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Bertrand competition with subcontracting

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We investigate how the possibility of subsequently subcontracting production to each other influences rivals' initial competition for a contract or a market as a two-stage game. In its first stage, the two firms engage in price competition to supply a contract or a market. In the second stage, the firms may subcontract production to each other. It is supposed that the firms produce the identical product with the same strictly convex cost function. The incentive for subcontracting comes from the strictly convex production costs. A firm is obliged to supply the entire quantity demanded at its quoted price. Our analysis discloses that if the winner of the game's first stage determines the terms of the subcontract in its second stage, there exists a unique, subgame perfect Nash equilibrium (SPNE) in pure strategies in which the firms bid the same price in the first stage and both receive zero profits. On the other hand, if the loser of the game's first stage sets the terms of the subcontract in the second stage, there exists a unique SPNE in pure strategies in which the firms bid the same price in the first stage and both receive positive profits. The presence of the possibility of subcontracting supports a unique SPNE in pure strategies, even though no actual subcontracting may occur. The SPNE price is below the socially-optimal price in the first case and is above it in the second case. We also consider other modes of sharing the gains from subcontracting between the two firms, such as the Nash bargaining solution.

1. Introduction

■ Subcontracting is a commonly used method of reducing production or service costs. Marine insurance companies compete for contracts to insure the hulls of fishing vessel fleets, and the winning bidder then sells off parts of the contract to the very rivals against whom he bid. Small architectural firms engage in a similar practice after competing for a major design contract. The same is true for securities underwriters. Subcontracting also occurs among suppliers to the government. Lockheed is now building wing components for C-17 Air Force transport planes under a $1.3 billion subcontract from the McDonnell-Douglas

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Corporation. In a recent dispute, Hughes Electronics alleged that had it, instead of IBM, been awarded the contract to supply the FAA with a new computer system, it would have purchased IBM computers to fulfill the contract. In the manufacturing sectors, especially in the electronics and automobile industries, subcontracting for the supply of components is extensive. According to Ikeda (1989), General Motors and Daimler-Benz each have as many as 10,000 subcontractors. In the Japanese automobile industry, about 75% of the final product is produced by subcontractors; in the United States and Europe, the figure is about 50%. Indeed, in Japan, subcontractors of the various automobile manufacturers are organized into tiers, with lower-tier subcontractors supplying higher-tier subcontractors. In the electronics industry, Apple Computer subcontracts both for the components and the assembly of its final products, as do its competitors.

Subcontracting serves to reduce the costs of supplying a product or a service. For a manufacturer, the opportunity to subcontract enables him to maintain a smaller productive capacity than he would have to otherwise. Expanding capacity to avoid subcontracting may involve increasing costs. For an insurance company, the opportunity to subcontract may lower its operating costs by enabling it to reduce its reserves against bad outcomes. The presence of subcontracting also implies that the purchaser of the final product is unconcerned about how the supplier produces the contracted quantity to be delivered, as long as the product meets the specifications.

It should be noted that while we recognize that subcontracting is commonly for components rather than for the entire product, we assume for the sake of simplicity that it is for that latter in our subsequent analysis. The main point is that subcontracting serves to reduce costs. We also suppose that the winning bidder is required to supply the entire quantity demanded. This is a natural assumption in the case of competition for a contract, and in the case of competition for a market, it implies severely high costs for outages.

In both the automobile and the electronics industries, subcontracts are awarded on the basis of competitive bids. However, rival subcontractors, in turn, subcontract production among each other after a subcontract has been awarded to one of them. Our focus here is on how the firms' initial bids for a contract or a market are influenced by their foresight of the possibility of subsequent subcontracting. Specifically, we are interested in the strategic relationship between the respective roles played by the winner and loser of the contract, or market, in setting the subcontracting terms (i.e., how much will be subcontracted and at what price) and his rivals' initial bids. We are also interested in the social welfare implications of subcontracting among rivals. Does this practice lead to higher or lower prices than are socially optimal?

We address these issues in terms of a two-stage game that involves two firms producing the identical product according to an identical, increasing, strictly convex cost function. It is the strict convexity of the cost function that creates the incentive for subcontracting, as total costs are reduced when production is split between the two firms. In the game's first stage, each firm submits a price at which it is willing to supply the entire quantity demanded, which is given by the product's demand function. Bidding for a contract may be regarded as a special case in which the demand function is vertical at a positive quantity, with a reservation price above which the quantity demanded is zero. In the event of a tied price, each firm supplies one-half of the quantity demanded at that price, or one firm is selected to supply the entire quantity demanded by the toss of a fair coin. The former tie-breaking rule may be more appropriate when the firms compete for a market, and the latter may be more appropriate if they compete for a contract. Thus, the firms are engaged in Bertrand competition in the game's first stage.

In its second stage, the firms may subcontract production to each other. There are any number of ways that the amount to be subcontracted and the price to be paid can be determined. We focus on two polar cases. In the first case, the winner of the game's first stage acts as a Stackelberg leader in its second stage. That is, he determines the quantity to be subcontracted and the price to be paid to the loser to maximize his own profit subject to the loser's opportunity cost, which is zero. In the second case, the loser of the game's first stage is the Stackelberg leader in its second stage. That is, he chooses the quantity to be subcontracted to the winner and its price to maximize his own profit subject to the winner's opportunity cost, which is the profit the winner can realize if he produces the entire quantity demanded. We also indicate what happens if the terms of the subcontract are determined by other means, such as the Nash bargaining solution.

We employ the subgame perfect Nash equilibrium (SPNE) as our solution concept. Thus, we use our analysis of the game's second stage to analyze its first stage. Our analysis discloses that for a downward-sloping demand function, each version of the two-stage game exhibits a unique, pure-strategy SPNE in which both firms choose the identical price in the first stage. Whether or not subcontracting occurs depends upon which tie-breaking rule is used. Subcontracting can occur in bidding for a contract under either tie-breaking rule.

