ and to $\bar{\pi}$ if $x \geq \bar{\pi}$. Proposition 2 below characterizes the optimal band when Assumption 1 fails.

Fig. 2 illustrates $\pi^D(x)$. When x falls inside the permissible range of rates of changes in the exchange rate, a dependable policymaker chooses $\pi = x$ just like his opportunistic counterpart would. When x falls between $\underline{\pi}$ and $\underline{\pi} - \delta$ or between $\bar{\pi}$ and $\bar{\pi} + \delta$, the

[15] This is a limited commitment in the spirit of Flood and Isard (1989) and Lohmann (1992).

dependable policymaker can no longer make this choice since this would involve a realignment of the band and entail a cost that is too high from his perspective. Consequently, the policymaker sets the nominal exchange rate at the boundaries of the band, implying that $\pi^D(x) = \underline{\pi}$ for relatively large negative realizations of x and $\pi^D(x) = \overline{\pi}$ for relatively large positive realizations of x. However, when x falls below $\underline{\pi} - \delta$ or above $\overline{\pi} + \delta$, the gain from realignment outweighs the cost c, so the dependable policymaker chooses $\pi^D(x) = x$ once again by adjusting the center rate downward if $x < \underline{\pi} - \delta$, and upward if $x > \overline{\pi} + \delta$. Thus, the exchange rate band gives rise to a positive *range of effective commitment* (REC), whenever $x \in [\overline{\pi},\ \overline{\pi} + \delta]$, and to a negative REC, when $x \in [\underline{\pi} - \delta,\ \underline{\pi}]$. When x lies in either of these RECs, the dependable policymaker is constrained by the band and sets π at the boundary of the band rather than at its ex post optimal level. The two RECs are illustrated in Fig. 3 in the traditional framework of the target zones literature in which the exchange rate is plotted as a function of time.

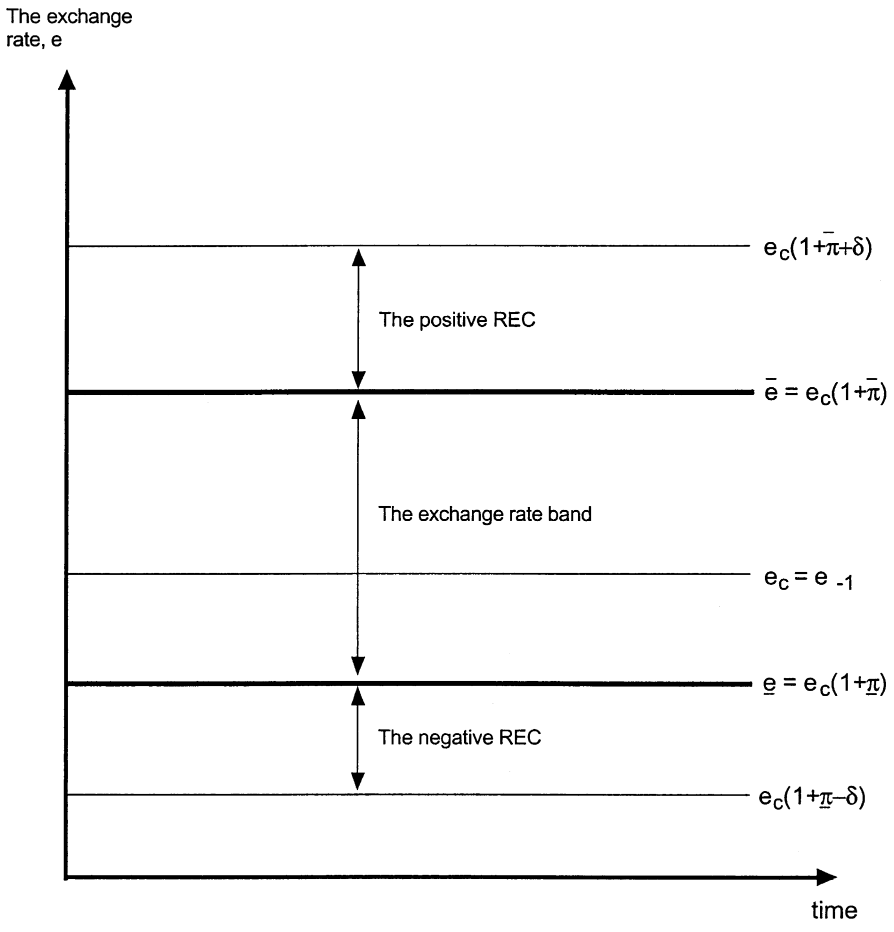

Fig. 3. The exchange rate band and the ranges of effective commitment (REC).

It is important to note that the width of the two RECs is equal to δ and is therefore beyond the control of the policymaker. What the policymaker can do is to shift the two RECs either closer to or away from 0 by choosing $\overline{\pi}$ and $\underline{\pi}$. This determines in turn the width of the range $[\underline{\pi}, \overline{\pi}]$ within which the policymaker can adjust the nominal exchange rate optimally ex post without incurring the cost c. Assumption 1 implies that the upper limit of the positive REC is lower than $\overline{x}$ and the lower limit of the negative REC is above $\underline{x}$.

3.2. Expectations about the rate of change in the exchange rate

Expectations are formed in stage 2 after the public observes the realization of x. At this point, the only remaining uncertainty concerns the policymaker's type. In a rational expectations equilibrium, the public's expectations are consistent with the equilibrium strategies of the two types of policymakers. Recalling that the public believes that the policymaker is dependable with probability α and opportunistic with probability $1-\alpha$, it follows that the expected value of π is given by

$$\pi^e(x) = \alpha\pi^D(x) + (1-\alpha)x. \tag{3}$$

Using (2) and (3) yields:

$$\pi^e(x) = \begin{cases} x, & \text{if} x < \underline{\pi} - \delta, \\ \alpha\underline{\pi} + (1-\alpha)x, & \text{if} \underline{\pi} - \delta \leq x \leq \underline{\pi}, \\ x, & \text{if} \underline{\pi} < x < \overline{\pi}, \\ \alpha\overline{\pi} + (1-\alpha)x, & \text{if} \overline{\pi} \leq x \leq \overline{\pi} + \delta, \\ x, & \text{if} x > \overline{\pi} + \delta. \end{cases} \tag{4}$$

Eq. (4) is illustrated in Fig. 4. When x lies outside the two RECs, the public correctly anticipates that under both policymaker types, the rate of change in the exchange rate will be x. By contrast, if x falls in one of the RECs, the public knows that with probability α, the policymaker is dependable and will therefore keep the exchange rate at the boundary of the band, and with probability $(1-\alpha)$, the policymaker is opportunistic and will change the exchange rate at a rate x.

The choice of band affects the public's expectations only in the positive and the negative RECs. In these ranges, the band has two main effects on expectations. First, when the band becomes wider, the two RECs shift further away from 0 although their width remains equal to δ. Second, and perhaps more importantly, as Fig. 4 illustrates, within the two RECs, the band moderates the public's expectations about the rates of change in the exchange rate, and as a result, it reduces the variability of expected depreciations or appreciations. For a given band width, this reduction becomes more substantial, the larger are the RECs (i.e., the larger is δ) and the higher is the reputation of policymakers (i.e., the larger is α).[16] That is, when the policymaker becomes more credible either because it is more costly for him to realign the band or because he has a higher reputation for being

[16] The moderating effect of the band on the public's expectations is reminiscent of the Krugman type 'honeymoon effect' stressed in the target zone literature of the early 90s. In both cases the band moderates expectations. But, unlike in the Krugman framework where the exchange rate is determined by market forces, here the exchange rate is fully managed by policymakers. As a consequence, in spite of the fact that it moderates expectations the band has no effect on the exchange rate and its volatility within the band.

