Available online at www.sciencedirect.com
SCIENCE DIRECT

Journal of Economic Theory 110 (2003) 213–233

JOURNAL OF Economic Theory

http://www.elsevier.com/locate/jet

Between liberalism and democracy

Dov Samet[a,*] and David Schmeidler[a,b]

[a] *Faculty of Management, Tel Aviv University, Tel Aviv 69978, Israel*
[b] *Department of Economics, Ohio State University, OH, USA*

Received 1 December 2001; final version received 16 April 2002

Abstract

We study and characterize axiomatically a class of voting rules, called consent rules, that incorporate aspects of majoritarianism and liberalism. An outcome of the vote specifies who among the voters are eligible to a certain right or qualification. Each outcome serves also as a permissible ballot. Consent rules are parameterized by the weights given to individuals in determining their own qualification. In one of these rules, the liberal rule, each individual's qualification is determined by her. In other rules, an individual's qualification requires social consent in various degrees. We also show the relation between self-determination and the liberal rule.

JEL classification: D70; D71; D72

Keywords: Consent rules; Liberalism; Democracy; Majority; Voting; Self-determination

1. Introduction

1.1. Liberalism and democracy

The liberal and the democratic principles dominate modern political thought. The first requires that decisions on certain matters rest with the individual and not with society. The second assigns the power of decision making to majorities. The question of the right balance between these two principles is an ongoing subject of debate in the public at large and among students of political thought. An effort is continually made to draw the line between the domains in which each principle applies: when

*Corresponding author. Fax: +972-3-640-9983.
E-mail addresses: dovs@tauex.tau.ac.il (D. Samet), schmeid@tau.ac.il (D. Schmeidler).

0022-0531/03/$ - see front matter
doi:10.1016/S0022-0531(03)00080-2

social involvement in an individual's affairs is justified, and when the person is allowed to make the decision alone.

Thus, for example, it is acceptable nowadays in liberal democracies that questions regarding the reading of certain books should not be decided by a majority of any form, and should be left to each individual's discretion. It is also obvious, that although all citizens have the same political rights, the question who holds the highest political position (say the president) is decided, roughly speaking, by a simple majority. But in between these two cases—the majoritarian rule, and the liberal rule—there are decisions, concerning an individual, that on the one hand do not require a majority, and on the other are not determined by the individual either. Consider, for example, the right to be a *candidate* for the top position in society, or some other political position. Exercising such a right by an individual is usually not up to him or her. But one does not need a majority decision to be included on the list of candidates; in most liberal democratic regimes one needs only a relatively small group of supporters to be considered as a candidate.

In some cases more than a simple majority is required for an individual to exercise certain rights. For example, in some cases, changes in privately owned real estate can be prevented by any individual of the community, that is, a unanimous approval of the change is required.

It is our purpose here to study and characterize, in the framework of a social choice model, a family of social procedures, called here consent rules, that allows different levels of social intervention in one's decisions. Thus, the liberal rule in which no social intervention is allowed, is an extreme case in this family. The unanimity requirement is another extreme. Simple majority rule has a special standing in this family. The dichotomy between liberalism and democracy has been much discussed and debated in liberal thought since its conception. Liberalism and majoritarianism, separately, have also been extensively studied in the social choice literature. But as far as we know, presenting them as two points in a whole spectrum of rules is new.

1.2. The model

In order to study the relation between the liberal and the majoritarian rules in the same model, we follow a recent work by Kasher and Rubinstein [10] who studied group identification. We present the model against the background of the first and most familiar social choice model of liberalism introduced in Sen's [15] pioneering work. Sen studied liberalism in terms of Arrow's [1] social choice functions. Such a function maps any profile of preference orders of individuals to a social preference order. An individual is said to be *decisive* on two alternatives, for a given social choice function, if the function orders these alternatives in the same way as the individual does. Sen's minimal liberalism axiom requires that there be at least two individuals each of whom is decisive on two alternatives. He shows, then, that this axiom contradicts Pareto optimality, referring to this contradiction as the liberal paradox.

The following three features of Sen's framework should be emphasized.

- Social alternatives, as in Arrow's model, have no distinguishable characteristics. As a result there is no endogenous way to associate an individual with specific alternatives. Thus, the alternatives over which an individual is decisive, according to Sen's liberal axiom, must be given exogenously.
- A social welfare function maps lists of individuals' preferences, which are non-observable, to social preferences, which are non-observable as well. The mechanism by which alternatives are actually selected is disregarded. In particular, the social welfare function is not interpreted as a voting rule.
- Liberalism is defined as the ability of an individual to guarantee that his *preference* over certain outcomes prevails, as opposed to the ability to guarantee a certain subset of the outcomes themselves.

Many social choice models of liberalism differ from that of Sen on one or another of these points. The model used here differs on each of them. First of all, social alternatives here are structured. More specifically, each alternative is a subset of individuals. Although, as we said, no such structure exists in Sen's framework, the example he used to illustrate the liberal paradox can be interpreted as having this structure. In this example society has to decide who of its members is allowed to read Lady Chatterley's Lover. Obviously, an answer to this question, that is, the social alternative chosen, would be a subset of society. As in Sen's example we study one right at a time. Social alternatives are subsets of individuals, where each subset is interpreted as the group of individuals eligible to the said right.

Second, unlike Sen, we do not deal with social welfare functions. We study, instead, a voting rule, in which each individual proposes a subset of individuals and, given the profile of the proposed subsets, the rule determines the eligible individuals.

Finally, liberalism is defined here as the ability of an individual to guarantee certain aspects of the outcomes. If we think of a voting rule as a game form in which the various ballots are the strategies of each individual, then liberalism means here that each individual has a strategy that enables her to determine certain aspects of the social outcome. This variant of liberalism says, in the example above, that the question of whether an individual reads a certain book or does not, can be determined by her, by choosing appropriate ballots, irrespective of others' ballots. Preferences over outcomes play no role in this definition of liberalism.

The model studied here, can be used in problems of endowments of rights on issues that are considered private and personal matter, like the books one reads and the dress one wears. But it can be applied to the endowment of rights that have social implications, like the right to drive a car in the public domain. It is applicable not only to endowment of rights but also to assignment of obligations. In ancient Athens individuals casted ballots (ostraca) listing the people who should be ostracized and deported from the city. The model also describes certain procedures for elections. Thus, when it is required to compose an agreed upon list of candidates for the election of a dean, it is natural to ask each faculty member to propose a list and then apply some aggregating rule to form a final list of candidates. In Kasher and

Rubinstein [10], the model is applied to the problem of socially defining the extension of a given nationality.

The feature common to all these problems is that a certain qualification of persons is considered. A qualification, as seen in the examples above, may have a significant subjective component; individuals may have different views concerning who is qualified. Therefore, a rule needs to be found that transforms the various views concerning qualification into a socially defined group of qualified individuals.

Qualification is a dichotomous distinction. It distinguishes between the qualified and the unqualified. The two sides of this dichotomy are usually not symmetric in any sense. Thus, marking a person as being qualified for having a driver's license is socially and personally very different from marking her as unqualified to have it. Later on we discuss this asymmetry extensively. It should be emphasized, though, that qualified and unqualified are used here just to describe the dichotomy; the terms themselves are neutral and carry no value judgment.[1]

1.3. The main result

We consider three axioms, which in one form or another are very standard in theories of social choice. The special twist of the rules that are characterized by these axioms stems from a particular feature of the model: a ballot is a subset of the very same group of individuals who cast the ballots. In the language of social choice theory, social alternatives are subsets of voters.