The equilibrium price when the winner of the game's first stage is assumed to be the Stackelberg leader in its second stage equals the average cost of producing the first one-half of the quantity demanded, and both firms receive zero profits. On the other hand, if the loser of the game's first stage is the Stackelberg leader in its second stage, the equilibrium price equals the average cost of producing the second one-half of the entire quantity demanded, and both firms receive positive profits. The intuitive reason for this difference is that in equilibrium, the winner's and the loser's profits must be equal. If the winner were to receive more than the loser, each firm would have an incentive to lower its bid to become the winner. Thus, if the loser of the first stage is to receive zero profits, the price in the first stage must be bid down to the level at which the winner's profit also equals zero. However, if the loser of the first stage gets to set the terms of the subcontract in the second stage, then he is assured a positive profit, and the price in the first stage must be bid down only to the level that assures both firms that profit level. The interesting result here is that it is advantageous to both firms to let the loser of the first stage have the power to determine the subcontracting terms in the second stage. Yet this arrangement does not, in general, yield the maximum profit that could be realized from complete cooperation—the monopoly profit of a single firm with two identical production facilities. The cooperative profit level is achievable if the monopoly price is between the SPNE prices that emerge in the two-polar case, in which either the first-stage winner or the first-stage loser sets the subcontracting terms, and its attainment requires the firms to share power in setting the subcontract terms. This is demonstrated by an example. Our analysis of mixed-strategy possibilities indicates that no SPNE other than the pure SPNE exists.

For the duopoly case, a price equal to the marginal cost of producing one-half of the quantity demanded maximizes the consumer surplus plus the producer surplus. The SPNE price that occurs if the winner of the first stage sets the subcontract price is below the socially-optimal price; it is above the socially-optimal price if the loser of the first stage determines the terms of the subcontract. However, there does exist a means of sharing the gains from subcontracting such that the SPNE price is socially optimal. In particular, if the firms share the gains from subcontracting equally and if the cost function is quadratic, then the equilibrium price is socially optimal.

Finally, our analysis of competing for a contract to produce a fixed quantity discloses that if the loser of the first stage is the leader in the second stage, there exists an SPNE in which there is a single winner in the first stage, and subcontracting occurs even if ties are resolved by equal sharing of the market.

We are unaware of any previous work on Bertrand competition with subcontracting. The most closely related work appears to be that dealing with price competition in the presence of convex costs and/or capacity constraints, or Bertrand-Edgeworth competition; summaries of this work are provided by Allen and Hellwig (1986), Dixon (1984), and Maskin (1986). In this work, only the existence of a mixed-strategy Nash equilibrium is established, whereas here the potential for subcontracting allows the existence of pure-strategy SPNE. This result is obtained even though there may be no actual subcontracting at the SPNE. Stahl (1988), in a paper dealing with price competition for inputs, also showed the existence of a pure-strategy Nash equilibria.

In Section 2, we present the model for the case of competition for a market. In Section 3, the case of competition for a fixed quantity is presented. A brief summary follows in Section 4.

2. Competition to supply a market

■ We posit two firms that produce an identical, divisible product, the demand function for which is $Q(P)$, with the same cost function, $C(Q)$. The assumed properties of $Q(P)$ and $C(Q)$ are given below.

Assumption 1. The demand function, $Q(P)$, is defined for $P \geq 0$ and is nonnegative, differentiable, and downward-sloping ($Q'(P) < 0$). Also, $\lim_{P \to \infty} Q(P) = 0$.

Assumption 2. The cost function, $C(Q)$, is defined for $Q \geq 0$ and is differentiable, strictly increasing ($C'(Q) > 0$), and strictly convex ($C''(Q) > 0$). Also, $C(0) = 0$.

Assumption 3. Production is profitable; that is, $Q(C'(0)) > 0$.

The game involves two stages. In its first stage, the two firms choose prices and compete for the production of the total quantity demanded at that price. The firm with the lowest price wins and is obliged to provide the entire quantity demanded at that price. In the event of a tie, each firm produces one-half of the total quantity demanded at that price, or one firm is chosen at random to supply the entire market demand. In the second stage of the game, we allow subcontracting to take place. The cost reduction realizable through subcontracting equals $C(Q) - 2C(Q/2)$, the difference between the cost of producing the entire quantity in one facility and the social cost of splitting its production between two facilities. One of the firms is a Stackelberg leader, who offers the other a quantity to produce and the unit price to be paid. We begin by focusing on two extreme versions of the game. In the first, game Γ_w, the leader is the firm that won in the first stage. In the second version, game Γ_l, the loser of the first stage is the second-stage leader. We also assume the absence of uncertainty regarding the cost functions or the demand function and that all strategic aspects of the game are common knowledge.

As we seek a SPNE in pure strategies, we begin with the second stage. In game Γ_w, the leader chooses the quantity to subcontract to the loser, q, and the price to offer, p, to solve

$$\max_{p,q} PQ(P) - pq - C(Q(P) - q) \tag{1}$$

$$\text{subject to} \qquad pq - C(q) \geq 0 \tag{2a}$$

and

$$p, q \geq 0. \tag{2b}$$

P and $Q(P)$ are fixed from the first stage, and $Q(P) - q$ is the amount to be produced by the winner. Constraint (2a) represents the loser's profit from subcontracting and cannot be driven below zero, as that is his opportunity cost. For $q > 0$, Constraint (2a) can be rewritten as

$$p \geq C(q)/q. \tag{3}$$

Since the objective function, (1), is strictly decreasing in p, this price is set equal to its lowest value, namely, $p = C(q)/q$. Thus, the maximum is attained by solving

$$\max_q PQ(P) - C(q) - C(Q(P) - q)$$

$$\text{subject to} \qquad q \geq 0, \tag{4}$$

which, upon being differentiated with respect to q, yields

$$C'(q) = C'(Q(P) - q). \tag{5}$$

Since the objective function in (4) is strictly concave in q, the first-order condition is a necessary and sufficient condition for a maximum. As the cost function is strictly convex, (5) implies that $q = Q(P)/2$. Thus, the winner subcontracts one-half of the total output at the unit price, $p = C(Q(P)/2)/(Q(P)/2)$, the average cost of producing the first one-half of the total quantity. The payoffs to the winner and loser, respectively, as a function of the winning price, P, are

$$W_1(P) = PQ(P) - 2C(Q(P)/2) \tag{6}$$

and

$$L_1(P) = 0. \tag{7}$$

Thus, the winner realizes the entire benefit, $C(Q(P)) - 2C(Q(P)/2)$, of subcontracting in this case.

If the first stage ends with a tie and the tie-breaking rule is equal sharing of the total quantity to be supplied, then each player realizes $PQ(P)/2 - C(Q(P)/2)$. If the tie is broken by a toss of a fair coin, then the payoff to each player is $.5W_1(P) + .5L_1(P)$. Since these two quantities are the same, we conclude that under either rule

$$T(P) = .5W_1(P) + .5L_1(P). \tag{8}$$

Before proceeding to the analysis of the game's first stage, we present the following result.

