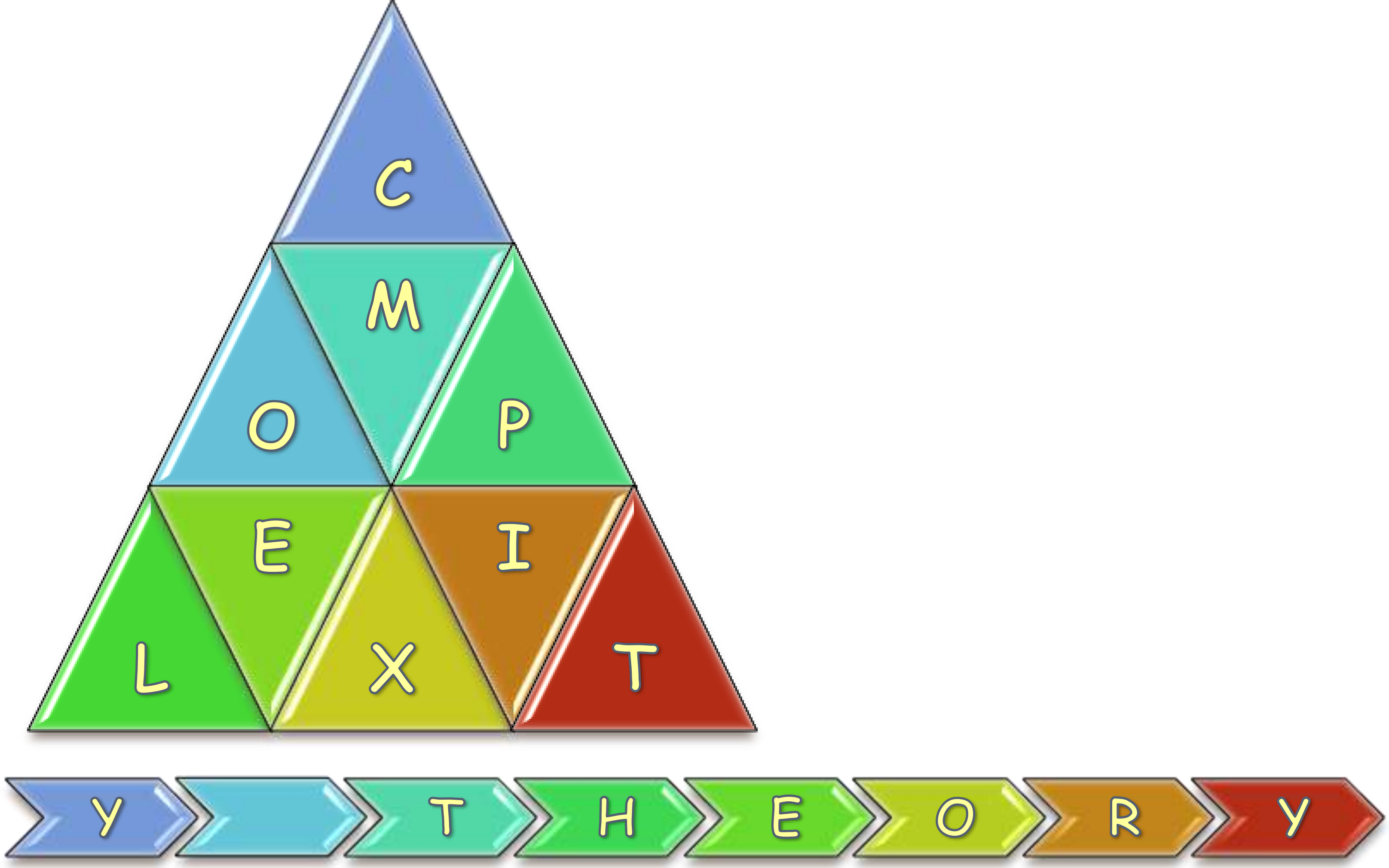
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GOAL:

- Introduce basic concepts in Complexity Theory.