The first axiom requires *monotonicity*. Consider the set of the socially qualified individuals given certain ballots. If some voters change their mind and add more names to their ballots, then all the individuals previously qualified remain qualified.

Next we require *independence*. The social qualification of an individual should depend only on the views held by all individuals concerning *this* particular individual and not others.

Finally, the *symmetry* axiom requires that a rule does not depend on the names of individuals. In our model, where social outcomes are subsets of voters, this innocent looking axiom has a special bite (see Section 4.1).

These axioms characterize a family of rules which we call *consent rules*. As we explain shortly, these rules, like the liberal rule, put special weight on one's opinion regarding herself, and like the majority rule require social approval. The relation to the majority rule is not surprising: similar axioms characterize the majority rule in various models (the first one is due to May [11]). The relation to the liberal rule is somewhat less obvious.

Consent rules are parameterized by *consent quotas* s and t, which are positive integers, the sum of which does not exceed $n+2$.[2] These quotas determine the level of social consent required to make an individual's vote for herself socially acceptable. Given the parameters s, t, one's qualification of oneself is adopted by society only if

[1] We use in the sequel the verbs qualify and disqualify. In this paper the latter means marking a person as unqualified and not changing the status of a person who was previously qualified.

[2] The reason for this bound is explained in the next section.

there are at least $s-1$ other individuals who qualify her, and one's disqualification of oneself is adopted by society only if there are at least $t-1$ other individuals who disqualify her.

In the next subsection we elaborate two contrasts expressed by the consent quotas s and t. The first is the contrast between individual and social power in determining the individual's status. This contrast is reflected in the size of the quotas. The larger the quota s is, the greater is the social power to act against an individual's vote for self-qualification. The case of the quota t is completely analogous to that of s. The other contrast is represented by the difference $|s-t|$. The larger this difference is, the more unevenly the rule treats qualification versus non-qualification.

1.4. The domain of consent quotas

Consider first the extreme cases where $s=1$ or $t=1$. In the first case society has no power to prevent qualification from an individual who votes to qualify herself. Similarly, in the second case, when an individual votes against her qualification, her vote is decisive. In case both hold, $s=1$ and $t=1$, the power to qualify and disqualify an individual rests completely in the hands of the individual. The consent rule, $f^{1,1}$, in which social power is minimal, is termed the *liberal rule*. Its axiomatic characterization is presented in Section 3.

When the quotas s or t increase, an individual needs more support from others to have his way. Although the case of both s and t being large is excluded, the extreme case where either of them separately may equal $n+1$ is possible. The rule $f^{1,n+1}$ is the only rule for which $t=n+1$. According to this rule, when an individual votes for her qualification she is qualified. When she votes for her disqualification, she needs the support of $n+1$ individuals, which exceeds the size of the population. Therefore, this rule is the constant rule, according to which each individual is qualified, independently of the profile of votes. Similarly, $f^{n+1,1}$ is the constant rule which disqualify all individuals independently of their votes. The more interesting rules are those where either $s=n$ or $t=n$, in which case unanimity is required to accept self-qualification or self-disqualification.

We consider next the difference between the quotas. Being qualified and being disqualified may have different social implications. This is why we can expect rules f^{st} in which $s \neq t$. Consider, for example, the qualification of individuals as having the right to drive a car in the public domain. Having such a right certainly impinges on different rights of others, like being able to cross the street safely.[3] Thus, when one wishes to drive in the public domain, social consent is required. But if one wishes not to, the consent of others is not needed. A rule of the form f^{s1} for some $s>1$ seems to be most appropriate here, as in other cases of licensing (practicing medicine, selling food). Rules f^{st} with $s>t$ are typical of situations where qualification means having a

[3] We do not refer here to externalities, namely, the impinging on the same right. In the latter case, society may wish to limit the number of licenses. Such a rule is excluded by the axiom of independence as we explain in Section 4.2.

right that partially impinges on the rights of others, while not having it hardly affects them.

Rules f^{st} with $s<t$ will be appropriate when qualification means imposing an obligation or duty. Very little if any social consent is required when an individual qualifies herself, i.e., volunteers. It is disqualification, which means a release from duty in this case, that may require broad social consent.

In some cases the dichotomy qualified–disqualified is socially neutral. Consider a stylized exhaustive dichotomy like Democrat–Republican in the US, or Catholic–Protestant in Germany. If qualification of an individual means labeling her as a Democrat, then disqualifying her means labeling her as a Republican. In such a case, if we were to employ a consent rule to classify individuals, we would use a rule with $s = t$, which is neutral with respect to the dichotomy qualified–disqualified.

The liberal rule f^{11} is an extreme case in the family of the neutral rules f^{ss}. The other extreme case in this family, for an odd n, is when s and t are each $(n+1)/2$. It is easy to see that this voting rule is the (simple) majority rule. Whether a person is qualified or not is determined, according to this rule, by a simple majority in which this person's vote counts as every other vote. This is the *only* nontrivial consent rule[4] in which one's vote concerning one's qualification has no special weight.[5]

1.5. Related works

Structuring social alternatives, in order to study the liberal paradox, was proposed by Gibbard [8]. Each individual, in his model, is associated with certain *issues*. An issue can be the color of John's shirt, or whether Mary reads book X. A social alternative specifies how all issues are resolved. Gibbard's liberal axiom requires that an individual should be decisive on *any* two alternatives that differ on only one issue, which is associated with that individual. The alternatives in our model can be viewed as a special case of those in Gibbard's model. For each individual there is one issue, which can be resolved in one of two ways: it is the question whether or not the individual is qualified. Gibbard shows that there can be no social choice function that satisfies the liberal axiom. In contrast, in our model, where liberalism is outcome based, liberal rules obviously exist; these are the simple rules which allow individuals to resolve the issues related to them as they wish.

The procedural, game theoretic, aspects of social choice were first studied by Farquharson [6]. He characterized families of voting procedures axiomatically and analyzed their game theoretic aspects. Gibbard's [9] manipulability result, further highlighted these game theoretic aspects.

Barbera et al. [2] studied voting procedures in which the subsets of some fixed finite set serve both as the alternatives voted for, and as the ballots. The model here is a special case where this finite set is the set of individuals. They characterize the rules of voting by committees as those rules which are strategy-proof and satisfy voter

[4] Obviously, in the trivial consent rules $f^{n+1,1}$ and $f^{1,n+1}$ all votes are equally ineffectual.

[5] When n is even, the extreme case is $s = t = n/2 + 1$. In this case one's vote on one's status does not count at all, and a simple majority of all individuals *other* than j is required to determine j's status.

sovereignty over separable preferences. In our terminology, voting by committees are rules that satisfy monotonicity and independence.

The distinction between the two variants of liberalism, preference based and outcome based, has been discussed by many authors. Nozick [12] criticized Sen's preference based liberalism. Gärdenfors [7] formalized outcome-based liberalism by a game theoretic model. Liberalism as it is understood here corresponds to the notion of dichotomous veto power in Deb et al. [5]. A definition of liberalism, applied to a game forms, is discussed in Riley [13].