Lemma 1. There exists a unique price, P_1, such that $P_1 = C(Q(P_1)/2)/(Q(P_1)/2)$, $W_1(P) < 0$ for each $P < P_1$, $W_1(P) \geq 0$ for each $P > P_1$, and $dW_1(P)/dP > 0$ for $P \leq P_1$.

Proof. Let $\bar{P}$ be the lowest price for which $Q(P) = 0$. If Q is positive for all P, then $\bar{P} = \infty$. For $P < \bar{P}$, $Q(P) > 0$, and (6) can be rewritten as

$$W_1(P) = Q[P - C(Q/2)/(Q/2)], \tag{9}$$

where we write Q for $Q(P)$.

Consider the function

$$G(P) = P - C(Q/2)/(Q/2). \tag{10}$$

Differentiating with respect to P, we get

$$G'(P) = 1 - Q'[C'(Q/2) - C(Q/2)/(Q/2)]/Q. \tag{11}$$

As C is strictly convex and $Q'(P) < 0$, $G'(P) > 0$ for all P. Also, $G(0) < 0$, and $\lim_{P \to \bar{P}} G(P) = \bar{P} - C'(0) > 0$ by Assumptions 2 and 3. Thus, there exists a P_1, $0 < P_1 < \bar{P}$, such that $G(P_1) = 0$, $G(P) < 0$ for each $P < P_1$, and $G(P) > 0$ for $P > P_1$. From (9), it follows that $W_1(P_1) = 0$ and $W_1(P) < 0$ for each $P < P_1$, while $W_1(P) \geq 0$ for $P > P_1$. From (10), we have that $P_1 = C(Q(P_1)/2)/(Q(P_1)/2)$. Differentiating $W_1(P)$ gives $dW_1(P)/dP = Q + Q'[P - C'(Q/2)]$. But, $P - C'(Q/2) < G(P)$, and therefore for $P \leq P_1$, $P - C'(Q/2) < 0$, which, together with $Q'(P) < 0$, shows that for such P, $dW_1(P)/dP > 0$. Clearly the properties of P_1 imply that there can be at most one such point. *Q.E.D.*

Proposition 1. In game Γ_w, there exists a unique first-stage SPNE in pure strategies, in which both firms bid the price P_1, where $P_1 = C(Q(P_1)/2)/(Q(P_1)/2)$, the average cost of producing the first one-half of the total quantity demanded, $Q(P_1)$. Each firm produces one-half of the total quantity demanded at that price, and they both receive zero profits.

Proof. Note that $T(P) = W_1(P)/2$. Let $\underline{P}$ be the lowest price offered in the first stage. Suppose that $\underline{P} < P_1$; then, both $T(\underline{P})$ and $W(\underline{P})$ are negative, and therefore, a firm that offered that price would be better off bidding a higher price and being the loser. If $\underline{P} > P_1$, then either there is a losing firm which makes zero profits, or there is a tie, in which case both firms receive $T(\underline{P})$. In the first case, the loser makes himself better off by choosing a price P such that $P_1 < P < \underline{P}$, becoming the winner, and obtaining $W_1(P) > 0$, since $dW(P_1)/dP(P_1) > 0$. In the second case, one of the firms could bid a price $\underline{P} - \epsilon$ for small enough $\epsilon > 0$ that satisfies $W_1(\underline{P} - \epsilon) > W_1(\underline{P})/2 = T(\underline{P})$. Thus, $\underline{P} = P_1$. If there is a winner in the game, he can gain by choosing a winning price, $P > P_1$, that is close enough to P_1 and getting $W_1(P) > W_1(\underline{P})$. So, the only possible SPNE is one in which both firms offer the price P_1. It is easy to see that this is indeed a SPNE. *Q.E.D.*

We turn next to game Γ_l in which the loser of the game's first stage is the Stackelberg leader in its second stage. The loser determines the subcontract terms in the second stage by solving the problem

$$\max_{p,q} pq - C(q) \tag{12}$$

$$\text{subject to} \quad PQ(P) - pq - C(Q(P) - q) \geq PQ(P) - C(Q(P)) \tag{13a}$$

and

$$p, q \geq 0. \tag{13b}$$

Constraint (13a) indicates that the winner's profit cannot be reduced below the level that he could realize by supplying the total quantity demanded alone. For $q > 0$, it can be rewritten as

$$p \leq [C(Q(P)) - C(Q(P) - q)]/q. \tag{14}$$

As the objective function, (12), increases with p, the subcontract price is set at its upper limit. Thus, (12), (13a), and (13b) reduce to

$$\max_{q} C(Q(P)) - C(Q(P) - q) - C(q)$$

$$\text{subject to} \quad q \geq 0, \tag{15}$$

upon substituting for p from (14). Taking the derivative of (15) with respect to q and setting it equal to zero yields the intuitive result that $q = Q(P)/2$. Thus, the payoffs to the winner and the loser, respectively, as a function of the winning price, P, are

$$W_2(P) = PQ(P) - C(Q(P)) \tag{16}$$

$$L_2(P) = C(Q(P)) - 2C(Q(P)/2). \tag{17}$$

The loser realizes the entire benefit of subcontracting in this case. If a tie occurs, the profit function, $T(P)$, is given as before by (8) for either tie-breaking rule. The counterpart of Lemma 1, in this case, is given below.

Lemma 2. There exists a unique price, P_2, such that $P_2 = [C(Q) - C(Q/2)]/(Q/2)$, where $Q = Q(P_2)$, $W_2(P) < L_2(P)$ for all $P < P_2$, $W_2(P) \geq L_2(P)$ for all $P > P_2$, $dW_2(P)/dP > 0$ for all $P < P_2$, and $dL_2(P)/dP \leq 0$ for all P.

Proof. Let $\bar{P}$ be the lowest price for which $Q(P) = 0$. ($\bar{P} = \infty$ if there is no such price.) For each such $P < \bar{P}$, we can write, for $Q = Q(P)$,

$$W_2(P) - L_2(P) = PQ - 2[C(Q) - C(Q/2)]$$
$$= Q\{P - [C(Q) - C(Q/2)]/(Q/2)\} = QH(P), \tag{18}$$

where

$$H(P) = P - [C(Q) - C(Q/2)]/(Q/2). \tag{19}$$

Differentiating $H(P)$ with respect to P, we get

$$H'(P) = 1 - 2Q'\{C'(Q) - C'(Q/2)/2 - [C(Q) - C(Q/2)]/Q\}/Q. \tag{20}$$

Recall that by the strict convexity of $C(Q)$, $[C(Q) - C(Q/2)]/Q < C'(Q)/2$, and thus, by substitution