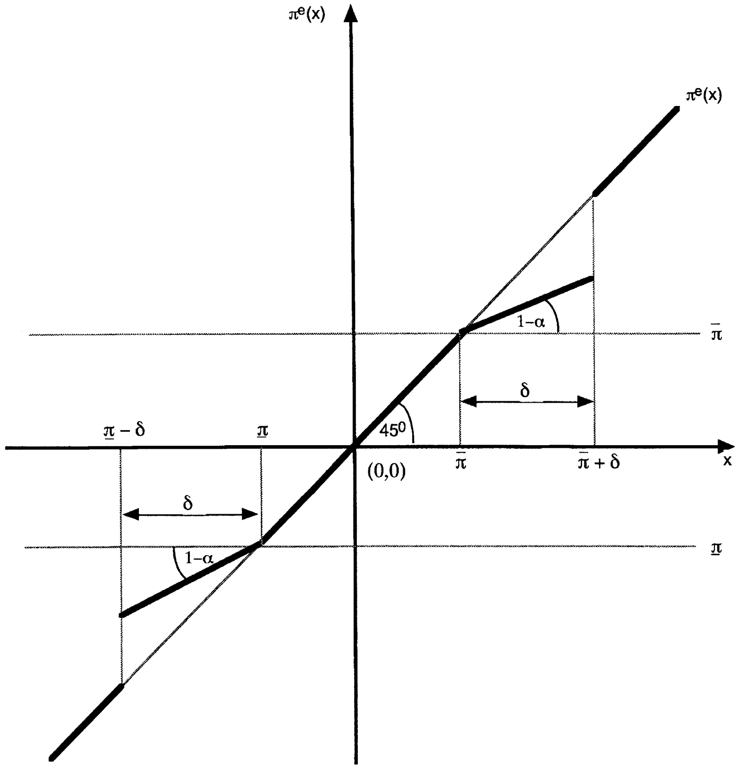

Fig. 4. The public's expectations about changes in the exchange rate as a function of the degree of misalignment, as measured by x.

dependable, expectations about the rate of change in the exchange rate become more stable since whenever x falls within the RECs, the public believes that it is more likely that the exchange rate will be kept at the boundaries of the exchange rate band.

Note that whenever the realization of x falls within the RECs, dependable policymakers are subject to a kind of 'Peso problem' in the sense that the expected rate of change in the exchange rate is larger than the actual rate. In spite of the fact that the dependable policymaker intends to maintain the band, the public does not rule out the possibility of a realignment. To illustrate, suppose that x falls in the positive REC so that $\overline{\pi} \leqslant x \leqslant \overline{\pi} + \delta$. In such a case a dependable policymaker maintains the band and devalues at the rate $\overline{\pi}$, so that the exchange rate reaches the upper limit of the band. But prior to this policy action the public's expectation is given, from Eq. (4), by $\alpha\overline{\pi} + (1 - \alpha)x$. In the absence of perfect reputation this expression is obviously larger than $\overline{\pi}$.

3.3. Choosing the exchange rate band

An opportunistic policymaker does not plan to honor his commitment to the band since he can exit from it at no cost. But he prefers not to reveal this fact at the announcement stage because once his identity is revealed, he totally loses the ability to move the real exchange rate in the desired direction by means of unanticipated depreciations or appreciations. Therefore the opportunistic policymaker announces the same band that a dependable policymaker would have announced. Hence we only need to characterize the band announced by a dependable policymaker. From the perspective of a dependable policymaker, the choice of the band involves a tradeoff between the moderating impact of the band on the public's expectations (credibility) and between his ability to move the exchange rate freely in accordance with the realization of the desired real exchange rate as proxied by the value of x (flexibility). To study this tradeoff, we substitute from (2) and (4) into (1), and express the policymaker's objective in terms of the random shock x:

$$V(x) \equiv V\left(\pi^D(x), \pi^e(x)\right) = \begin{cases} -\dfrac{x^2}{2} - c, & \text{if } x < \underline{\pi} - \delta, \\ x(1-\alpha)(\underline{\pi} - x) - \dfrac{\underline{\pi}^2}{2}, & \text{if} \underline{\pi} - \delta \le x \le \underline{\pi}, \\ -\dfrac{x^2}{2}, & \text{if} \underline{\pi} < x < \overline{\pi}, \\ x(1-\alpha)(\overline{\pi} - x) - \dfrac{\overline{\pi}^2}{2}, & \text{if} \overline{\pi} \le x \le \overline{\pi} + \delta, \\ -\dfrac{x^2}{2} - c, & \text{if} x > \overline{\pi} + \delta. \end{cases} \tag{5}$$

Using this equation, the expected payoff of the policymaker before x is realized is given by,

$$\begin{aligned} EV(x) &= -\int_{\underline{x}}^{\underline{\pi}-\delta} \left[\frac{x^2}{2} + c\right] \mathrm{d}F(x) + \int_{\underline{\pi}-\delta}^{\underline{\pi}} \left[x(1-\alpha)(\underline{\pi} - x) - \frac{\underline{\pi}^2}{2}\right] \mathrm{d}F(x) \\ &\quad - \int_{\underline{\pi}}^{\overline{\pi}} \frac{x^2}{2} \mathrm{d}F(x) + \int_{\overline{\pi}}^{\overline{\pi}+\delta} \left[x(1-\alpha)(\overline{\pi} - x) - \frac{\overline{\pi}^2}{2}\right] \mathrm{d}F(x) \\ &\quad - \int_{\overline{\pi}+\delta}^{\overline{x}} \left[\frac{x^2}{2} + c\right] \mathrm{d}F(x) \\ &= -\int_{\underline{x}}^{\overline{x}} \frac{x^2}{2} \mathrm{d}F(x) - cF(\underline{\pi} - \delta) - c(1 - F(\overline{\pi} + \delta)) \\ &\quad - \int_{\underline{\pi}-\delta}^{\underline{\pi}} \left[\frac{(\underline{\pi} - x)^2}{2} + \alpha x(\underline{\pi} - x)\right] \mathrm{d}F(x) \\ &\quad - \int_{\overline{\pi}}^{\overline{\pi}+\delta} \left[\frac{(\overline{\pi} - x)^2}{2} + \alpha x(\overline{\pi} - x)\right] \mathrm{d}F(x), \end{aligned} \tag{6}$$

where $F(x)$ is the cumulative distribution of x. To interpret this expression, note that if the policymaker does not commit to an exchange rate band, he chooses $\pi = x$ in stage 3, and this is what the public expects him to do. Therefore the value of the policymaker's objectives in that case is $V(x, x) = -x^2/2$. Thus, the first term in (6) is the policymaker's expected loss under full discretion. The second and third terms in (6) represent the expected cost that the policymaker incurs when he realigns the band. The fourth and fifth terms represent the expected increment to the policymaker's payoff (over states of nature in which x falls in the negative and positive RECs) from announcing the exchange rate band. This expected increment (which could be positive or negative) is composed of two components. The first term in each integrand represents the expected cost of lost flexibility, stemming from the fact that in the two RECs, the policymaker is constrained by the boundaries of the band and cannot adjust π optimally in line with the realization of x. The second term in each integrand represents the expected benefit associated with the moderating effect that the announcement of the band has on the public's expectations about π in the two RECs.

Note that as δ approaches zero, both policymaker types become opportunistic as none of them incurs a cost for realigning the band. In this case, all but the first term in (6) vanish, so the choice of $\underline{\pi}$ and of $\overline{\pi}$ becomes indeterminate. That is, the exchange rate band becomes irrelevant since it is common knowledge that the policymaker is going to ignore it in stage 3. Assumption 1 which requires that $\delta > 0$ rules out this possibility.

The policymaker chooses the bounds of the permissible range of rates of change in the exchange rate, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize $EV(x)$. Recalling that $\delta = \sqrt{2c}$, the first order conditions for an interior solution to the policymaker's problem (i.e., for $-\infty < \underline{\pi} < \overline{\pi} < \infty$) are:

$$\begin{aligned}\frac{\partial EV(x)}{\partial \underline{\pi}} &= \alpha\delta(\underline{\pi}-\delta)f(\underline{\pi}-\delta) + \int_{\underline{\pi}-\delta}^{\underline{\pi}} [(x-\underline{\pi}) - \alpha x]\mathrm{d}F(x) \\ &= \int_{\underline{\pi}-\delta}^{\underline{\pi}} (x-\underline{\pi})\mathrm{d}F(x) - \alpha\left[\int_{\underline{\pi}-\delta}^{\underline{\pi}} x\mathrm{d}F(x) - \delta(\underline{\pi}-\delta)f(\underline{\pi}-\delta)\right] = 0,\end{aligned} \tag{7}$$

and,

$$\begin{aligned}\frac{\partial EV(x)}{\partial \overline{\pi}} &= \alpha\delta(\overline{\pi}+\delta)f(\overline{\pi}+\delta) + \int_{\overline{\pi}}^{\overline{\pi}+\delta} [(x-\overline{\pi}) - \alpha x]\mathrm{d}F(x) \\ &= \int_{\overline{\pi}}^{\overline{\pi}+\delta} (x-\overline{\pi}) - \alpha\left[\int_{[\pi}^{\overline{\pi}+\delta} x\mathrm{d}F(x) - \delta(\overline{\pi}+\delta)f(\overline{\pi}+\delta)\right] = 0.\end{aligned} \tag{8}$$

The first term in the last line of (8) is positive. It represents the marginal benefit from a slight increase in $\overline{\pi}$ due to the fact that, at a higher $\overline{\pi}$, there is a narrower gap between the ex post optimal rate of depreciation, x, and $\overline{\pi}$ (which is the rate that a dependable policymaker is committed to over the REC). In other words, this term is the marginal benefit from increasing the flexibility to respond ex post to real exchange rate misalignments.