The liberal rule, as defined above, was introduced and axiomatically characterized by Kasher and Rubinstein [10]. One of their axioms, which they call the liberal axiom, requires that individuals can force certain outcomes. It says that if there is an individual who considers herself qualified, then there must be someone who is socially qualified, and if there is an individual who considers herself *un*qualified, then there must be someone who is socially *un*qualified. Thus, the special status of one's qualification of oneself is stated explicitly in the liberal axiom. In our model, none of the axioms requires explicitly either that individuals have any power to determine certain outcomes or that one's qualification of oneself is more significant than others' qualification of one.

Our contribution to the study of majority rules is the introduction of the 'liberal component' into these rules. Namely, granting each individual a special weight in determining her own qualification. Majority rules at large have been extensively discussed in the literature. Of special importance is the book entitled *The Calculus of Consent* by Buchanan and Tullock [3] in which majority rules with different quotas as well as their costs and benefits, are analyzed.

2. The main characterization

2.1. Preliminaries and notations

Let $N = \{1, \dots, n\}$ be a set of individuals. These individuals are facing the problem of collectively choosing a certain subset of N. The input for this collective choice is the personal views that individuals have concerning who is qualified, that is who should be a member of this set. These views are summarized by a *profile* which is an $n \times n$ matrix $P = (P_{ij})$, the elements of which are 0's and 1's. When $P_{ij} = 1$, we say individual i qualifies individual j. Thus, row i in the matrix P describes the group of qualified individuals in i's view; it is the set $\{j \mid P_{ij} = 1\}$. Column j tells us who are the individuals i who qualify j.

A *rule* is a function f which associates with each profile P, a vector $f(P) = (f_1(P), \dots, f_n(P))$ of 0's and 1's, which is the indicator function of the group of socially qualified individuals, $\{j \mid f_j(P) = 1\}$.

For $x \in \{0, 1\}$ we use the standard notation $\bar{x} = 1 - x$. Accordingly, $\bar{P} = (\bar{P}_{ij})$, and $\bar{f}(P) = (\overline{f_j(P)})$. For arrays A and B (matrices or vectors) of the same dimension we write $A \geqslant B$ if this inequality holds coordinatewise.

2.2. The axioms

Our first axiom requires that the social rule f is weakly increasing with respect to the partial order on profiles. For a pair of profile P and P', $P \geqslant P'$ means that every individual who was qualified by someone in the profile P' is qualified by the same person also in P. The axiom says that in this case every individual who is socially qualified in P' is qualified in P as well.

Monotonicity. If $P \geqslant P'$, then, $f(P) \geqslant f(P')$.

A weaker version of monotonicity which suffices for our purposes is discussed in Section 4.3.

We require, next, that the social qualification of individual j is independent of what individuals think about the qualification of individuals other than i.

Independence. If P and P' are profiles such that for some $j \in N$, $P_{ij} = P'_{ij}$ for all $i \in N$, then $f_j(P) = f_j(P')$.

We require that social qualification should not change if individuals switch their names. Name switching is described by a permutation π of N. We think of $\pi(i)$ as the *old* name of the person whose *new* name is i. For a profile P, given in terms of the old names, we denote by πP the profile after the name switching. To say that i qualifies j, using the new names, means that the individual whose old name is $\pi(i)$ qualifies the person whose old name is $\pi(j)$. Thus, $(\pi P)_{ij} = P_{\pi(i)\pi(j)}$. The axiom requires that given the profile πP, in terms of the new names, individual i is socially qualified, if and only if individual $\pi(i)$ was qualified when the profile was P. Denoting by $\pi f(P)$ the vector $(f_{\pi(1)}(P), \ldots, f_{\pi(n)}(P))$, the axiom can be succinctly stated as follows.

Symmetry. For any permutation π of N, $f(\pi P) = \pi f(P)$.

2.3. Consent rules and their characterization

We define now a family of rules which is characterized by the axioms in the previous subsection. The rules in this family are parameterized by two positive integers s and t. According to such a rule an individual who qualifies herself is socially qualified if she has the consent of at least s voters (including herself). Similarly, an individual who disqualifies herself is socially disqualified if she has the consent of at least t voters (including herself).

Definition 1. Let s and t be positive integers. A *consent rule* with *consent quotas s* and *t* is a social rule f^{st} such that:

(1) for each individual j and profile P,
  if $P_{jj} = 1$, then $f^{st}_j(P) = 1 \Leftrightarrow |\{i \mid P_{ij} = 1\}| \geqslant s$
  if $P_{jj} = 0$, then $f^{st}_j(P) = 0 \Leftrightarrow |\{i \mid P_{ij} = 0\}| \geqslant t$.

(2) $s + t \leqslant n + 2$.

Our main result is:

Theorem 1. *A rule satisfies the axioms of monotonicity, independence, and symmetry if and only if it is a consent rule. Moreover, all three axioms are independent.*

The proof that consent rules satisfy the axioms is, as usual, quite simple. The role that the various axioms play in the characterization of the consent rules is as follows. Independence implies that the social qualification of j, $f_j(P)$, depends only on column j in P, namely on the votes for and against qualification of j. By symmetry, the outcome for j does not change when we permute the votes of all other individuals concerning j. Thus, given j's vote, $f_j(P)$ depends only on the distribution of the votes of other individuals. Monotonicity, ensures that this dependence takes the form of quotas, one for the case j votes for her qualification, and one for the case she votes against it. Invoking symmetry once again we conclude that the quotas are the same for all individuals. The condition $s + t \leqslant n + 2$ is intimately related to monotonicity. It also reflects a restriction on the power of society that we explain now.

Given a profile P and an individual j, denote by P^j the profile with entries the same as in P, except that $P^j_{jj} = \bar{P}_{jj}$. That is, P and P^j differ only in j's vote on herself. For a rule f, there are four possibilities.

- $f_j(P) = f_j(P^j) = 1$,
- $f_j(P) = f_j(P^j) = 0$,
- $f_j(P) = P_{jj}$ and $f_j(P^j) = \bar{P}_{jj}$,
- $f_j(P) = \bar{P}_{jj}$ and $f_j(P^j) = P_{jj}$.

In the first and second cases the rule allows society to determine j's status paying no respect to her vote. In the third, the rule grants j the right to choose between being qualified and disqualified. In the fourth alternative, the rule allows for a spiteful vote. It neither determines j's social status, nor does it allow j the choice; it only guarantees that her vote, whatever it is, is socially disapproved. This leads to the following definition.

Definition 2. A rule f is *non-spiteful* if there exist no profile P and individual j such that $f_j(P) = \bar{P}_{jj}$ and $f_j(P^j) = P_{jj}$.

The next result states formally that condition 2 in Definition 1 is equivalent to requiring non-spitefulness or monotonicity.

Proposition 1. *Consider a rule f that satisfies condition* (1) *in Definition* 1 *with respect to some positive integers $s, t \leqslant n + 1$. Then the following three conditions are equivalent.*

(i) $s + t \leqslant n + 2$,
(ii) f *is monotonic*,
(iii) f *is non-spiteful*.