PLAN:

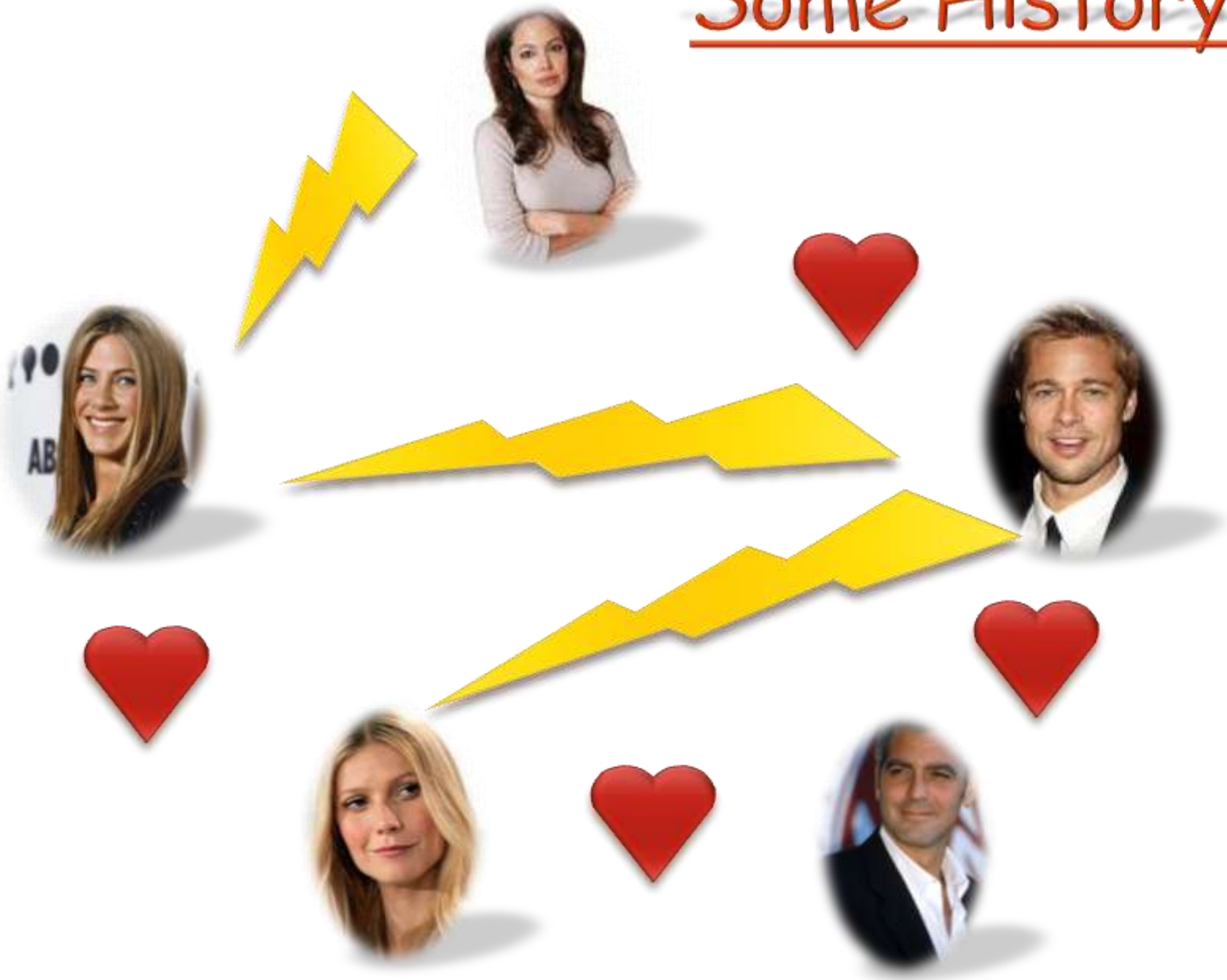
- Meet Celebrities and Computations
- Growth Rate and Tractability
- Reducibility
- … etc. …