The two terms inside the brackets in the last line of (8) represent the total effect of the increase in $\overline{\pi}$ on $EV(x)$ through the associated change in expectations. This total effect is composed of two parts that affect expectations, and therefore expected objectives, in opposite directions. The first term inside the brackets in (8) is due to the fact that following the increase in $\overline{\pi}$, the public expects that whenever x falls in the positive REC, a dependable policymaker will choose a depreciation level $\overline{\pi}$ which is slightly higher than before. This partial effect reduces the value of expected objectives of policymakers.

But the increase in $\overline{\pi}$ also triggers a moderating effect on expected depreciation and exerts, therefore, a partial positive influence on $EV(x)$. This partial effect is reflected by the second term inside the brackets in Eq. (8). It is due to the fact that, although a small increase in $\overline{\pi}$ shifts both the upper and the lower bounds of the positive REC away from 0 and hence affects expectations in opposite directions, the first effect dominates. The reason is that $\pi^e(x)$ is continuous at the lower bound but has an upward jump at the upper bound (see Fig. 4). As a consequence, the former shift has only a negligible impact on $\pi^e(x)$, whereas the latter shift has a non-negligible moderating effect on $\pi^e(x)$. This moderating effect arises because, after the increase in $\overline{\pi}$, the public expects that the rate of change in the exchange rate at $x = \overline{\pi} + \delta$ will be $\alpha\overline{\pi} + (1 - \alpha)(\overline{\pi} + \delta)$ rather than $\overline{\pi} + \delta$.

Although at first blush it appears from the preceding discussion that the total effect of an increase in $\overline{\pi}$ on expected objectives is generally ambiguous, at an internal solution for $\overline{\pi}$, the total effect is unambiguously negative. This implies that, overall, expected depreciation goes up when the upper bound of the band is raised. The intuitive reason is that, at an internal maximum, the marginal benefits in terms of better flexibility of a wider upper bound for the band have to be balanced at the margin by the cost of a higher expected depreciation.

Eq. (7) has a similar interpretation. In particular, a widening of the band below the center rate (a decrease in $\underline{\pi}$) raises expected objectives at the margin by increasing flexibility and (provided the lower bound of the band is internal) reduces them at the margin by raising expected appreciation. Thus both first order conditions essentially state that, at the margin, the two bounds of the band are determined by equating the marginal benefit of better credibility (lower expected depreciation or appreciation achieved by a marginal narrowing of the band) to the marginal cost of less flexibility.

4. Determinants of the width of the band

This section examines the characteristics of the optimal exchange rate system. It begins by identifying the conditions under which it is optimal to have a free float, a peg, or a band, and in the latter case whether the band is symmetric or asymmetric. It then continues by investigating how the width of the band (in cases where it is optimal to set a band) responds to changes in the two main exogenous parameters of the model which are the policymaker's reputation, α, and the cost of realignment, δ.

4.1. Is the optimal system a peg, a band, or a free float?

Since it nests a peg and a free float as particular cases, the exchange rates band framework presented here can also be used to identify conditions under which each of those extreme systems is desirable. This subsection shows that the answer to the question posed in the heading of the subsection largely depends on the characteristics of the distribution of the shock x. The main characterization results appear in the first proposition below. Loosely speaking, the proposition states, inter alia, that if the distribution of the shock is uniform or has fatter tails than a uniform distribution (i.e., $f(x)$ is increasing for $x > 0$ and decreasing for $x < 0$) a free float is optimal. On the other hand, a non degenerate band (positive but finite width) is optimal only if $f(x)$ increases for some $x < 0$ and decreases for some $x > 0$.

Using (7) and (8), we establish the following proposition (the proof, along with all other proofs, is in the Appendix):

Proposition 1. *Given Assumption 1, the solution to the policymaker's problem has the following properties:*

(i) The boundaries of the exchange rate band are such that $\underline{\pi} < 0 < \overline{\pi}$; i.e., a fixed exchange rate is never optimal.
(ii) If f(x) is nonincreasing for $x < 0$ (i.e., the range over which the target for the real exchange rate is lower than the preexisting level), the lower bound of the exchange rate band will be such that $\underline{\pi} = \underline{x}$, so that, below the center rate, the exchange rate is completely flexible. Likewise, if f(x) is nondecreasing for $x > 0$ (the range for which the target for the real exchange rate exceeds the preexisting level), the upper bound of the exchange rate band will be such that $\overline{\pi} = \overline{x}$, so that, above the center rate, the exchange rate is completely flexible.
(iii) If $\underline{\pi}$ attains an interior solution so that $\overline{\pi} > \underline{x}$, then f(x) must be increasing over at least part of the negative REC. If $\overline{\pi}$ attains an interior solution so that $\overline{\pi} < \overline{x}$, then f(x) must be decreasing over at least part of the positive REC.
(iv) If f(x) is symmetric around 0, so that $f(-x)=f(x)$ for all $x > 0$, the band will be symmetric around 0 in the sense that $-\underline{\pi} = \overline{\pi}$.

To understand the first part of Proposition 1, recall that a fixed exchange rate is not fixed under all circumstances since even a dependable policymaker is willing to exit from the band or the peg ex post if the perceived divergence between the desired and the real exchange rate is such that a realignment increases his payoff by more than c. What a fixed exchange rate does is to push the two ranges of effective commitment all the way towards 0 so under a dependable policymaker, the exchange rate is truly fixed only when x is between $-\delta$ and δ. It turns out that under Assumption 1, the marginal benefit from lowering $\underline{\pi}$ (raising $\overline{\pi}$) slightly below (above) 0 and gaining some flexibility, exceeds the corresponding marginal loss from raising the public's expectations for appreciation (depreciation) when x falls in the negative (positive) REC. Hence, under Assumption 1, a fixed exchange rate is never optimal (but as we shall see below, when Assumption 1 fails, a peg is optimal when the policymaker's reputation is perfect).

While a fixed exchange rate is never optimal under Assumption 1, a completely flexible exchange rate might be optimal. In particular, part (ii) of Proposition 1 shows that a sufficient condition for such a regime is that the distribution of x will be everywhere nonincreasing for $x < 0$ and nondecreasing for $x > 0$. Thus, for instance, if x is distributed uniformly over the support $[\underline{x}, \overline{x}]$ (i.e., every possible real exchange rate misalignment is equally likely), then it is optimal to set an exchange rate band that never binds, so that the exchange rate is fully flexible. Indeed, part (iii) of Proposition 1 shows that necessary conditions for internal solutions for $\underline{\pi}$ and for $\overline{\pi}$ (the nominal exchange rate is not fully flexible) are that $f(x)$ is increasing on at least part of the range of negative values of x, and decreasing on at least part of the range of positive values of x. That is, it must be the case that (some) small deviations of actual from desired real exchange rates are more likely than some larger ones. It appears reasonable to presume that in reality smaller deviations of actual from desired real exchange rates are more likely than larger deviations. Under this presumption the distribution of x has a single mode at zero and part (iii) of the proposition implies that there is an interior solution for the optimal band.

Part (iv) of Proposition 1 states that a sufficient condition for a symmetric band is that the distribution of perceived real exchange rate misalignments is symmetric around 0 (i.e., a pressure for depreciation is as likely as the pressure for appreciation). Although this condition is not necessary, the optimal band is generally not symmetric when the distribution of x is not symmetric. This is illustrated for the case of a non symmetric triangular distribution in Section 4.3 below.

Proposition 1 relies on Assumption 1 which implies that even a dependable policy-maker will realign under *some* circumstances. The following result examines the case in which Assumption 1 fails.