Remark. Without the condition $s, t \leqslant n+1$ in the proposition above, conditions (ii) and (iii) do not imply (i). The problem is that we deal with rules that satisfy only condition (1) in Definition 1. In this case the quotas $(1, n+1)$ and $(n+1, 1)$ describe the same rules as $(1, m)$ and $(m, 1)$, respectively, for any $m > n+1$. Thus, the rules corresponding to $(1, m)$ and $(m, 1)$ are monotonic and non-spiteful, but they do not satisfy $s + t \leqslant n + 2$. The constant rules are the only ones that have a non-unique representation.

2.4. Duality

Consider a social vote described by a profile P. Suppose we ask all individuals to name the unqualified individuals rather than the qualified ones. The profile describing this vote is $\bar{P}$. Applying a rule f to $\bar{P}$ gives us the set of individuals who are socially qualified in the terms of $\bar{P}$, or unqualified in terms of P. Therefore, $\overline{f(\bar{P})}$ describes a set of qualified individuals, in the P terminology. We call this rule for social qualification, the *dual* of f and denote it by $\bar{f}$. In the case that the rule $\bar{f}$ is the same as f we say that f is self-dual. This is summarized in the following definition.

Definition 3. The *dual* of the rule f is the rule $\bar{f}$ defined by, $\bar{f}(P) = \overline{f(\bar{P})}$. The rule f is *self-dual* if $f = \bar{f}$.

The following proposition characterizes the action of duality on consent rules and identifies the self-dual rules among them.

Proposition 2. *For any consent rule* f^{st}, $\overline{f^{st}} = f^{ts}$. *A consent rule* f^{st} *is self-dual if and only if* $s = t$.

The set of self-dual consent rules can be characterize by adding the following axiom.

Self-duality. The rule f is self-dual.

In view of Theorem 1, the following theorem can be easily proved.

Theorem 2. *A social rule satisfies the axioms of monotonicity, independence, symmetry, and self-duality if and only if it is a consent Rule with equal consent quotas. Moreover, all four axioms are independent.*

3. Self-determination

The political principle of self-determination says that a group of people recognized as a nation has the right to form its own state and choose its own government. One of the main difficulties in applying self-determination is that it grants the right to exercise sovereignty to well-defined national identities; it assumes that the self is well defined. In many cases the very distinct national character of the group is under dispute. Such disputes can be resolved, at least theoretically, by a voting rule. Here we want to examine rules which grant the self the right to determine itself. For brevity, we refer to this property of voting rules as self-determination and not determination of the self, despite the new meaning we give it.

Suppose we want to define the nationality of Hobbits. On first examination the requirement that Hobbits determine who are Hobbits seems to be circular. But this circularity can be avoided in two ways. In the first, we require that after defining Hobbits using the rule f, changing non-Hobbits' opinion about Hobbits and applying the rule again will result in the same definition of Hobbits. We call this axiom *exclusive self-determination*, because it is expressed in terms of excluding non-Hobbits from those who have the power to define Hobbits. We call the second axiom *affirmative self-determination,* because it states directly the right of Hobbits to define Hobbits. In order to formulate it we use the rule f to not only to find out who are Hobbits, but also to find out who qualify Hobbits. Affirmative self-determination says that the two groups, of Hobbits and of qualifiers of Hobbits, should coincide. We show that each of these axioms combined with monotonicity, independence and nondegeneracy characterize the liberal rule. That is, the right of a collective to define itself is reduced to the right of each individual to consider herself as part of this collective. Note in particular that the symmetry axiom is not required for this characterization.

The formulation of the first version of self-determination is straightforward.

Exclusive self-determination. For a profile P, let

$$H = \{j \mid f_j(P) = 1\}.$$

If Q is a profile such that $P_{ij} \neq Q_{ij}$ only if $i \notin H$ and $j \in H$, then $f(Q) = f(P)$.

For our characterization we need the following nondegeneracy axiom, which excludes voting rules in which qualification of a certain individual is independent of the votes.

Nondegeneracy. For each individual i there are profiles P and Q such that $f_i(P) = 1$, and $f_i(Q) = 0$.

Theorem 3. *The liberal rule is the only one that satisfies the axioms of monotonicity, independence, nondegeneracy, and exclusive self-determination.*

For the formulation of affirmative self-determination we need to be able to identify those individuals who have the authority to determine who is a Hobbit. For this purpose, we view a rule as an aggregator. A rule takes as input the rows of a profile and aggregates them into a single row. The set of individuals indicated by the aggregate row has the same meaning, i.e., represents the same property, as the sets indicated by the rows of the profile. There is however a difference. The property indicated by a row of the profile is indexed by this row. Since the aggregation is over rows, these indices disappear in the aggregate set. In our interpretation of profiles, the rows of the profile are indexed by members of society, and the aggregate row which does not correspond to any single individual is interpreted as indexed by "society".

For example, let P be a profile in which row i lists all persons that i considers Hobbits. Here, the property of an individual in the set indicated by a row of the profile is *being* a Hobbit. That is, $p_{ij} = 1$ means that j is a Hobbit (according to i). The aggregate row indicates the same property. That is, if j's coordinate is 1, then j is a Hobbit (according to society).

Let P^{t} be the transpose of the profile matrix P. That is, $P^{\mathrm{t}}_{ij} = P_{ji}$. The property of an individual in the set indicated by a row of the *transposed* profile is *defining* someone as a Hobbit. That is, $p^{\mathrm{t}}_{ij} = 1$ means that j defines someone as a Hobbit (individual i in this case). The aggregate row indicates the same property. That is, if j's coordinate is 1, then j defines Hobbits. Thus, applying an aggregation rule f to P results in a set of Hobbits, while applying the same rule to P^{t} results in a set of definers of Hobbits.

The affirmative self-determination axiom requires that Hobbits and only Hobbits determine who are Hobbits. In our formulation this is expressed by:

Affirmative self-determination. $f(P) = f(P^{\mathrm{t}})$.

Theorem 4. *The liberal rule is the only one that satisfies monotonicity, independence, nondegeneracy and affirmative self-determination.*

4. Discussion

4.1. The symmetry axiom

In our model a society N is required to qualify its own individuals. A more general model is one in which society is required to qualify individuals or objects of some set M, disjoint of N. A social profile, in this case, would be a matrix P, not necessarily a square one, where the rows are labeled by individuals in N, and columns are labeled by individuals in M. The axioms of monotonicity and independence can be stated in the same way, and equally motivated for the more general case.

The symmetry axiom is special, though, to our model. In the general model there are two axioms that are related to symmetry. First, we can require *anonymity* of the individuals of society. That is, if the names of the members of N—the qualifying individuals—are permuted, then the socially qualified group remains the same. Formally, for any permutation τ of N, we require that $f(\tau P) = f(P)$, where $(\tau P)_{ij} = P_{\tau(i)j}$.

Second, we can require *neutrality* of the qualified persons, as follows. Let σ be a permutation of M, and P a profile. Suppose, now, that we change the profile such that each individual i in the society qualifies j iff he qualified $\sigma(j)$ in P. We require that socially qualified persons in the new profile are those obtained by permuting the qualified persons in the original profile. Formally, it is required that $f(\sigma P) = \sigma f(P)$, where $(\sigma P)_{ij} = P_{i\sigma(j)}$.