$$\{C'(Q) - C'(Q/2)/2 - [C(Q) - C(Q/2)]/Q\} > [C'(Q) - C'(Q/2)]/2 > 0.$$

Since $Q'(P) < 0$, it follows that $H'(P) > 0$. But, $H(0) < 0$, and

$$\lim_{P \to \bar{P}} H(P) = \bar{P} - C'(0) > 0.$$

Therefore, there exists a P_2, $0 < P_2 < \bar{P}$, such that $H(P_2) = 0$, $H(P) < 0$ for $P < P_2$, and $H(P) > 0$ for $P > P_2$. Clearly, $P_2 = [C(Q) - C(Q/2)]/(Q/2)$, where $Q = Q(P_2)$. From (19), $W_2(P) < L_2(P)$ for $P < P_2$, and $W_2(P) \geq L_2(P)$ for $P > P_2$. Finally, to see that $dW_2(P)/dP > 0$ for $P \leq P_2$, observe that

$$dW_2(P)/dP = Q + Q'[P - C'(Q)] > 0, \tag{21}$$

since for each $P \leq P_2$, $P - C'(Q) < H(P) < 0$ and $Q' < 0$. To see that $dL_2(P)/dP \leq 0$, simply observe that

$$dL_2(P)/dP = Q'[C'(Q) - C'(Q/2)] \leq 0, \tag{22}$$

since $C(Q)$ is strictly convex and $Q'(P) < 0$. *Q.E.D.*

We can now state the counterpart to Proposition 1.

Proposition 2. In game Γ_l, there exists a unique SPNE in pure strategies, in which both firms bid the price P_2, where

$$P_2 = [C(Q(P_2)) - C(Q(P_2)/2)]/(Q(P_2)/2) \tag{23}$$

in the first stage. P_2 equals the average cost of producing the second one-half of the total quantity sold. In equilibrium, each firm supplies one-half of the total quantity demanded at that price, and they both receive a positive profit, $C(Q(P_2)) - 2C(Q(P_2)/2)$.

Proof. The proof is similar to the proof of Proposition 1. We observe first that $T(P) = (W_2(P) + L_2(P))/2$. The lowest price in the first stage, $\underline{P}$, cannot, in equilibrium, be lower than P_2, since if only one firm bids $\underline{P}$, it can benefit by increasing its bid a little (as W_2 is increasing below P_2). If both firms bid $\underline{P}$, then a firm can gain by increasing its bid, thereby becoming the loser as $L_2(\underline{P}) > T(\underline{P})$. This lowest price, $\underline{P}$, cannot exceed P_2, since if there is a losing player, he can choose P close enough to P_2, where $P_2 < P < \underline{P}$, become the winner, and gain (as $W_2(P) > W_2(P_2) = L_2(P_2) \geq L_2(\underline{P})$ by Lemma 2).

Next, consider a tie situation. Two cases are possible. In the first case, $W_2(\underline{P}) = L_2(\underline{P})$. By deviating to $P > P_2$, where P is close enough to P_2, a player can become the winner and obtain $W_2(P) > W_2(P_2) = L_2(P_2) \geq L_2(\underline{P}) = T(\underline{P})$. In the second, $W_2(\underline{P}) > L_2(\underline{P})$. By deviating to $P < \underline{P}$, where P is close enough to $\underline{P}$, a player can become the winner and obtain $W_2(P) > (W_2(P) + L_2(\underline{P}))/2 = T(\underline{P})$. Therefore, the lowest price is P_2. There cannot be a winning player, since either player could still win by bidding $P > P_2$, where P is close enough to P_2, and obtain $W_2(P) > W_2(P_2)$. So, the game can only end in a tie, where they both bid P_2. It is easy to show that this is indeed a SPNE. *Q.E.D.*

In the Appendix, we analyze mixed-strategy equilibria in games Γ_w and Γ_l. We show that even in this wider set of strategies, there exists a unique SPNE in each game, which is the pure-strategy SPNE of Propositions 1 and 2.

We now turn to the social welfare implications of subcontracting in terms of the maximization of consumer plus producer surplus.

Proposition 3. $P_1 < P^* = C'(Q(P^*)/2) < P_2$, where P^* is the price that maximizes consumer plus producer surplus.

Proof. The price P^* and the quantities to be produced in the two identical production facilities are determined from the problem

$$\max_{Q^*,q} \int_0^{Q^*} P(Q)dQ - C(q) - C(Q^* - q), \tag{24}$$

where $P(Q)$ is the inverse demand function, Q^* is the total quantity to be produced, and q is the amount to be produced in one of the facilities. Partial differentiation with respect to Q^* and q yields

$$P(Q^*) - C'(Q^* - q) = 0 \tag{25a}$$

and

$$-C'(q) + C'(Q^* - q) = 0. \tag{25b}$$

Thus,

$$P(Q^*) = C'(Q^*/2). \tag{26}$$

But, by the strict convexity of $C(Q)$, it follows that

$$P_1 = C(Q(P_1)/2)/(Q(P_1)/2) < C'(Q^*/2)$$
$$< P_2 = [C(Q(P_2)) - C(Q(P_2)/2)]/(Q(P_2)/2). \tag{27}$$

Thus, from society's standpoint, P_1 is too low and production is too high if the winner of the game's first stage gets to set the terms of the subcontract in the second stage, and P_2 is too high and production is too low if the loser in the first stage is the Stackelberg leader in the second stage. *Q.E.D.*

Remark 1. Suppose that the terms of the subcontract are determined by a bargaining process in which the winning and the losing firms share the potential gains from subcontracting, $C(Q(P)) - 2C(Q(P)/2)$, in the proportions s and $1 - s$, respectively, where $0 \leq s \leq 1$. (In game Γ_w, $s = 1$, and in game Γ_l, $s = 0$.) Then, it can be shown that
(a) the quantity subcontracted is always $Q(P)/2$,
(b) the SPNE price in pure strategies is

$$P_s = \frac{sC(Q(P_s)/2) + (1 - s)[C(Q(P_s)) - C(Q(P_s)/2)]}{Q(P_s)/2}, \tag{28}$$

and (c) firm profits in the event of a tie are $T(P_s) = .5W(P_s) + .5L(P_s)$ for all s. The Nash bargaining solution occurs at $s = ½$ and $P_s = C(Q(P_s))/Q(P_s)$, the average cost of producing the entire quantity demanded. Note that the average cost of producing the entire quantity, Q, is just the equally weighted convex combination of the average costs of producing the first one-half and the second one-half of the quantity demanded. It can be shown that there exists an s, say s^*, such that $P_{s^*} = P^*$, the socially optimal price, and an s, say s^{**}, such that $P_{s^{**}} = P_m$, the monopoly price, as long as $P_m \leq P_2$.