Drama At the Oscars

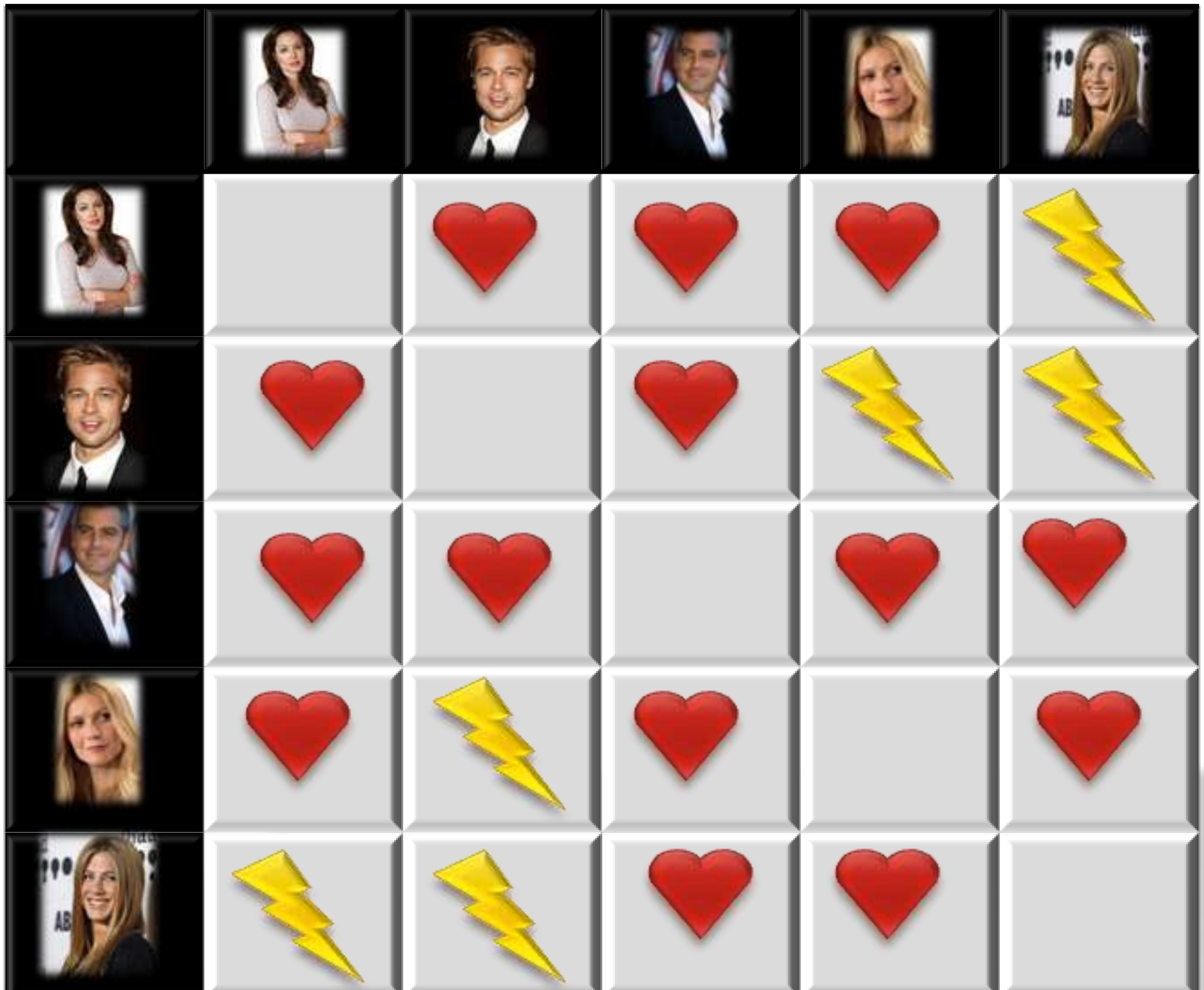
Problem:

- seat all guests around a table, so people who sit next to each other <u>get along</u>.

Some History…



How Can a Catastrophe be Avoided?