Requiring anonymity in our model implies the disregarding of the natural identification of society with the set of qualified individuals. In particular, the linkage between the individual and the issue to which her qualification is considered becomes socially irrelevant. Among the consent rules only the simple majority rule, for odd n, satisfies this.

Requiring neutrality, on top of independence, implies that the same rule is used by society to determine the qualification of each individual. Again, only the simple majority rule, for odd n, satisfies this, among all the consent rules.

We conclude, then, that by adding either of these two axioms, or both, to the three axioms in Theorem 1 results in a unique mechanism, the simple majority rule, when n is odd, and no possible rule for even n. Observe also, that these two axioms imply the symmetry axiom, and therefore omitting symmetry and adding anonymity and neutrality gives the same characterization as adding both on top of symmetry.

The emergence of the weight given to one's qualification of oneself, in our model, depends on the combination of all three axioms. But we would like to highlight the role of the symmetry axiom in this respect. Symmetry of any object refers to the transformations of that object that leave it the same. Indeed, symmetry defines sameness. It defines what is essential to the object and what is not. Social symmetries define the meaningful features of society. In our model, the symmetry axiom says that the naming of individuals is insignificant. And as rules are expressed in terms of names, this means that individuals are indistinguishable, which reflects the idea of the equality that underlies the democratic principle.

However, as social alternatives in our model are subsets of individuals, the symmetry axiom says more. It allows name swapping only as long as the same permutation is carried out both for the individuals as voters, and as members of the social alternatives. Thus, breaking the linkage between an individual and a certain issue is not allowed by the symmetry axiom, or, in other words, the axiom allows such a linkage to be socially meaningful. In some cases this linkage seems to be necessary. Suppose, for example that the issue is the reading of book X. If Adam changes his name to Barry, then Adam's reading book X, now becomes Barry's reading of it. The linkage between one and one's reading X cannot be broken. This sounds almost tautological: it is impossible to think of *my* reading of a certain book as an issue which in some transformation of society is not linked to me, let alone linked to someone else. But the linkage between me and *my* apple—the one that I consider my property—is less obvious. We can easily think about transformation of the social environment in which my apple is no longer mine. Indeed, some of the arguments made to justify property rights try to establish a logical link between individuals and the objects over which they have property rights. Thus, for example, Locke's argument is based on the work and effort invested by the individuals in their property.

Our result seems to indicate, then, that rights are not necessarily primitive notions. Rights can be based, among other things, on the more primitive assumption that certain linkages between individuals and issues are considered socially meaningful and relevant. This assumption is more primitive, since it does not say how these meaningful linkages should be reckoned with: they can be liabilities, for example, rather than rights.

4.2. The independence axiom

In certain social situations the independence axiom is untenable. Consider for example the election of a committee of a certain fixed size from a list of candidates. The list may consist of all voters, like the model we discuss in this paper. Suppose, moreover, that each individual votes by specifying a subset of individuals (who are also the candidates). A social rule, in this case, will be defined exactly as it is defined here, with one difference. The range of such a rule should be restricted to subsets of the size of the committee. In this case the independence axiom is not reasonable. The question whether a certain individual should qualify as a member of the committee should depend not only on what voters think about her, but also on what they think about others, and how the others compare to her. A well-known rule for such a problem is approval voting (see Brams and Fishburn [4]), in which the elected committee consists of the individuals with the highest score, where the score of an individual is the number of voters who include her in their votes. Obviously, approval voting does not satisfy independence.

Another example where independence is an inappropriate and impossible restriction on a voting rule is when it is required that the number of men and women who are qualified should be the same. In general, if some restriction is imposed on the subsets of the socially qualified individuals, then independence may be inappropriate. The proposition below shows, that independence indeed implies a relatively full range.

Proposition 3. *Let f be a rule that satisfies the axiom of independence. Consider the partition of N into the three subsets (T_0, T_1, S) as follows. Individual j is in T_0 iff for all P, $f_j(P) = 0$. Individual j is in T_1 iff for all P, $f_j(P) = 1$. Finally, $j \in S$ iff there exist Q_0 and Q_1 such that $f_i(Q_0) = 0$ and $f_i(Q_1) = 1$. Then for each subset $T \subseteq S$ there exists a profile Q, such that for each $j \in S$, $f_j(Q) = 1$ iff $j \in T$.*

Moreover, if the rule f also satisfies symmetry, then its range is either $\{0,1\}^N$, or $(1,\ldots,1)$, or $(0,\ldots,0)$. That is, exactly two of the subsets in the partition (T_0, T_1, S) are empty.[6]

A less transparent example of a rule that does not satisfy independence but satisfies monotonicity and symmetry appears in the proof of Theorem 1. The range

[6] Rubinstein and Fishburn [14] also record a connection between independence and restrictions on the range of a rule. They studied models in which the rows of P and the range of f can be neither $(1,\ldots,1)$ nor $(0,\ldots,0)$. They show that the only rules that satisfy the axioms of independence and Consensus are the dictatorial rules. The requirement that the range of a rule is full plays an important role in Barbera et al. [2] and is called voter sovereignty.

of the rule consists of all subsets of individuals. This shows that the inverse of Proposition 3 does not hold.

4.3. Monotonicity

The axiom of monotonicity can be weakened. If an individual is qualified, and certain voters change their mind and qualify her, then she is still qualified.

4.4. An alternative representation of consent rules

We represent now consent rules in terms of weighted majority games. Such games are described by a list of $n+1$ numbers $[q; w_1, \ldots, w_n]$, where $q>0$ is the quota and $w_i \geqslant 0$ is the weight of player i. A coalition S is winning iff $\Sigma_{i \in S} w_i \geqslant q$. Representing consent rules in terms of such games enables us to express the special role played by individuals in determining their own qualification by the weight they have in these games.

Define for each pair (s,t) of consent quotas a pair of weighted majority games v_j^s and v_j^t. The quota in each of the games is n. The weight of all individuals in both games is 1, except for j whose weight is $n-s+1$ in the first game and $n-t+1$ in the second. Consent rules can be redefined in terms of these games as follows. The simple proof is omitted.

Proposition 4. *For any profile P and individual j, if $P_{jj}=1$, then $f_j^{st}(P)=1$ iff $\{i \mid P_{ij}=1\}$ is a winning coalition in v_j^s, and if $P_{jj}=0$, then $f_j^{st}(P)=0$ iff $\{i \mid P_{ij}=0\}$ is a winning coalition in v_j^t.*

4.5. The liberal rule and the liberal axiom

We assume, now, that each individual i has a strict preference relation $\succ_i$ over subsets of individuals (or equivalently over $\{0,1\}^N$). For given preferences of individuals we can select a subset of individuals by applying the liberal rule, $f^{1,1}$, to the profile of most preferred subsets. We may ask, now, whether the liberal rule satisfies the liberal axiom.