□ **Examples.** If the cost function is linear quadratic, or $C(Q) = bQ + cQ^2$ where b, $c > 0$, and $s = 1/2$, then the first-stage SPNE price is socially optimal, i.e.,

$$P_s = C(Q(P_s))/Q(P_s) = C'(Q(P_s)/2),$$

the marginal cost of producing one-half of the total quantity demanded, $Q(P_s)$. This follows from the fact that for the above cost function, the average cost is $C(Q)/Q = b + cQ$ and the marginal cost evaluated at $Q/2$ (as, in equilibrium, each firm produces one-half of the total quantity demanded) is $C'(Q/2) = b + cQ$.

We can also illustrate the circumstances under which the monopoly price that would be charged by a single firm with two production facilities operating with the identical, strictly convex cost function is below P_2, the SPNE, when the loser sets the subcontracting terms. The sharing rule, s, that will enable the two independent firms to share equally in their monopoly profits can also be derived. We do this by positing, in addition to the linear quadratic cost function, an inverse linear demand function, $P = a - Q$ where $a > b$. The last assumption ensures the profitability of production at a positive level. For the posited cost function, $P_2 = b + 3cQ(P_2)/2$, the average cost of producing the second one-half of $Q(P_2)$. But, $Q(P_2) = a - P_2$, from the posited inverse demand function. Thus, substituting and collecting terms gives

$$P_2 = (2b + 3ac)/(2 + 3c). \tag{29}$$

For the monopolist with two identical production facilities, the monopoly price is determined from the maximization problem

$$\max_Q \, (a - Q)Q - 2C(Q/2).$$

His total cost is $2C(Q/2)$ because he splits production evenly between the two facilities. As $2C(Q/2) = bQ + cQ^2/2$, the first-order condition for a maximum yields that the monopoly output is

$$Q_m = (a - b)/(2 + c); \tag{30}$$

the corresponding monopoly price is

$$P_m = (a + ac + b)/(2 + c). \tag{31}$$

(It is easy to show that $P_1 = (ac + 2b)/(2 + c)$ and that, therefore, $P_m > P_1$.) To determine when $P_m \leq P_2$, we consider $P_m - P_2$ and find, after substituting from (29) and (31) and using some algebra, that

$$\text{sign}\,(P_m - P_2) = \text{sign}\,(a - b)(2 - c). \tag{32}$$

As $a > b$ by assumption, it follows that $P_m \leq P_2$ if and only if $c \geq 2$. This condition amounts to the requirement that $2C''(Q/2) = c \geq 2$, i.e., that the slope of the monopolist's marginal cost function be greater than or equal to two. To determine the sharing rule, s, that induces the two firms to choose P_m as the SPNE price in the first stage, we substitute P_m in the left-hand side of (28) and evaluate the terms on the right-hand side in accordance with the posited cost function. This yields

$$s = [3cQ_m - 2(P_m - b)]/2cQ_m.$$

But, from (30) and (31), $P_m - b = (1 + c)Q_m$. Thus, upon substituting and collecting terms, we get

$$s = 1/2 - 1/c, \qquad c \geq 2. \tag{33}$$

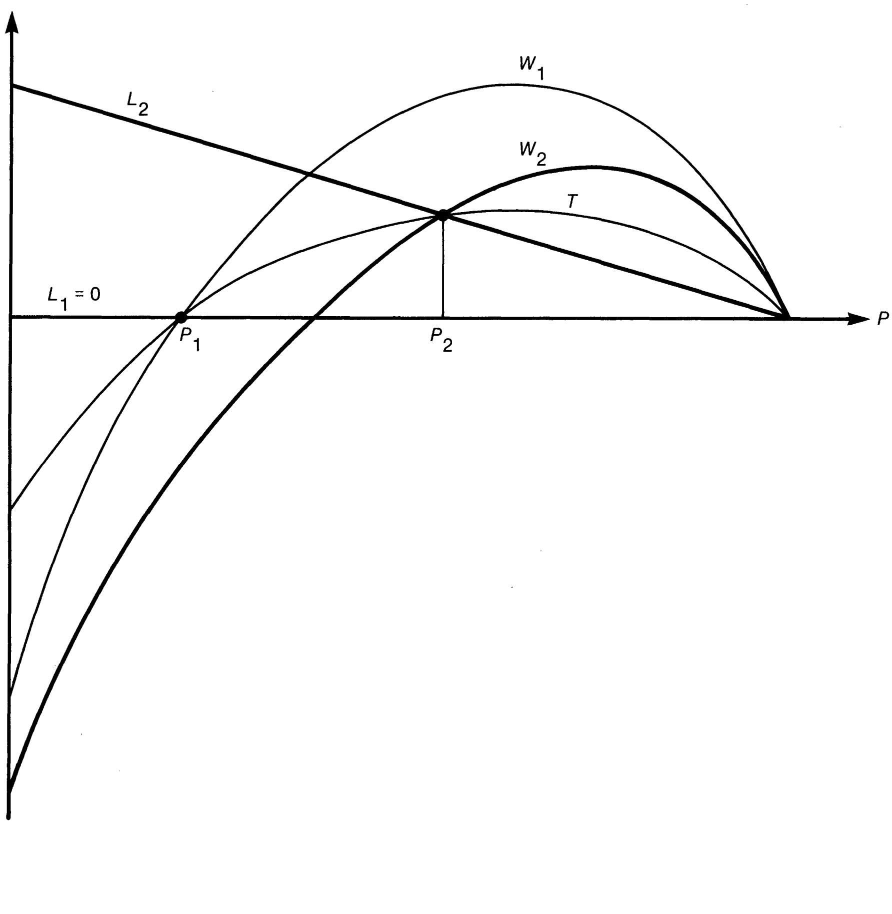
What this little exercise indicates is that if the joint monopoly profits are achievable within the range of the possible SPNE prices specified by (28), then the loser of the game's first stage should, according to (33), realize at least one-half of the cost saving from subcontracting. With the sharing rule specified by (33) for the winner and the loser in the game's second stage, each firm will bid the monopoly price, P_m, in the first stage, and they will share the monopoly profits equally because each produces exactly one-half of the monopoly output, Q_m.