Proposition 2. *Suppose that $\delta > Max[-\underline{x}, \bar{x}]$. Then the optimal exchange rate system is a peg if reputation is perfect, i.e., $\alpha = 1$, but a nondegenerate band with $\underline{x} < \underline{\pi} < 0 < \bar{\pi} < \bar{x}$ otherwise.*

Proposition 2 shows that when the political cost that a dependable policymaker bears in case of a realignment is sufficiently high, it is never optimal to have a free float. However, a peg is also not optimal unless the policymaker's reputation is perfect. Taken together, Propositions 1 and 2 imply that a peg is chosen when the policymaker has perfect reputation in the sense that following the announcement of the peg, it is common knowledge that the exchange rate will indeed remain fixed *under all circumstances*. An immediate implication of this result is that 'hard pegs' or tight currency boards are optimal only when reputation is perfect. The currency board of Hong Kong, which has never been abandoned since its inauguration in 1984, appears to be a close real life counterpart to this corner solution. But when the public believes that there are circumstances under which even a dependable policymaker will prefer to break his commitment to the previously announced policy, the optimal system is a nondegenerate band.

4.2 *The effects of reputation and of the cost of realignment on optimal band width*

In this subsection we report comparative statics results regarding the optimal band under the assumption that the band is non degenerate (i.e., $\underline{x} < \underline{\pi} < 0 < \bar{\pi} < \bar{x}$). We report first how changes in the policymaker's reputation, α, affect the width of the band.

Proposition 3. *The optimal exchange rate band is wider the lower the reputation, α, of policymakers. In the limit, as α approaches 0, $\underline{\pi} = \underline{x}$, and $\bar{\pi} = \bar{x}$ so the exchange rate becomes completely flexible.*

Proposition 3 implies that a policymaker with a higher reputation for being dependable can afford to set a narrower band. Since a narrower band means that there is a higher likelihood that the band will be realigned, we get the interesting, and somewhat counterintuitive, prediction that the more reputable is the policymaker himself (i.e., the higher is α), the less credible is his commitment to the band in the sense that he is more likely to realign it. An intuitive explanation of this seeming puzzle is as follows. When the policymaker has a better reputation, the band has a greater moderating influence on the public's expectations for depreciation (when $x > 0$) or appreciation (when $x < 0$) so the benefit from setting a narrower band is larger. Consequently, the policymaker is more willing to take the risk that the realization of the shock x will be such that the band will have to be realigned and he will, therefore, incur the cost c.[17] In the limit as α approaches 0, the policymaker totally loses the ability to use the exchange rate band to moderate the public's expectations. The reason, of course, is that the public expects him to completely ignore the band and set $\pi = x$. In that case, the policymaker might as well set a band that covers the entire support of x, thereby retaining the flexibility to adjust π optimally ex post without incurring a political cost for doing so.

Next we consider how the width of the exchange rate band depends on the cost that a dependable policymaker incurs when he exits the band. It would seem at first blush that, since an increase in δ raises the credibility of the band in the sense that the RECs become wider, δ will affect the width of the band just like α. The following proposition shows however that this is not necessarily so.

Proposition 4. *Let $\eta(x) \equiv -f'(x)x/f(x)$ be the elasticity of the distribution function f(x). Then, when δ increases, π shifts downward if $\eta(\underline{\pi} - \delta) < (1+\alpha)/\alpha$ and upward otherwise, while π shifts upward if $\eta(\overline{\pi} + \delta) < (1+\alpha)/\alpha$ and downward otherwise.*

Proposition 4 shows that although an increase in δ widens the two RECs, it may or may not make the exchange rate band itself wider, depending on the elasticity of the distribution of the shocks evaluated at the extreme bounds of the two RECs. An interesting implication of Proposition 4 is that an increase in c (which leads to an increase in δ) might induce the policymaker to set a narrower band, in which case the likelihood that the policymaker will realign the band and incur the cost c increases. The reason why an increase in c may induce the policymaker to act in a way that puts him at a greater risk of incurring this cost is that the increase in c may have a sufficiently large beneficial moderating effect on the public's expectations which outweighs the risk of setting a narrower band.

The intuition underlying the proposition for the upper bound of the band, $\overline{\pi}$, is as follows. (The intuition for the lower bound is analogous). On one hand, an increase in δ raises the marginal flexibility benefit of an increase in $\overline{\pi}$ and, taken alone, tends to widen the band (the partial derivative of the first term on the right side of the last line in (8) is positive). On the other hand, by raising the REC, an increase in δ raises the marginal cost of increasing $\overline{\pi}$ through the resulting increase in expected depreciation. This effect tends to tighten the band (the partial derivative of the second term on the right side of the last line in (8) is negative). In addition, depending on the slope of $f(x)$ at the upper bound of the REC, an increase in δ may contribute an additional upward or downward jolt to the marginal effect of an increase in $\overline{\pi}$ via expected depreciation (the sign of the partial derivative of the

[17] The result in Proposition 3 is analogous to a result in Cukierman and Liviatan (1991) which states, in the context of Barro (1986) type monetary policy games, that dependable policymakers with better reputation commit to more ambitious inflation targets. In both frameworks the origin of the result is that, with better reputation, the policymaker's announcement has a stronger marginal impact on expectations.

last term in (8) is generally ambiguous). The proposition reveals that the combined impact of those three effects depends on the relative magnitudes of $\eta(x)$ and α.

4.3. *A particular distribution of misalignments*

To illustrate Propositions 1 and 2, let us consider the case where x has a triangular distribution on the interval $[\underline{x}, \overline{x}]$, so that:

$$f(x) = \begin{cases} \dfrac{2}{\overline{x} - \underline{x}}\left(1 - \dfrac{x}{\underline{x}}\right) & \text{if } x \leq 0, \\ \dfrac{2}{\overline{x} - \underline{x}}\left(1 - \dfrac{x}{\overline{x}}\right) & \text{if } x > 0. \end{cases} \tag{9}$$

The distribution is symmetric only if $-\underline{x} = \overline{x}$. Substituting for $f(x)$ in (7) and (8), solving for $\underline{\pi}$ and $\overline{\pi}$ and dividing by $\underline{x}$ and $\overline{x}$, respectively, yields the following expressions:

$$\frac{\underline{\pi}}{\underline{x}} = \begin{cases} \dfrac{1+\alpha}{1+2\alpha} + \dfrac{2}{3}\dfrac{\delta}{\underline{x}}, & 0 > \dfrac{\delta}{\underline{x}} > -1, \\ \dfrac{1+\alpha}{1+2\alpha} - \dfrac{2}{3}, & \dfrac{\delta}{\underline{x}} \leq -1, \end{cases} \qquad \frac{\overline{\pi}}{\overline{x}} = \begin{cases} \dfrac{1+\alpha}{1+2\alpha} - \dfrac{2}{3}\dfrac{\delta}{\overline{x}}, & 0 < \dfrac{\delta}{\overline{x}} < 1, \\ \dfrac{1+\alpha}{1+2\alpha} - \dfrac{2}{3}, & \dfrac{\delta}{\overline{x}} \geq 1. \end{cases} \tag{10}$$

Eq. (10) expresses the boundaries of the band relative to the maximal possible shocks and thereby provides a natural metric for band width. As $\overline{\pi}/\overline{x}$ and $\underline{\pi}/\underline{x}$ decrease from 1 to 0, the band gradually becomes tighter and tends towards a peg. The extreme case in which $\overline{\pi}/\overline{x} = \underline{\pi}/\underline{x} = 0$ corresponds to a peg and the opposite extreme case in which $\overline{\pi}/\overline{x} = \underline{\pi}/\underline{x} = 1$ corresponds to a free float (under a free float, policymakers retain the flexibility to set π ex post at any level they wish).

Since Eq. (10) implies that $\overline{\pi}/\overline{x}$ and $\underline{\pi}/\underline{x}$ are less than 1, so a free float is never optimal. This is consistent with part (ii) of Proposition 1 and with Proposition 2 since the triangular distribution is strictly increasing for $x < 0$ and strictly decreasing for $x > 0$. Moreover, consistent with part (i) of Proposition 1 and Proposition 2, a peg is optimal only when $\alpha = 1$ and $\delta > \text{Max}[-\underline{x}, \overline{x}]$ (in which case, $\delta/\overline{x} \geq 1$ and $\delta/\underline{x} \leq -1$) since only then $\overline{\pi}/\overline{x} = \underline{\pi}/\underline{x} = 0$. Otherwise, (10) shows that $\underline{x} < \underline{\pi} < 0 < \overline{\pi} < \overline{x}$, so the optimal regime is a nondegenerate band. Eq. (10) also shows that a symmetric band is optimal only when $f(x)$ is symmetric. Otherwise, the band is wider above the center rate if positive realizations of x (for which the desirable policy absent a band is depreciation) are more likely than negative realizations of x (for which the desirable policy absent a band is appreciation), i.e., whenever $\overline{x} > -\underline{x}$. Conversely, the band will be wider below the center rate if negative realizations of x are more likely than positive realizations of x.