Since social alternatives in our model are a special case of those in Gibbard [8], it is most appropriate to consider Gibbard's version of the liberal axiom. In terms of our setup, a rule f satisfies this axiom when each individual is decisive on certain alternatives as follows. If x and y are two elements of $\{0,1\}^N$, such that $x_j = y_j$, for each $j \neq i$, and i prefers x to y, then $f(P) \neq y$. As is shown in Gibbard [8] there is no rule, defined over all preference orders, that satisfies this axiom. However, he shows that there are rules that satisfy a *restricted liberal axiom*, in which decisiveness of individuals is required only when preferences are *unconditional*. Individual i's preference is said to be unconditional (for his issue), when for any x and y as above, if i prefers x to y, then he also prefers x' to y', whenever, $x'_i = x_i$, $y'_i = y_i$ and $x'_j = y'_j$

for all $j \neq i$. In terms of subsets this means that if for some S where $i \notin S$, $S \cup \{i\} \succ_i S$, then for all T such that $i \notin T$, $T \cup \{i\} \succ_i T$, and if for some S where $i \notin S$, $S \succ_i S \cup \{i\}$, then for all T such that $i \notin T$, $T \succ_i T \cup \{i\}$.

It is straightforward to see that $f^{1,1}$ satisfies the restricted liberal axiom. Moreover, Gibbard's proof for the existence of a social choice function that satisfies the restricted liberal axiom is carried out by constructing a function that extends the liberal rule to the more general case studied in Gibbard [8].

4.6. Sincerity

Individual preferences combined with the rule $f^{1,1}$ define a game. We can ask, then, under which conditions will it be an equilibrium for the individuals to be *sincere* (as defined by Farquharson [6]), i.e., to propose their most preferred subset. It is easy to see that if an individual's preferences are unconditional, then proposing his most preferred subset is a dominant strategy. Indeed, an individual's vote determines only whether he belongs to the socially selected subset of qualified persons or not. If individual i belongs to her most preferred set, then by unconditionality she prefers to join any subset. This she achieves by voting her most preferred subset. The argument is similar when she does not belong to her most preferred subset.

The model here is a special case of the one in Barbera et al. [2], where individuals vote for a subset of a given set K. A preference order $\succ$ on subsets of K is called *separable*, in that work, when for any subset T and $x \notin T$, $T \cup \{x\} \succ T$ if and only if x is an element of the most preferable subset. In our model, separability implies unconditionality, and therefore guarantees that proposing the most preferred subset of individual is a dominant strategy for f^{11}. It is easy to see also that the stronger condition of separability guarantees that truth telling is dominant for every rule which satisfies monotonicity and independence (and hence in all consent rules). This is the "easy" part of the characterization of voting by committees in Barbera et al. [2].

4.7. The liberal and the Pareto axioms

As noticed by Gibbard, even the restricted liberal axiom contradicts Pareto efficiency. Indeed, suppose there are two individuals who have to determine who reads X. Individual 1 cares foremost for 2's education: he prefers any alternative in which 2 reads X to any alternative in which 2 does not. In the second place, 1 cares for his own education: other things being equal (i.e., given 2's behavior) he would rather read X than not. Similarly, 2 cares foremost for 1's moral fiber, and therefore she prefers any alternative in which 1 does not read X to any alternative in which 1 does. Likewise, other things being equal she prefers refraining from reading salacious X. Proposing their most preferred alternatives (1 proposes that both read it, 2—that both do not), the liberal rule allows 1 to read X and 2 not to. Yet both prefer that 1 does not read it, and 2 does.

Each individual preference in this case is unconditional for his/her issue. The reason for the failure of Pareto efficiency is due to the excessive nosiness of the individuals: each individual minds foremost the other's business. If we require the opposite, that each individual cares in the first place for his/her issue and only in the second place for others', then the liberal rule satisfies Pareto efficiency. We say that i's preference is *moderately nosy* if when i prefers x to y he also prefers x' to y', whenever $x'_i = x_i$, and $y'_i = y_i$. When individuals are moderately nosy, there cannot be an alternative x which is unanimously preferred to $f^{1,1}(P)$. Indeed, if $x \neq f^{1,1}(P)$, then for some j, $x_j \neq f_j^{1,1}(P) = P_{jj}$. As $(P_{j1}, \ldots, P_{jn})$ is the alternative most preferred by j, it follows by moderate nosiness, that it is also preferred to x.

5. Proofs

Proof of Theorem 1. *Necessity*. We show that consent rules necessarily satisfy the three axioms. It is easy to see that any such rule satisfies independence. To prove that it satisfies monotonicity, assume that $P' \geqslant P$ and let $f = f^{st}$. Suppose that $f_j(P) = 1$. Then either $P_{jj} = 1$ and $|\{i \mid P_{ij} = 1\}| \geqslant s$, or $P_{jj} = 0$ and $|\{i \mid P_{ij} = 0\}| < t$. The latter is equivalent to $|\{i \mid P_{ij} = 1\}| \geqslant n - t + 1$. If $P'_{jj} = P_{jj}$ then in either case $f_j(P') = 1$, since $|\{i \mid P'_{ij} = 1\}| \geqslant |\{i \mid P_{ij} = 1\}|$. If $P_{jj} = 0$ and $P'_{jj} = 1$, then $|\{i \mid P'_{ij} = 1\}| \geqslant n - t + 2 \geqslant s$ (the last inequality follows from condition 2 in the definition of consent rules) and therefore $f_j(P') = 1$ in this case too.

To show that symmetry holds for $f = f^{st}$, let π be a permutation of N. Suppose that $(\pi P)_{jj} = P_{\pi(j)\pi(j)} = 1$. Then, $f_j(\pi P) = 1$ iff $|\{i \mid (\pi P)_{ij} = (\pi P)_{jj}\}| \geqslant s$, i.e., $|\{i \mid P_{\pi(i)\pi(j)} = P_{\pi(j)\pi(j)}\}| \geqslant s$. As π is one-to-one, the latter condition is equivalent to $|\{\pi(i) \mid P_{\pi(i)\pi(j)} = P_{\pi(j)\pi(j)}\}| \geqslant s$. But this is exactly the necessary and sufficient condition that $f_{\pi(j)}(P) = P_{\pi(j)\pi(j)}$ $(= 1)$. Thus, $f_j(\pi P) = f_{\pi(j)}(P)$ in this case. Similar argument follows when $(\pi P)_{jj} = P_{\pi(j)\pi(j)} = 0$.

Sufficiency. We show that any social rule f that satisfies the three axioms is a consent rule. By the independence axiom, $f_j(P)$ depends only on column j in P. Therefore, for each j there exists a function $h_j : \{0,1\}^N \to \{0,1\}$ such that $f_j(P) = h_j(P_{1j}, \ldots, P_{nj})$.

From monotonicity it follows that if $P \geqslant P'$, then

$$h_j(P_{1j}, \ldots, P_{nj}) = f_j(P) \geqslant f_j(P') = h_j(P'_{1j}, \ldots, P'_{nj}).$$

Thus, h_j is monotonic, that is, for $x, y \in \{0,1\}^N$, if $x \geqslant y$, then $h_j(x) \geqslant h_j(y)$.