Thus, comparing Propositions 1 and 2, it is evident that were the firms able to choose the means of subcontracting, they would both prefer that the loser in the first stage be the one that sets the terms of the subcontract in the second stage. The intuitive reason for this is that if the winner of the first stage gets to set the subcontract in the second stage, the loser receives zero profits, and therefore, being the loser is very disadvantageous. Bidding in the first stage will be aggressive to the point that both firms receive zero profits in equilibrium. On the other hand, if the loser of the first stage gets to set the terms of the subcontract in the second stage, then being the loser is not disadvantageous, and both firms bid less aggressively in the first stage. While both firms are better off if the loser of the first stage is the leader in the second stage, these profits do not necessarily coincide with those that occur under full cooperation, i.e, the profits of a monopolist with two identical production facilities.

The results of Propositions 1 and 2 and Equation (28) are illustrated in Figure 1. L_1 and W_1 refer to the respective profits of the loser and winner in game Γ_w, while L_2 and W_2 are their respective profits in game Γ_l. All the prices in the interval $[P_1, P_2]$ are potential SPNE in pure strategies, as the sharing rule, s, ranges between zero and one. Figure 1 also leads to the following remark.

FIGURE 1:

SYMMETRIC BERTRAND DUOPOLY WITH SUBCONTRACTING



Remark 2. If the firms share the market equally in the event of a tied price and subcontracting is not allowed, then under the posited assumptions regarding the cost function and the requirement that the winning firm supply the entire quantity demanded, any price in the interval $[P_1, P_2]$ is a Nash equilibrium price in pure strategies. This can be easily seen from Figure 1 by noting that in the absence of subcontracting, L_1 represents the loser's profits and W_2, the winner's profits. Recall that W_2 is the profits the winner can realize by supplying the entire quantity demanded, and T represents the profits in the event of a tie. Now, any tied price, P, in the interval $[P_1, P_2]$ is a Nash equilibrium at which each firm realizes a profit on the T curve. This is so because if a firm raises its price, it will become the loser and receive zero profits, and if it lowers its price, it will become the winner, and its profits will fall to a level along the W_2 curve. Thus, in the absence of subcontracting, there exists a multiplicity of Nash equilibria. However, if ties are resolved by the toss of a fair coin, then there is a unique Nash equilibrium in the absence of subcontracting. This is because the tie curve drops down to the level at which it intersects $W_2 = L_1$ and is below the W_2 curve to the right of this intersection. (Note that in the absence of subcontracting, the two tie-breaking rules do not yield the same $T(P)$ function.) At any price to the right of this intersection, each firm has the incentive to lower its price and become the winner, while for any price to the left of the intersection, each has an incentive to raise its price and become the loser. At the intersection, neither firm has the incentive to raise or lower its price. At the intersection of the W_2 and L_1 curves, the price equals the average cost of supplying the entire quantity demanded, since at this price, the winner's profits are zero. Thus, in the absence of subcontracting and with the winner being chosen at random in the event of a tie, the unique Nash equilibrium price equals the average cost of producing the entire quantity demanded, and both firms receive zero profits.

3. Competition to supply a fixed quantity

■ The description of games Γ_w and Γ_l remain the same in this section, but we assume now that the demand function is of the form

$$Q(P) = \begin{cases} 0 & \text{if} \quad P > P_0 \\ Q_0 & \text{if} \quad P \leq P_0, \end{cases} \tag{34}$$

for the positive price P_0 and quantity Q_0. This is descriptive of the situation in which the first stage involves bidding for the production of a fixed quantity, as in a government contract. The analysis of the second stage of the game remains the same as before, since P is assumed to be fixed in this stage. Thus, the loser's and the winner's payoffs as a function of the winning price, P, are given by

$$W_1(P) = PQ_0 - 2C(Q_0/2),$$

$$L_1(P) = 0,$$

$$W_2(P) = PQ_0 - C(Q_0),$$

and

$$L_2(P) = C(Q_0) - 2C(Q_0/2) \qquad \text{if} \qquad P \leq P_0,$$

and

$$W_1(P) = L_1(P) = W_2(P) = L_2(P) = 0 \qquad \text{if} \qquad P > P_0.$$

The payoff for each player in the case of a tie is

$$T(P) = \begin{cases} PQ_0/2 - C(Q_0/2) & \text{if} \quad P \leq P_0 \\ 0 & \text{if} \quad P > P_0. \end{cases}$$

The prices

$$P_a = C(Q_0/2)/(Q_0/2),$$

$$P_b = C(Q_0)/Q_0,$$

and

$$P_c = [C(Q_0) - C(Q_0/2)]/(Q_0/2)$$

play an important role in describing the equilibria in games Γ_w and Γ_l. Note that W_1 is an increasing linear function that vanishes when $P = P_a$. W_2 is an increasing linear function that vanishes when $P = P_b$ and coincides with the constant function L_2 where $P = P_c$.

Note also that, as above, $T = (W_1 + L_1)/2 = (W_2 + L_2)/2$.

Proposition 4. In game Γ_w, the following hold.
(i) If $P_0 > P_a$, there exists a unique SPNE in which both firms bid the price $P_1 = P_a$ and end up with a payoff of zero.
(ii) If $P_0 < P_a$, then in any SPNE both firms bid prices that are higher than P_0 in the first stage. Moreover, any two bids higher than P_0 support an SPNE. In all these SPNEs, the firms produce zero and receive zero payoffs.
(iii) If $P_0 = P_a$, then all SPNE's in (ii) are still SPNE, and there is an additional SPNE in which both firms bid the price P_0, each produces $Q_0/2$, and each receives a zero payoff.

Proof.
(i) Conditions similar to those of Lemma 1 hold here for $P_1 = C(Q_0/2)/(Q_0/2)$, and the proof now follows that of Proposition 1.
(ii) For any $P \leq P_0$, $W_1(P) < T(P) < 0$. Thus, the lower price in the first stage in an SPNE cannot be lower than P_0, since in this case at least, one of the firms receives a negative payoff, and it can always guarantee a zero payoff. So, in any SPNE, the two bids are above P_0. Indeed, any such two prices constitute an SPNE, since the payoffs in this case are zero, and no one can improve upon them.
(iii) Clearly, bidding P_0 is also an equilibrium. *Q.E.D.*

Proposition 5. In game Γ_l, i.e., if the loser of the first stage is the leader in the second stage, the following hold.
(i) If $P_0 \geq P_c$, then there exists a unique SPNE in which the firms bid the same price, P_c, in the first stage, and each receives the positive payoff $C(Q_0) - 2C(Q_0/2)$.
(ii) If $P_b < P_0 < P_c$, then all the SPNE's are of the following type. One firm wins by bidding P_0, while the other bids a higher price. This results in subcontracting in the second stage. Both firms enjoy a positive profit, the loser's being higher.
(iii) If $P_0 = P_b$, then all SPNE's of (ii) are also SPNE here. In addition, any pair of prices higher than P_0 are also SPNE prices that result in zero payoffs for both firms.
(iv) If $P_0 < P_b$, any pair of prices higher than P_0 is an SPNE. These are all the SPNEs.