Eq. (10) also shows that, consistent with Propositions 3 and 4, the optimal band becomes tighter as α and δ increase.[18] Fig. 5 illustrates this by depicting $\overline{\pi}/\overline{x}$ for alternative values of α

[18] It can be verified that in the triangular distribution case, as long as $\underline{\pi} - \delta > \underline{x}$ and $\overline{\pi} + \delta < \overline{x}$, the conditions $\eta(\underline{\pi} - \delta) > (1+\alpha)/\alpha$ and $\eta(\overline{\pi} + \delta) > 1 + \alpha/\alpha$ hold. Hence Proposition 4 implies that the band tightens as δ increases.

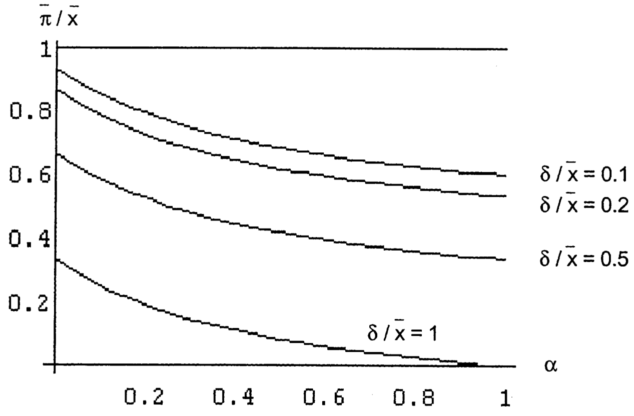

Fig. 5. Illustration of the effects of α and δ on the exchange rate band when x has a triangular distribution and $\bar{x} = 1$.

and δ (the figure for $\underline{\pi}/\underline{x}$ is analogous). The figure shows that as reputation tends to 1, the band becomes tighter although this occurs at a diminishing rate. In addition, the figure shows that the band tightens uniformly as $\delta/\bar{x}$ increases from 0 to 1.

Finally, Eq. (10) shows that the band widens as the support of x increases. This means that a wide band is associated with a large uncertainty about exchange rate misalignments, x. This result is consistent with the observation that during the 1990s many countries with exchange rate bands widened their bands, in some cases, following exchange rate crises.[19] This mounting perception of wider spreads in potential exchange rate misalignment is probably due to a combination of several factors. But there is little doubt that an important contributing factor is the substantial increase in the volume of international financial flows following the removal of restrictions on such flows. Eq. (10) implies that this trend is likely to be accompanied by a flexibilization of the exchange rate system.

5. The case where π^e is set before x is realized leads to one sided bands and pegs

Thus far we assumed that in stage 2, the public forms expectations about π *after* the shock x was realized. Although this simplification makes it possible to examine several issues related to the choice of band more sharply, it largely excludes the possibility of stabilization policy. To allow for this possibility, we invert the timing of the model by assuming that expectations are formed *before* x is realized. Under this alternative timing, π^e reflects not only political uncertainty (i.e., whether the policymaker is dependable or opportunistic) but also uncertainty about real exchange rates misalignments. As a result,

[19] Further analysis in a previous version of the paper (available upon request) shows that this result applies more generally. Practically all the countries mentioned at the beginning of the Introduction recently raised the flexibility of their exchange rate systems. Brazil, The Czech Republic, Ecuador, Mexico and Sweden even instituted fully flexible rates.

the expected rate of change in the exchange rate is derived by taking the expectation of $\pi^e(x)$ over the entire support of x. Using (4) we, therefore, have:

$$\begin{aligned}\pi^e \equiv \quad & E\pi^e(x) = \int_{\underline{x}}^{\underline{\pi}-\delta} x\,\mathrm{d}F(x) + \int_{\underline{\pi}-\delta}^{\underline{\pi}} [\alpha\underline{\pi} + (1-\alpha)x]\mathrm{d}F(x) + \int_{\underline{\pi}}^{\overline{\pi}} x\,\mathrm{d}F(x) \\ & + \int_{\overline{\pi}}^{\overline{\pi}+\delta} [\alpha\overline{\pi} + (1-\alpha)x]\mathrm{d}F(x) + \int_{\overline{\pi}+\delta}^{\overline{x}} x\,\mathrm{d}F(x) \\ = \quad & \hat{x} + \alpha\int_{\underline{\pi}-\delta}^{\underline{\pi}} (\underline{\pi}-x)\mathrm{d}F(x) + \alpha\int_{\overline{\pi}}^{\overline{\pi}+\delta} (\overline{\pi}-x)\mathrm{d}F(x),\end{aligned} \tag{11}$$

where $\hat{x}$ is the expected value of x.

Substituting (11) and (2) into (1), the expected payoff of the policymaker when he chooses the band is,

$$\begin{aligned}EV(x) = & \int_{\underline{x}}^{\underline{\pi}-\delta} \left[\frac{x^2}{2} - x\pi^e - c\right]\mathrm{d}F(x) + \int_{\underline{\pi}-\delta}^{\underline{\pi}} \left[x(\underline{\pi}-\pi^e) - \frac{\underline{\pi}^2}{2}\right]\mathrm{d}F(x) \\ & + \int_{\underline{\pi}}^{\overline{\pi}} \left[\frac{x^2}{2} - x\pi^e\right]\mathrm{d}F(x) + \int_{\overline{\pi}}^{\overline{\pi}+\delta} \left[x(\overline{\pi}-\pi^e) - \frac{\overline{\pi}^2}{2}\right]\mathrm{d}F(x) \\ & + \int_{\overline{\pi}+\delta}^{\overline{x}} \left[\frac{x^2}{2} - x\pi^e - c\right]\mathrm{d}F(x) = \int_{\underline{x}}^{\overline{x}} \frac{x^2}{2}\mathrm{d}F(x) - \hat{x}\pi^e - cF(\underline{\pi}-\delta) \\ & - c(1-F(\overline{\pi}+\delta)) - \int_{\underline{\pi}-\delta}^{\underline{\pi}} \frac{(\underline{\pi}-x)^2}{2}\mathrm{d}F(x) - \int_{\overline{\pi}}^{\overline{\pi}+\delta} \frac{(\overline{\pi}-x)^2}{2}\mathrm{d}F(x).\end{aligned} \tag{12}$$

Eq. (12) shows that if $\hat{x} = 0$ (on average, the target for the real exchange rate coincides with the preexisting level), the public's expectations, π^e, have no effect on the policymaker's objective. Hence, the policymaker cannot benefit from restricting his flexibility to respond ex post to misalignments and, as a consequence, the optimal system is a free float.[20] This is analogous to the well known result, in the literature on strategic monetary policy, that a central banker who targets the natural rate of employment does not produce any systematic inflation bias (see e.g., McCallum, 1995; Blinder, 1998). Hence there is no need to confront the central bank with a Walsh (1995), or any other type, of optimal contract since it provides the socially optimal level of stabilization without any bias in the first place. Similarly, when neither depreciations nor appreciations are desirable, a free float is optimal since it enables policymakers to provide the appropriate amount of stabilization of the real exchange rate without triggering any systematic expectation of a depreciation or appreciation.

The picture changes when $\hat{x} \neq 0$. For example, if $\hat{x} > 0$, the public expects on average a pressure for depreciation. Then, imposing ex ante restrictions on exchange rate variability affects expectations in a desirable direction and the familiar tradeoff between the credibility

[20] Indeed, straightforward differentiation of $EV(x)$ with respect to $\underline{\pi}$ and $\overline{\pi}$ reveals that $\partial EV(x)/\partial\overline{\pi} > 0 > \partial EV(x)/\partial\underline{\pi}$, implying that at the optimum, $\underline{\pi} = \underline{x}$ and $\overline{\pi} = \overline{x}$.

needed to reduce the depreciation bias and the flexibility required for the stabilization of the real exchange rate reappears. The following proposition characterizes the optimal band for this case and shows that it will be one sided. The case where $\hat{x}<0$ is completely analogous.