Let π be a permutation of N. Then,

$$f_j(\pi P) = h_j((\pi P)_{1j}, \ldots, (\pi P)_{nj}) = h_j(P_{\pi(1)\pi(j)}, \ldots, P_{\pi(n)\pi(j)}).$$

and

$$f_{\pi(j)}(P) = h_{\pi(j)}(P_{1\pi(j)}, \ldots, P_{n\pi(j)}).$$

Since by the symmetry axiom, $f_j(\pi P) = f_{\pi(j)}(P)$ it follows that,

$$h_j(P_{\pi(1)\pi(j)}, \ldots, P_{\pi(n)\pi(j)}) = h_{\pi(j)}(P_{1\pi(j)}, \ldots, P_{n\pi(j)}). \quad (1)$$

Assume, now, that $\pi(j) = j$. Then, for such a permutation, equation (1) yields $h_j(P_{\pi(1)j}, \dots, P_{\pi(n)j}) = h_j(P_{1j}, \dots, P_{nj})$. Therefore, for any such permutation, and $x \in \{0,1\}^N$, $h_j(x_{\pi(1)}, \dots, x_{\pi(n)}) = h_j(x)$. It follows that if x and y are such that $x_j = y_j$, and $\sum_{i \neq j} x_i = \sum_{i \neq j} y_i$, then $h_j(x) = h_j(y)$. Thus, there exists a function $g_j(a,b)$, where $a \in \{0,1\}$ and $b \in \{0, \dots, n-1\}$, such that $h_j(x) = g_j(x_j, \sum_{i \neq j} x_i)$. Moreover, by (1), for any permutation π, $h_j(x_{\pi(1)}, \dots, x_{\pi(n)}) = h_{\pi(j)}(x)$. But then $g_j(x_{\pi(j)}, \sum_{i \neq j} x_{\pi(i)}) = g_{\pi(j)}(x_{\pi(j)}, \sum_{i \neq \pi(j)} x_i)$, and this means that $g_j = g_{\pi(j)}$. Therefore, we can write g for all g_j.

Assume first that g is constant. If $g \equiv 0$ then $f = f^{n+1,1}$, and if $g \equiv 1$ then $f = f^{1,n+1}$. If g is not constant, then there exist integers a and b such that $g(a,b) = 1$, and integers c and d such that $g(c,d) = 0$. As h_j is monotonic, g is monotonic in both arguments. Thus, $g(1,b) = 1$ and $g(0,d) = 0$. Let $s-1$ be the first integer b in $\{0, \dots, n-1\}$ for which $g(1,b) = 1$, and r the last integer d in $\{0, \dots, n-1\}$ such that $g(0,d) = 0$.

Consider an individual j and a profile P. If $P_{jj} = 1$ then $f_j(P) = g(1, \sum_{i \neq j} P_{ij})$. By the monotonicity of g, $f_j(P) = 1$ iff $\sum_{i \neq j} P_{ij} \geqslant s-1$, which is equivalent to $\sum_i P_{ij} \geqslant s$. Thus,

$$\text{If } P_{jj} = 1 \text{ then } f_j(P) = 1 \Leftrightarrow |\{i \mid P_{ij} = 1\}| \geqslant s. \tag{2}$$

If $P_{jj} = 0$ then $f_j(P) = g(0, \sum_{i \neq j} P_{ij})$ and therefore $f_j(P) = 0$ iff $\sum_{i \neq j} P_{ij} = \sum_i P_{ij} \leqslant r$. Equivalently, defining $t = n - r$,

$$\text{If } P_{jj} = 0 \text{ then } f_j(P) = 0 \Leftrightarrow |\{i \mid P_{ij} = 0\}| \geqslant t. \tag{3}$$

Thus, f satisfies condition (1) of Definition 1 with respect to quotas s and t. To conclude the proof of sufficiency we show that s and t satisfy condition (2) of this definition. Assume to the contrary that $s + t > n + 2$. Define a profile P with $P_{i1} = 0$ for $i \leqslant t-1$ and $P_{i1} = 1$ for $i \geqslant t$. (By the negation assumption, $t \geqslant 3$.) Hence, by (3), $f_1(P) = 1$. Let P' be a profile with the same entries as P, except that $P'_{11} = 1$. Then, $P' \geqslant P$. The number of 1's in column 1 of P' is $n - t + 2$ which is smaller than s, and thus by (2) $f_1(P') = 0$ which contradicts monotonicity. Hence, $f = f^{st}$.

Independence of the axioms. We describe for each axiom a rule that does not satisfy the axiom but does satisfy all other axioms. We omit the detail of the proof.

Monotonicity. Let f be the rule defined for each P by $f(P) = f^{11}(\bar{P})$ (or equivalently $f(P) = \overline{f^{11}(P)}$). For any j, $f_j(0) = 1$, where 0 here is the zero matrix. Choose j and a profile P' with $P'_{jj} = 1$. Then $P' \geqslant 0$, while $f_j(P') = 0$.

Note also that if we omit condition (2) in the definition of consent rules, then for s and t with $s + t > n + 2$, f^{st} satisfies all axioms but monotonicity. This can be easily seen from the proof of the sufficiency part above.

Symmetry. This axiom is obviously violated by dictatorial rules, like $f(P) = (P_{11}, P_{12}, \ldots, P_{1n})$.

Independence. Define $f(P)$ as follows. For each individual i and profile P,

$$\text{if } P_{ii} = 1, \text{ then } f_i(P) = 1 \Leftrightarrow |\{j \mid P_{ij} = 1\}| \geqslant n/2,$$

$$\text{if } P_{ii} = 0, \text{ then } f_i(P) = 0 \Leftrightarrow |\{j \mid P_{ij} = 0\}| \geqslant n/2.$$

Unlike consent rules, what matters for the qualification of i is the number of individuals whom i qualifies rather then the number of individuals who qualify i. □

Proof of Proposition 1. By Theorem 1, (i) implies that f is monotonic. By the definition of spitefulness, a spiteful rule is not monotonic. Thus (ii) implies (iii). We show that (iii) implies (i), by negation. Suppose $s + t \geqslant n + 3$. Since $s, t \leqslant n + 1$ it follows that $s, t \geqslant 2$. Fix an individual j and define a profile P such that $|\{i \neq j \mid P_{ij} = 1\}| = s - 2$ and $|\{i \neq j \mid P_{ij} = 0\}| = n - s + 1$. If $P_{jj} = 1$ then $|\{i \mid P_{ij} = 1\}| = s - 1$ and therefore $f_j(P) = 0$. If $P_{jj} = 0$ then $|\{i \mid P_{ij} = 0\}| = n - s + 2 < t$ and therefore $f_j(P) = 1$. □

Proof of Proposition 2. Suppose $P_{jj} = 1$. Then $\bar{P}_{jj} = 0$, and therefore, $f^{st}(\bar{P}) = 0$ iff $|\{i \mid \bar{P}_{ij} = 0\}| \geqslant t$, or equivalently $|\{i \mid P_{ij} = 1\}| \geqslant t$. Hence $\overline{f^{st}(\bar{P})} = 1$ iff $|\{i \mid P_{ij} = 1\}| \geqslant t$. Similarly, if $P_{jj} = 0$ then $\overline{f^{st}(\bar{P})} = 0$ iff $|\{i \mid P_{ij} = 0\}| \geqslant s$. Thus, for each profile P, $\overline{f^{st}(\bar{P})} = f^{ts}(P)$, that is $\overline{f^{st}} = f^{ts}$. In particular, $\overline{f^{ss}} = f^{ss}$ and therefore f^{ss} is self-dual. Conversely, suppose f^{st} is self-dual. For each profile P, if $P_{ii} = 1$, then $f_i^{st}(P) = 1$ iff $|\{i \mid P_{ij} = 1\}| \geqslant s$, and $f_i^{ts}(P) = 1$ iff $|\{i \mid P_{ij} = 1\}| \geqslant t$. But as f^{st} is self-dual, $f^{ts} = \overline{f^{st}} = f^{st}$. Thus, for each profile P, if $P_{ii} = 1$, then $|\{i \mid P_{ij} = 1\}| \geqslant s$ iff $|\{i \mid P_{ij} = 1\}| \geqslant t$. This can be true for each profile P only when $s = t$. □

Proof of Theorem 2. By Theorem 1 it follows that self-dual consent rules satisfy monotonicity, independence, and symmetry. By Proposition 2 they also satisfy self-duality.