Proof.
(i) Conditions similar to those of Lemma 2 hold here, and the proof is similar to that of Proposition 2.
(ii) Similar to Proposition 2, the lowest bid, $\underline{P}$, cannot be below P_0. $\underline{P}$ cannot exceed P_0, since in this case, both firms realize zero profits and a firm can gain by bidding P_0, become the winner, and receive $W_2(P_0) > 0$. Thus, $\underline{P} = P_0$. It is impossible in equilibrium that both bid P_0, since by increasing its bid and becoming the loser, a firm receives $L_2(P_0) > T(P_0)$. Therefore, one firm bids P_0 and receives $W_2(P_0)$, while the other bids a price higher than P_0 and receives $L_2(P_0)$. To see that this is indeed an equilibrium, we note that the loser

receives the highest payoff in the game and would not deviate, and the winning firm can only lower its payoff by bidding a lower price or receive zero by increasing its bid.

The proofs of (iii) and (iv) are simpler and, therefore, are left to the reader. *Q.E.D.*

The interesting feature of this analysis of competition for a contract is that in game Γ_l, subcontracting may occur in equilibrium (Part (ii) of Proposition 5), even if the tie-breaking rule entails equal sharing of the quantity to be supplied. Such an equilibrium does not arise in the case of competition for a market with the same tie-breaking rule.

4. Summary

■ We have investigated a Bertrand duopoly with the potential for subcontracting production to the rival as a two-stage game. The presence of strictly convex production costs creates the incentive for subcontracting. Our analysis of the SPNE of the game discloses the sensitivity of the final product price and of the firms' profits to the subcontracting terms. If the terms of the subcontract favor the loser of the game's first stage, then both rivals are better off than if they favor the winner. (A result that should be of some comfort to all the losers in the world.) Indeed, if the winner of the first stage sets the terms of the subcontract in the second stage, then competition to be the winner dissipates away all of the profits. On the other hand, if the first-stage loser sets the terms of the subcontract, then both firms realize the full benefits of subcontracting. The presence of subcontracting possibilities does not, in general, assure that the SPNE price will be socially optimal. However, from the standpoint of the party seeking bids for a contract, or for consumers in the case of competition for a market, it is preferable that the winner set the subcontracting terms, for consumers then realize the lowest price. Indeed, were the party to ask for bids on the contract, or were the consumers able to specify the subcontracting arrangements, they would require that the winner set the subcontracting terms.

There are a large number of possible extensions of our analysis. These include consideration of more than two firms and of firms with different cost functions. The analysis of these situations in Kamien and Li (1989) reveals that with three or more firms, SPNE in pure strategies exist if the losers set the subcontracting terms, but do not exist if the winners set them. The exact opposite is true in the case of two asymmetric firms, i.e., one with lower production costs than the other. Repeated play of these games, the presence of uncertainty, the possibility of sharing information, and the expansion of these games to include the choice of the mode of subcontracting as a strategic variable are all extensions that would lend more realism to our models.

Appendix

■ We consider games Γ_w and Γ_l (in the case of competition to supply a market) when the firms use mixed strategies in the first stage. (Allowing mixed strategies in the second stage does not change the analysis, since in this stage, the leader has a dominant strategy.) We show that the results of Section 2 do not change. The pure-strategy equilibria in Propositions 1 and 2 remain the only equilibria in the bigger set of mixed strategies. We study a single game, Γ, which generalizes to both game Γ_w and game Γ_l.

Consider a two-person game, Γ, in which pure strategies for both players are prices in the interval $[0, \infty)$. We denote the players by 1 and 2, and when we use i and j to refer to the players, we always assume $i \neq j$. The payoff function to player i is $v_i(P_i, P_j)$, where the price chosen by i always appears as the first argument. The payoff functions are given by three continuous functions on $[0, \infty)$, L, T, and W, as follows.

$$v_i(P_i, P_j) = \begin{cases} L(P_j) & \text{if} \quad P_i > P_j, \\ T(P_i) & \text{if} \quad P_i = P_j, \\ W(P_i) & \text{if} \quad P_i < P_j. \end{cases}$$

Clearly, v_i is continuous at each point (P_i, P_j) when $P_i \neq P_j$. We assume that the functions L, T, and W satisfy the following conditions.

Assumption A1. There exists a $P^0 > 0$ such that
(a) for $P < P^0$, $L(P) > T(P) > W(P)$;
(b) for $P > P^0$, $W(P) \geq T(P) \geq L(P)$, where inequality holds strictly in a neighborhood of P^0, and both inequalities are strict whenever one of them is;
(c) $W(P)$ is not decreasing for $P < P^0$;
(d) $L(P)$ is not increasing for $P > P^0$;
(e) W and L are differentiable at P^0.

Observe that by the continuity of L, T, and W, $L(P^0) = T(P^0) = W(P^0)$ by Assumptions A1(a) and A1(b). A mixed strategy is a measure μ on $[0, \infty)$, which we represent by its cumulative distribution function, F. We identify each $P \in [0, \infty)$ with the mixed strategy with all mass at P. The payoff functions, v_i, are extended naturally to mixed strategies. For a pair of mixed strategies, F_1 and F_2,

$$v_i(F_i, F_j) = \int_0^\infty \int_0^\infty v_i(P_i, P_j)dF_i(P_i)dF_j(P_j).$$

The support of F, denoted supp (F), contains all points which do not have a neighborhood of measure zero (i.e., where F is constant). The set of atoms of F (i.e., where F has a jump) is denoted by $J(F)$. For $P \in J(F)$, $\mu(P)$ is the measure of P. A pair (F_1, F_2) is an equilibrium if for each player i, $v_i(F_i, F_j) = \max_F v_i(F, F_j)$. Note that since v_i is continuous whenever $P_i \neq P_j$, it follows that $v_i(P, F_j) = \int_0^\infty v_i(P, P_j)dF_j(P_j)$ is continuous in P whenever $P \notin J(F_j)$. This remark can be easily used to prove the following proposition. (Formal proofs of this proposition and the following corollary are available from the authors.)

Proposition A1. Let (F_1, F_2) be an equilibrium. Then, for each price P and player i, $v_i(P, F_j) \leq v_i(F_i, F_j)$. Moreover, equality holds for each $P \in$ supp (F_i) whenever either $P \in J(F_i)$ or $P \notin J(F_j)$.