Proposition 5. (One sided band) *Suppose that π^e is formed before the realization of x. Then, if f(x) is increasing for $x<0$ and decreasing for $x>0$ (i.e., f(x) has a single mode at 0) and if $\hat{x}>0$, the lower bound of the exchange rate band will be such that $\underline{\pi}=\underline{x}$, so the exchange rate will be completely flexible below the center rate, while the upper bound of the band will be such that $\overline{\pi}<\overline{x}$. In addition, if $\int_0^\delta xf(x)\mathrm{d}x/(F(\delta)-F(0))>\alpha\hat{x}$ (the expectation of x conditional on it being positive but less than δ exceeds $\alpha\hat{x}$), the upper bound of the band will exceed zero so the policymaker will set up a one-sided non degenerate band above the center rate.*

Proposition 5 states that when expectations are formed before the public observes *x*, then, if on average the public expects a depreciation, the policymaker will let the exchange rate be completely flexible under the center rate. Establishing a band under the center rate is undesirable on two counts. First, it raises the expectation of depreciation further above 0, thereby making it more costly for the policymaker to achieve any given unexpected depreciation (which on average is desirable ex post). Second, a band under the center rate involves a loss in the flexibility to appreciate the exchange rate if this is needed (i.e., if $x<0$). By contrast, in spite of the associated loss of flexibility, a band above the center rate or a one sided peg are always beneficial because they moderate the public's expectation of a depreciation to sufficiently lower the policymaker's overall cost of achieving a given rate of unexpected depreciation. Tightening the band above the center rate plays here the same role as an increase in the conservativeness of the central bank in Rogoff's (1985) framework. In both cases such changes moderate undesirable expectations at the cost of diminished flexibility to optimally stabilize the economy ex post.

When will the policymaker opt for a one sided peg and when for a one sided band? The proposition states that a sufficient condition for the latter case is that the expectation of *x* conditional on it being positive but less than δ exceeds $\alpha\hat{x}$. This sufficient condition is more likely to hold when the cost of realignment, δ, is large and the policymaker's reputation, α, is small, or when the expected value of x, $\hat{x}$, is small. In general, anything that reduces the impact of the band on expectations and raises the cost of foregone flexibility makes it more likely that the regime will be a one sided band rather than a one sided peg.[21]

5. Concluding remarks

This paper develops a framework for analyzing the choice of unilateral exchange rate bands by policymakers with varying degrees of reputation for dependability. Such bands are shown to emerge as the outcome of optimization under a policy objective function that

[21] For unimodal distributions with a mode at zero the condition $\int_0^\delta xf(x)\mathrm{d}x/(F(\delta)-F(0))>\alpha\hat{x}$ is satisfied for any $\alpha\leq 1$. It follows from Eq. (A.26) in the Appendix that for such distributions the exchange rate regime will never be a one sided peg.

weighs the cost of deviations of actual from desired real exchange rates against the variability of the nominal exchange rate.

The paper shows that reputation for dependability has systematic effects on the choice of band width. In particular, when price setters face only political uncertainty, the model implies that, unless reputation is perfect and the political cost of realignment is prohibitive, a fixed exchange rate is never optimal. A similar result arises when price setters face both political and economic uncertainty provided the distribution of economic shock is unimodal with a mode at zero. This carries with it the implication that, unless they will be maintained under all circumstances, 'hard pegs' should not be used.[22] Hong Kong's currency board appears to be a close real life counterpart to the case of a hard peg of this kind. In addition the relation between reputation and band type is monotonic. In particular, better reputation for dependability is generally conducive to the establishment of narrower bands.

Some of the discussion in the paper is consistent with the view that the recently observed shift toward more flexible exchange rate systems in various countries reflects policymakers' optimal response to a rise in the variability of shocks due to an increase in the degree of international capital mobility. In particular the paper shows that when the probability of large misalignments is larger than that of small misalignments the optimal exchange rate regime is a free float. But when the opposite is true, the optimal regime is a non degenerate band whose width is positively related, under reasonable conditions, to the spread of the distribution of misalignments.

Although the paper provides mostly a positive analysis, some of the results on optimal band width can also be applied prescriptively. For instance, given a distribution of potential misalignments, the framework of the paper can be used to instruct a policymaker with a given reputation for dependability and a given political cost of realigning the band, about the type of considerations involved in deciding whether or not to opt for a band, and when he does, how should the band look like (i.e., how wide should it be, whether it should be symmetric or not, one sided, and so on).

The framework of the paper can also be used to investigate the effects of various parameters on the expected size of realignments as well as on the expected rate of change in the exchange rate, and through it, on the interest rate differential. Although we have some results regarding those issues they have not been presented for reasons of brevity. In particular it is possible to characterize the effects of the distance of the current exchange rate from the center rate and of reputation and the cost of exiting the band on the interest rate differential and on the expected size of realignments.[23] The analytical framework can also be used to characterize conditions that are conducive to multiperiods bands but this too is beyond the scope of this paper.[24]

This paper has posited that, as is the case in a number of countries, policymakers manage the exchange rate. Since in some other countries the exchange rate is not managed at all, it is

[22] Or, alternatively, that other considerations than those analyzed here are involved in the choice of tight currency boards. Schuler (1996) has strongly advocated tight currency boards for developing countries.

[23] For example, it can be shown that for symmetric distributions, the public expects a depreciation or an appreciation depending on whether the current exchange rate is above or below the center rate. Furthermore, when it is above (below) the center rate, expected depreciation (appreciation) is lower the higher are reputation and the political costs of exiting the band.

[24] Frameworks with two periods' bands appear in Cukierman et al. (1994, 1996, Section 5).

desirable to examine the robustness of our results to the alternative assumption that, except within the ranges of effective commitment, the exchange rate is left to be determined by market forces. A preliminary analysis along those lines appears in Cukierman and Spiegel (2000). Finally, the presence of speculators that rush to attack the currency whenever they expect that other speculators will do so—thereby forcing a realignment—introduces additional considerations into the choice of exchange rate regime. A framework that incorporates such considerations is developed in Cukierman et al. (2002).

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Appendix

The policymaker's objective function (Eq. (1))

This part of the Appendix demonstrates that Eq. (1) in the text can be viewed as an approximation to the following objective function:

$$-L(R) - \frac{\pi^2}{2}, \qquad L(R) \equiv \frac{A(R - R^*)^2}{2}, \tag{A.1}$$

where R is the real exchange rate, R^* is the desired level of the real exchange rate, and π is the rate of change in the nominal exchange rate which is assumed to be set by the policymaker. Eq. (A.1) reflects the policymaker's desire to minimize the loss from deviating from R^* and from variability in nominal variables. The parameter A measures the relative importance that the policymaker assigns to the first goal vis-a-vis the second: The larger is A, the stronger is the policymaker's aversion to deviating from his desired real exchange rate.

Assuming that the economy is small and open (nominal) domestic prices (and wages) are determined by the (nominal) exchange rate. Assuming in addition that domestic prices and wages are temporarily sticky, current prices and wages, p, are set in advance on the basis of the expected nominal exchange rate, $\hat{e}$.[25] This implies (after normalizing exogenous foreign prices to 1) that $p = \hat{e}$, so that the real exchange rate is $R = e/\hat{e}$. Letting R_0 be last period's real exchange rate, and expanding $L(R)$ linearly around it we obtain

$$\begin{aligned} L(R) &\simeq \frac{A}{2}(R_0 - R^*)^2 + A(R_0 - R^*)(R - R_0) \\ &= -\frac{A}{2}(R_0^2 - R^{*2}) + A(R_0 - R^*)R. \end{aligned} \tag{A.2}$$

[25] To focus on the differential stickiness between the nominal exchange rate, on one hand, and prices and wages, on the other, we abstract from differences in stickiness between prices and wages.