It follows from Theorem 1 that any rule that satisfies the axioms in our theorem is a consent rule. Since it satisfies self-duality it must be, by Proposition 2 of the form f^{ss}.

To see that the axioms are independent, note that the three rules constructed to prove the independence of monotonicity, independence, and symmetry, in the proof of Theorem 1, satisfy self-duality. Thus, they also prove the independence of these axioms in this theorem. To show the independence of self-duality note that by Proposition 2 any rule f^{st} with $s \neq t$ does not satisfy self-duality, while, by Theorem 1, it satisfies the other three axioms. □

Proof of Theorem 3. Since the liberal rule is a self-dual consent rule, it satisfies, by Theorem 2, the axioms of monotonicity, independence and self-duality, and the latter implies nondegeneracy. It is easy to see that it also satisfies exclusive self-determination.

Assume now that f satisfies the axioms. We have to show that for any profile P and each individual k, $f_k(P) = P_{kk}$.

Observe, first, that by monotonicity, $f_j(Q)$ is monotonic in Q for each j, and by independence it depends only on column j. Hence, by nondegeneracy, if for all i, $Q_{ij} = 1$, then $f_j(Q) = 1$, and if for all i, $Q_{ij} = 0$, then $f_j(Q) = 0$.

Let Q be the profile with column k being the same as in P, and with all other columns being 0. By construction $P_{kk} = Q_{kk}$, and by independence, $f_k(Q) = f_k(P)$. Thus, it is enough to show that $f_k(Q) = Q_{kk}$. Note that by the above observation, $f_j(Q) = 0$ for all $j \neq k$.

Consider first the case where $Q_{kk} = 0$, and suppose to the contrary that $f_k(Q) = 1$. Let 0 be the zero matrix. It may differ from Q only for entries Q_{jk} with $j \neq k$. Thus, by exclusive self-determination, $f_k(Q) = f_k(0)$. But, by the first observation, $f_k(0) = 0$, which is a contradiction.

Next, consider the case $Q_{kk} = 1$. Let 1^k be the matrix with column k consisting of 1's and all other columns being 0. Then $f_k(1^k) = 1$. But, Q may differ from 1^k only in entries in column k and rows $j \neq k$. Hence, by exclusive self-determination $f_k(1^k) = f_k(Q)$. □

Proof of Theorem 4. It is easy to see that the liberal rule satisfies the axioms. Conversely, suppose that f satisfies the axioms. We show that for any profile P and each individual k, $f_k(P) = P_{kk}$. Assume first that $P_{kk} = 1$. Let Q be a matrix such that for all i, j, $Q_{ik} = P_{ik}$ and $Q_{kj} = 1$. Denote by 1 the $n \times n$ matrix of ones. Then, $f_k(P) = f_k(Q) = f_k(Q^t) = f_k(1)$, where the first and last equality hold by independence, and the second equality holds by affirmative self-determination. Clearly, monotonicity and nondegeneracy imply that $f_k(1) = 1$. Assume now that $P_{kk} = 0$. Let Q be a matrix such that for all i, j, $Q_{ik} = P_{ik}$ and $Q_{kj} = 0$. Then, by the same argument as above, $f_k(P) = f_k(Q) = f_k(Q^t) = f_k(0)$, where 0 is the $n \times n$ matrix of zeroes. Again, by monotonicity and nondegeneracy $f_k(0) = 0$. □

Proof of Proposition 3. Let S be the subset of all individuals j for which $f_j(P)$ is independent of P, and let $T = N \backslash S$. For every $j \in T$, there must be profiles Q^{j0} and Q^{j1} such that $f_j(Q^{j0}) = 0$ and $f_j(Q^{j1}) = 1$. Given any $x \in \{0,1\}^T$, construct a profile Q as follows. For $j \in T$, if $x_j = 0$, then $Q_{ij} = Q_{ij}^{j0}$ for all i, and if $x_j = 1$, then $Q_{ij} = Q_{ij}^{j1}$ for all i. The rest of Q can be defined arbitrarily. By independence, for all $j \in T$, $x_j = f_j(Q)$.

Suppose that f also satisfies the symmetry axiom. If $S = \emptyset$ then $T = N$, and the range of f is $\{0,1\}^N$. Assume, now, that $j \in S$. Then, $f_j(P)$ is independent of P. We show, moreover, that $f_k(P) = f_j(P)$ for each k and P, and hence the range of f is either $(1, \ldots, 1)$ or $(0, \ldots, 0)$. Indeed, let π be a permutation of N such that $\pi(j) = k$. Then, $f_j(P) = f_j(\pi P) = f_{\pi(j)}(P) = f_k(P)$, where the first equality follows from the definition of S and the second from symmetry. □

Acknowledgments

We are indebted to Itzhak Gilboa for suggesting an improvement of the axiomatic characterization, and to Ben Polak for many comments and suggestions that improved the paper. David Schmeidler gratefully acknowledges ISF Grant No. 790/00.

References

[1] K. Arrow, Social Choice and Individual Values, 2nd Edition, Wiley, New York, 1951.
[2] S. Barbera, H. Sonnenschein, L. Zhou, Voting by committees, Econometrica 59 (1991) 595–609.
[3] J.M. Buchanan, G. Tullock, The Calculus of Consent, University of Michigan Press, Ann Arbor, 1962.
[4] S.J. Brams, A.C. Fishburn, Approval voting, Amer. Polit. Sci. Rev. 72 (1978) 831–847.
[5] R.P. Deb, K. Pattanaik, L. Razzolini, Game forms, rights, and the efficiency of social outcomes, J. Econom. Theory 72 (1997) 74–95.
[6] R. Farquharson, Theory of Voting, Yale University Press, New Haven, 1969.
[7] P. Gärdenfors, Right games and social choice, Noûs 15 (1981) 341–356.
[8] A. Gibbard, Manipulation of voting schemes: a general result, Econometrica 41 (1973) 587–601.
[9] A. Gibbard, A pareto-consistent libertarian claim, J. Econom. Theory 7 (1973) 388–410.
[10] A. Kasher, A. Rubinstein, On the question 'Who is a j', a social choice approach, Logique Analyse 160 (1997) 385–395.
[11] K.O. May, A set of independent necessary and sufficient conditions for simple majority decision, Econometrica 20 (1952) 680–684.
[12] R. Nozick, Anarchy, State and Utopia, Basic Books, Addison-Wesley, New York, 1974.
[13] J. Riley, Rights to liberty in purely private matters, Econom. Philos. 5 (1989) 121–166.
[14] A. Rubinstein, P.C. Fishburn, Algebraic aggregation theory, J. Econom. Theory 38 (1986) 63–77.
[15] A.K. Sen, Collective Choice and Social Welfare, Holden-Day, San Francisco, USA, 1970.