Corollary A1. Let (F_1, F_2) be an equilibrium. If $P_i \in$ supp (F_i) and either $P_i \in J(F_i)$ or $P_i \notin J(F_j)$, then for each P, $v_i(P_i, F_j) \geq v_i(P, F_j)$.

Theorem A1. Γ has a unique equilibrium in mixed strategies, given by (P^0, P^0).

We note that the functions W_1, L_1 and T, as well as the functions W_2, L_2, and T, satisfy all of Assumptions A1(a)–A1(e), and therefore by Theorem A1, game Γ_w and game Γ_l each have a unique SPNE in mixed strategies.

Proof. We prove Theorem A1 using Lemmas A1 and A2. We assume throughout the proof that (F_1, F_2) is an equilibrium of Γ.

Lemma A1. min (supp $(F_1) \cup$ supp $(F_2)) \geq P^0$.

Proof. Let $\underline{P}$ = min (supp $(F_1) \cup$ supp (F_2)), and assume $\underline{P} < P^0$. Now, either $\underline{P} \in J(F_1) \cap J(F_2)$, or for at least one j, $\underline{P} \notin J(F_j)$. In either case, by Corollary A1, $v_i(\underline{P}, F_j) \geq v_i(P^0, F_j)$. We finish the proof by contradicting the last inequality. Indeed, for $P_j \geq P^0$, $v_i(\underline{P}, P_j) = W(\underline{P}) \leq W(P^0) = v_i(P^0, P_j)$ by Assumption A1(c). For $\underline{P} \leq P_j < P^0$, $v_i(\underline{P}, P_j) \leq T(P_j) < L(P_j) = v_i(P^0, P_j)$ by (1). Thus, $v_i(\underline{P}, F_j) = \int_{\underline{P}}^\infty v_i(\underline{P}, P_j)dF_j(P_j) < v_i(P^0, P_j)$. *Q.E.D.*

Lemma A2. For at least one player i, F_i is the pure-strategy P^0.

Proof. Let $\underline{P}$ = max (supp $(F_1) \cup$ supp (F_2)), and suppose $\underline{P} > P^0$. Consider the following three cases.

Case 1. $\underline{P} \in$ supp (F_i), and $\underline{P} \notin J(F_j)$.

By Corollary A1, $v_i(P^0 + \epsilon, F_j) - v_i(\underline{P}, F_j) \leq 0$. Evaluating this difference and assuming that $P^0 + \epsilon \notin J(F_j)$ and that $P^0 + \epsilon < \bar{P}$, we find, using Assumptions A1(d) and A1(b), that

$$\int_{P^0+\epsilon}^{P} [W(P^0 + \epsilon) - L(P_j)]dF_j(P_j) \geq \int_{P^0+\epsilon}^{P} [W(P^0 + \epsilon) - L(P^0 + \epsilon)]dF_j(P_j)$$

$$= [W(P^0 + \epsilon) - L(P^0 + \epsilon)] \int_{P^0+\epsilon}^{P} dF_j(P_j) \geq 0.$$

Thus, we conclude that $[W(P^0 + \epsilon) - L(P^0 + \epsilon)] \int_{P^0+\epsilon}^{P} dF_j = 0$. But, for small enough ϵ,

$$W(P^0 + \epsilon) - L(P^0 + \epsilon) > 0$$

by (2), and therefore, $\int_{P^0+\epsilon}^{P} dF_j = 0$. Letting $\epsilon \rightarrow 0$ and using Lemma A1, we conclude that F_j is P^0.

Case 2. $P \in J(F_1) \cap J(F_2)$, and $W(P) = L(P)$.

By (2), $T(P) = L(P)$, and $v_i(P, P) = L(P)$. The computations of Case 1 remains exactly the same.

Case 3. $P \in J(F_1) \cap J(F_2)$, but $W(P) > L(P)$.

We show that this case is impossible. By Assumption A1(b), $W(P) > T(P) > L(P)$. By Corollary A1, $v_i(P - \epsilon, F_j) - v_i(P, F_j) \leq 0$. But, this difference, when $P - \epsilon \notin J(F_j)$, is

$$\int_{[P-\epsilon,P)} [W(P-\epsilon) - L(P_j)]dF_j(P_j) + \mu_j(P)[W(P+\epsilon) - L(P)].$$

When $\epsilon \to 0$, the integral vanishes, and the other term converges to $\mu_j(P)[W(P) - L(P)] > 0$ by the stipulation of the case, which is a contradiction. *Q.E.D.*

We are now ready to prove Theorem 1.

By Lemma A2, there exists a player i such that F_i is P^0. By Lemma A1, supp $(F_j) \subseteq [P^0, \infty)$. Using Corollary A1, Assumption A1(d), and Assumption A1(e) to get $P^0 + \epsilon \notin J(F_j)$ gives

$$\begin{aligned}
0 &\geq v_i(P^0+\epsilon, F_j) - v_i(P^0, F_j) \\
&= \int_{P^0}^{P^0+\epsilon} L(P_j)dF_j(P_j) + \int_{P^0+\epsilon}^{\infty} W(P^0+\epsilon)dF_j(P_j) - L(P^0) \\
&\geq L(P^0+\epsilon)\mu_j((P^0, P^0+\epsilon)) + W(P^0+\epsilon)\mu_j([P^0+\epsilon, \infty)) \\
&\quad + \mu_j(P^0)L(P^0) - L(P^0) \\
&= [L(P^0) + \epsilon L'(P^0) + o(\epsilon)]\mu_j((P^0, P^0+\epsilon)) \\
&\quad + [W(P^0) + \epsilon W'(P^0) + o(\epsilon)]\mu_j([P^0+\epsilon, \infty)) \\
&\quad + \mu_j(P^0)L(P^0) - L(P^0) \\
&= \epsilon[L'(P^0)\mu_j((P^0, P^0+\epsilon)) + W'(P^0)\mu_j([P^0+\epsilon, \infty))] + o(\epsilon).
\end{aligned}$$

Since $\mu_j((P^0, P^0+\epsilon)) \to 0$ as $\epsilon \to 0$, it follows that

$$0 \geq \liminf_{\epsilon \to 0} [v_i(P^0+\epsilon, F_j) - v_i(P^0, F_j)]/\epsilon \geq W'(P^0)\mu_j((P^0, \infty)).$$

But, $W'(P^0) > 0$, and hence, $\mu_j((P^0, \infty)) = 0$; i.e., F_j is P^0. *Q.E.D.*

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