Substituting this expression into (A.1), noting that $R = e/\hat{e}$, and ignoring terms that do not depend on e "which is the policymaker's choice variable", the policymaker's objective becomes:

$$x\frac{e}{\hat{e}} - \frac{\pi^2}{2}, \qquad x \equiv -A(R_0 - R^*). \tag{A.3}$$

The parameter x is positive, zero, or negative, depending on whether R^* is larger than, equal to, or smaller than R_0. Moreover, x increases with the gap between the real exchange rate target and last period's real exchange rate, and with the policymaker's concern with deviations of the real exchange rate from the desired level. We assume that due to shocks to trade, capital flows, employment and other factors which are beyond the control of policymakers, R^* fluctuates randomly. R^* is realized before the public forms its expectations about the exchange rate and before the policymaker makes his policy choice. Hence, x is also a random variable that realizes prior to the formation of expectations and to the choice of the rate of change in the exchange rate by the policymaker. Now let $u \equiv e/\hat{e} - 1$ be the rate of unanticipated depreciation or appreciation. Assuming that u is not too large,

$$\log\frac{e}{\hat{e}} = \log(1+u) \cong u. \tag{A.4}$$

Letting e_0 and $\pi^e \equiv (\hat{e} - e_0)/e_0$ denote respectively last period's nominal exchange rate and the expected rate of change in the exchange rate from that point and on,

$$\log\frac{e}{\hat{e}} = \log\frac{e/e_0}{\hat{e}/e_0} = \log\frac{1+\pi}{1+\pi^e} = \log(1+\pi) - \log(1+\pi^e) \cong \pi - \pi^e. \tag{A.5}$$

Taken together, the last two equations imply that:

$$\frac{e}{\hat{e}} - 1 \equiv u \cong \pi - \pi^e. \tag{A.6}$$

Substituting from (A.6) into (A.3) and ignoring terms that do not depend on e, yields Eq. (1) in the text. □

Derivation of Eq. (2): Since $V(\pi, \pi^e)$ is strictly concave in π and since c is fixed, it is clear that if $x > \overline{\pi}$, the policymaker's optimal choice is either x if $V(x, \pi^e) - c > V(\overline{\pi}, \pi^e)$ or $\overline{\pi}$ if $V(x, \pi^e) - c < V(\overline{\pi}, \pi^e)$. But, given Eq. (1), $V(x, \pi^e) = x^2/2 - x\pi^e$ and $V(\overline{\pi}, \pi^e) = x(\overline{\pi} - \pi^e) - \overline{\pi}^2/2$. Hence, $\pi^D(x) = x$ if $x > \overline{\pi} + \delta$ and $\pi^D(x) = \overline{\pi}$ if $\overline{\pi} \le x \le \overline{\pi} + \delta$ where $\delta \equiv \sqrt{2c}$. Similarly, if $x < \underline{\pi}$ the policymaker's optimal choice is x if $V(x, \pi^e) - c > V(\underline{\pi}, \pi^e)$, or $\underline{\pi}$ if $V(x, \pi^e) - c < V(\underline{\pi}, \pi^e)$. From Eq. (1), $V(\underline{\pi}, \pi^e) = x(\underline{\pi} - \pi^e) - \underline{\pi}^2/2$, so $\pi^D(x) = x$ if $x < \underline{\pi} + \delta$ and $\pi^D(x) = \underline{\pi}$ if $\underline{\pi} - \delta \le x \le \underline{\pi}$. This establishes that the dependable policymaker's choices in stage 3 are given by (2). □

Proof of Proposition 1. (i) Evaluating Eq. (7) at $\underline{\pi} = 0$,

$$\left.\frac{\partial EV(x)}{\partial \underline{\pi}}\right|_{\underline{\pi}=0} = -\alpha\delta^2 f(-\delta) + \int_{-\delta}^{0} (1-\alpha)x \, \mathrm{d}F(x) < 0, \tag{A.7}$$

where the inequality follows because $0 < \delta < -\underline{x}$. Hence, at the optimum, $\underline{\pi} < 0$. Likewise, evaluating Eq. (8) at $\bar{\pi} = 0$,

$$\frac{\partial EV(x)}{\partial \bar{\pi}}\bigg|_{\bar{\pi}=0} = \alpha\delta^2 f(\delta) + \int_0^{\delta} (1-\alpha)x \, \mathrm{d}F(x) > 0, \tag{A.8}$$

where the inequality follows because $0 < \delta < \bar{x}$. Hence, at the optimum, $\bar{\pi} > 0$.

(ii) Suppose that $f(x)$ is nonincreasing for all $x < 0$. Then:

$$\begin{aligned}\frac{\partial EV(x)}{\partial \underline{\pi}} &= \alpha\delta(\underline{\pi}-\delta)f(\underline{\pi}-\delta) + \int_{\underline{\pi}-\delta}^{\underline{\pi}} [(x-\underline{\pi}) - \alpha x]\mathrm{d}F(x) \\ &= \int_{\underline{\pi}-\delta}^{\underline{\pi}} \left[(x-\underline{\pi}) - \alpha x + \alpha\underline{\pi}\frac{f(\underline{\pi}-\delta)}{f(x)} - \alpha\delta\frac{f(\underline{\pi}-\delta)}{f(x)}\right] f(x)\mathrm{d}x \\ &< \int_{\underline{\pi}-\delta}^{\underline{\pi}} \left[(1-\alpha)(x-\underline{\pi}) - \alpha\delta\frac{f(\underline{\pi}-\delta)}{f(x)}\right] f(x)\mathrm{d}x < 0,\end{aligned} \tag{A.9}$$

implying that the policymaker would like to push $\underline{\pi}$ downward as far as possible. The proof that the policymaker would like to push $\bar{\pi}$ upward as far as possible when $f(x)$ is nondecreasing for all $x > 0$ is analogous.

(iii) The proof follows immediately from part (ii). So long as $f(x)$ is decreasing over the whole negative REC, $\partial EV(x)/\partial \underline{\pi} < 0$, while if $f(x)$ is increasing over the whole positive REC, $\partial EV(x)/\partial \bar{\pi} > 0$.

(iv) If $f(x)$ is symmetric around 0, then for $0 < a < b$,

$$f(x) = f(-x), \qquad \int_{-b}^{-a} f(x)\mathrm{d}x = \int_a^b f(x)\mathrm{d}x, \quad \text{and}$$

$$-\int_{-b}^{-a} xf(x)\mathrm{d}x = \int_a^b xf(x)\mathrm{d}x. \tag{A.10}$$

Using these properties, Eq. (7) becomes:

$$\begin{aligned}&\alpha\delta(\underline{\pi}-\delta)f(\underline{\pi}-\delta) + \int_{\underline{\pi}-\delta}^{\underline{\pi}} [(x-\underline{\pi}) - \alpha x]\mathrm{d}F(x) \\ &\quad = \alpha\delta(\underline{\pi}-\delta)f(-\underline{\pi}+\delta) - \int_{-\underline{\pi}}^{-\underline{\pi}+\delta} [(x+\underline{\pi}) - \alpha x]f(x)\mathrm{d}x = 0.\end{aligned} \tag{A.11}$$

But now, if we replace $\underline{\pi}$ with $-\bar{\pi}$, we get exactly Eq. (8). Thus, both $\underline{\pi}$ and $-\bar{\pi}$ satisfy the second line in Eq. (A.11) and hence they must be equal. □

Proof of Proposition 2. (i) $\delta > \mathrm{Max}[-\underline{x}, \bar{x}]$ implies that $\underline{x} > \underline{\pi} - \delta$ and $\bar{x} < \bar{\pi} + \delta$. Hence, the expected payoff of the policymaker becomes:

$$\begin{aligned}EV(x) = &-\int_{\underline{x}}^{\bar{x}} \frac{x^2}{2}\mathrm{d}F(x) - \int_{\underline{x}}^{\underline{\pi}} \left[\frac{(\underline{\pi}-x)^2}{2} + \alpha x(\underline{\pi}-x)\right]\mathrm{d}F(x) \\ &- \int_{\bar{\pi}}^{\bar{x}} \left[\frac{(\bar{\pi}-x)^2}{2} + \alpha x(\bar{\pi}-x)\right]\mathrm{d}F(x).\end{aligned} \tag{A.12}$$

As a result,

$$\frac{\partial EV(x)}{\partial \underline{\pi}} = -\int_{\underline{x}}^{\underline{\pi}} [\underline{\pi} - (1-\alpha)x]\mathrm{d}F(x),$$

$$\frac{\partial EV(x)}{\partial \overline{\pi}} = -\int_{\overline{\pi}}^{\overline{x}} [\overline{\pi} - (1-\alpha)x]\mathrm{d}F(x). \tag{A.13}$$

If $\alpha = 1$, then $\partial EV(x)/\partial \underline{\pi} > 0 > \partial EV(x)/\partial \overline{\pi}$ for all $\underline{\pi} \in [\underline{x}, 0)$ and $\overline{\pi} \in (0, \overline{x}]$ implying that the optimal system is a peg with $\underline{\pi} = \overline{\pi} = 0$.

Next suppose that $\alpha < 1$. Then, evaluating $\partial EV(x)/\partial \overline{\pi}$ at $\overline{\pi} = 0$ reveals that

$$\left.\frac{\partial EV(x)}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} = \int_{0}^{\overline{x}} (1-\alpha)x\mathrm{d}F(x) > 0. \tag{A.14}$$

Moreover, at $\overline{\pi} = 0$, the slope of $v\partial EV(x)/\partial \overline{\pi}$ is negative since

$$\left.\frac{\partial^2 EV(x)}{\partial \overline{\pi}^2}\right|_{\overline{\pi}=0} = -(1 - F(0)) < 0. \tag{A.15}$$

On the other hand, at $\overline{\pi} = \overline{x}$, we have:

$$\left.\frac{\partial EV(x)}{\partial \overline{\pi}}\right|_{\overline{\pi}=\overline{x}} = \int_{\overline{x}}^{\overline{x}} (1-\alpha)x\mathrm{d}F(x) = 0, \tag{A.16}$$

and

$$\left.\frac{\partial^2 EV(x)}{\partial \overline{\pi}^2}\right|_{\overline{\pi}=\overline{x}} = \alpha\overline{x}f(\overline{x}) - (1 - F(\overline{x})) = \alpha\overline{x}f(\overline{x}) > 0, \tag{A.17}$$

where the last equality follows since $F(\overline{x}) = 1$. Since $\partial EV(x)/\partial \overline{\pi}$ is a continuous function, the Intermediate Value Theorem implies that there exists $\overline{\pi} \in (0, \overline{x})$ at which $\partial EV(x)/\partial \overline{\pi} = 0$. The proof that $-\underline{x} < \underline{\pi} < 0$ is analogous. □

Proof of Proposition 3. Using (7), applying the implicit function theorem, and using the fact that by the second order condition for maximization, $\partial^2 EV(x)/\partial \underline{\pi}^2 < 0$ yields:

$$\mathrm{Sign}\frac{\partial \underline{\pi}}{\partial \alpha} = \mathrm{Sign}\left[\delta(\underline{\pi} - \delta)f(\underline{\pi} - \delta) - \int_{\underline{\pi}-\delta}^{\underline{\pi}} x\mathrm{d}F(x)\right], \tag{A.18}$$

However, (7) implies that,

$$\delta(\underline{\pi} - \delta)f(\underline{\pi} - \delta) = -\frac{1}{\alpha}\int_{\underline{\pi}-\delta}^{\underline{\pi}} [(x - \underline{\pi}) - \alpha x]\mathrm{d}F(x). \tag{A.19}$$

Substituting into (A.18) and rearranging terms we get:

$$\mathrm{Sign}\frac{\partial \underline{\pi}}{\partial \alpha} = \mathrm{Sign}\left[-\frac{1}{\alpha}\int_{\underline{\pi}-\delta}^{\underline{\pi}}[(x-\underline{\pi})-\alpha x]\mathrm{d}F(x) - \int_{\underline{\pi}-\delta}^{\underline{\pi}} x\mathrm{d}F(x)\right]$$

$$= -\frac{1}{\alpha}\int_{\underline{\pi}-\delta}^{\underline{\pi}}(x-\underline{\pi})\mathrm{d}F(x) > 0. \tag{A.20}$$

Hence, as α falls, the policymaker wants to lower $\underline{\pi}$, thereby widening the band under the central parity line. The proof that the policymaker wants to raise $\bar{\pi}$ (i.e., widen the band above the central parity line) as α falls is analogous. As α approaches 0, the first terms in (7) and (8) and the second terms in the two integrals vanish. Thus, the derivative in (7) is everywhere decreasing and the derivative in (8) is everywhere increasing, so as a result, $\underline{\pi} = \underline{x}$ and $\bar{\pi} = \bar{x}$. □

Proof of Proposition 4. Using (7), applying the implicit function theorem, and using the fact that by the second-order condition for maximization, $\partial^2 EV(x)/\partial \underline{\pi}^2 < 0$, yields

$$\mathrm{Sign}\frac{\partial \underline{\pi}}{\partial \delta} = \mathrm{Sign}[-\delta(1+\alpha)f(\underline{\pi}-\delta) - \alpha\delta(\underline{\pi}-\delta)f'(\underline{\pi}-\delta)]$$

$$= \mathrm{Sign}\left[-\delta\alpha f(\underline{\pi}-\delta)\left[\frac{1+\alpha}{\alpha} - \eta(\underline{\pi}-\delta)\right]\right]. \tag{A.21}$$

Hence, $\underline{\pi}$ decreases with δ if $\eta(\underline{\pi}-\delta) < (1+\alpha)/\alpha$ and increases towards 0 if $\eta(\underline{\pi}-\delta) > (1+\alpha)/\alpha$. Likewise,

$$\mathrm{Sign}\frac{\partial \bar{\pi}}{\partial \delta} = \mathrm{Sign}[\delta(1+\alpha)f(\bar{\pi}+\delta) + \alpha\delta(\bar{\pi}+\delta)f'(\bar{\pi}+\delta)]$$

$$= \mathrm{Sign}\left[\delta\alpha f(\bar{\pi}+\delta)\left[\frac{1+\alpha}{\alpha} - \eta(\bar{\pi}+\delta)\right]\right]. \tag{A.22}$$

Hence, $\bar{\pi}$ increases with δ if $\eta(\bar{\pi}+\delta) < (1+\alpha)/\alpha$, and decreases towards 0 if $\eta(\bar{\pi}+\delta) > (1+\alpha)/\alpha$. □

Proof of Proposition 5. Given the assumption that $f(x)$ has a single mode at 0, the effect of the band on the public's expectations is given by:

$$\frac{\partial \pi^e}{\partial \underline{\pi}} = \alpha\int_{\underline{\pi}-\delta}^{\underline{\pi}}\mathrm{d}F(x) - \alpha\delta f(\underline{\pi}-\delta) = \alpha\int_{\underline{\pi}-\delta}^{\underline{\pi}}[f(x) - f(\underline{\pi}-\delta)]\mathrm{d}x > 0, \tag{A.23}$$

and

$$\frac{\partial \pi^e}{\partial \bar{\pi}} = \alpha\int_{\bar{\pi}}^{\bar{\pi}+\delta}\mathrm{d}F(x) - \alpha\delta f(\bar{\pi}+\delta) = \alpha\int_{\bar{\pi}}^{\bar{\pi}+\delta}[f(x) - f(\bar{\pi}+\delta)]\mathrm{d}x > 0. \tag{A.24}$$

Hence, lowering the lower bound of the band lowers π^e while raising the upper bound of the band raises π^e.

The partial derivatives of $EV(x)$ with respect to $\underline{\pi}$ and $\overline{\pi}$ are given by

$$\frac{\partial EV(x)}{\partial \underline{\pi}} = -\hat{x}\frac{\partial \pi^e}{\partial \underline{\pi}} - \int_{\underline{\pi}-\delta}^{\underline{\pi}} (\underline{\pi} - x)\mathrm{d}F(x),$$

$$\frac{\partial EV(x)}{\partial \overline{\pi}} = -\hat{x}\frac{\partial \pi^e}{\partial \overline{\pi}} - \int_{\overline{\pi}}^{\overline{\pi}+\delta} (\overline{\pi} - x)\mathrm{d}F(x). \tag{A.25}$$

If $\hat{x} > 0$, the first terms in both derivatives are negative. Since the second term in $\partial EV(x)/\partial \underline{\pi}$ is also negative, it follows that $\partial EV(x)/\partial \underline{\pi} < 0$. Hence it is optimal to set $\underline{\pi} = \underline{x}$. On the other hand, the second term in $\partial EV(x)/\partial \overline{\pi}$ is positive. To determine the width of the band above the center rate, note that

$$\left.\frac{\partial EV(x)}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} = -\hat{x}\left.\frac{\partial \pi^e}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} + \int_0^{\delta} x\,\mathrm{d}F(x) = \int_0^{\delta} [(x - \alpha\hat{x})f(x)]\mathrm{d}x + \alpha\hat{x}\delta f(\delta)$$

$$= (F(\delta) - F(0))\left[\frac{\int_0^{\delta} xf(x)\mathrm{d}x}{F(\delta) - F(0)} - \alpha\hat{x}\right] + \alpha\hat{x}\delta f(\delta),$$

and

$$\left.\frac{\partial EV(x)}{\partial \overline{\pi}}\right|_{\overline{\pi}=\overline{x}} = -\hat{x}\left.\frac{\partial \pi^e}{\partial \overline{\pi}}\right|_{\overline{\pi}=\overline{x}} < 0. \tag{A.27}$$

Eq. (A.27) indicates that a completely flexible exchange rate above the center rate is never optimal. By contrast a one-sided peg, $\overline{\pi} = 0$, cannot be ruled out in general. But when the square bracketed term in (A.26) is positive, a non degenerate one-sided band, $\overline{\pi} > 0$, is surely optimal. Note that this is only a necessary condition since it might be optimal to set $\overline{\pi} > 0$ even if this condition fails. □

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