Getting It Right

Naive Algorithm

Observation:

- Given a seating one can efficiently check if all guests get along with their neighbors

```
For each seating arrangement:
Check if all guests are OK with neighbors
Stop if a good arrangement is found
```

How much time would it take? (worse case)

Naive Algorithm

```
For each seating arrangement:
Check if all guests are OK with neighbors
Stop if a good arrangement is found
```

How much time would it take? (worse case)

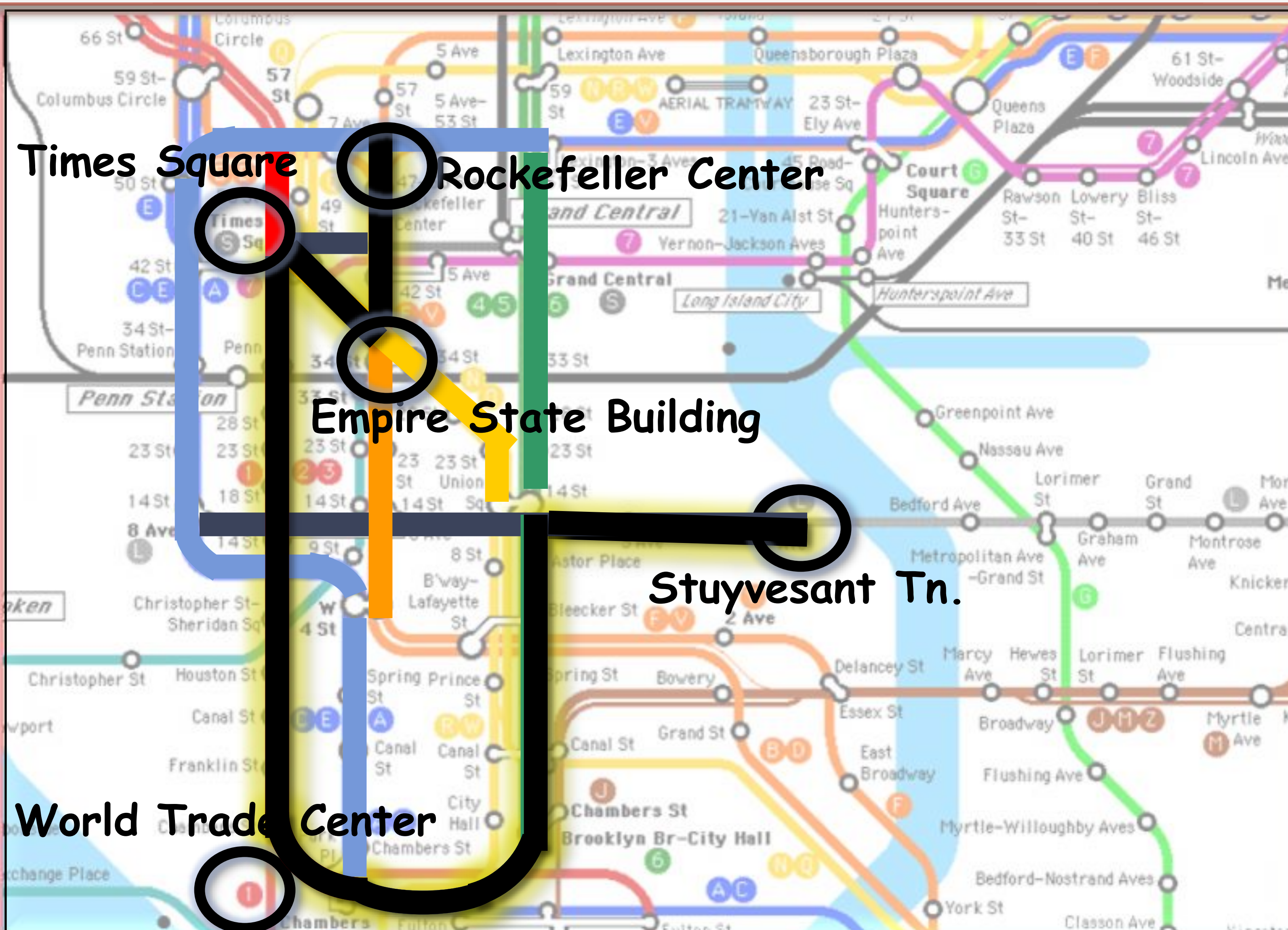
Guests	Steps
N	(N-1)!
5	24
15	87178291[illegible]0
100	$\approx 9 \cdot 10^{155}$

say our computer is capable of 10^{10} instructions per second, this will still take $\approx 3 \cdot 10^{138}$ years!

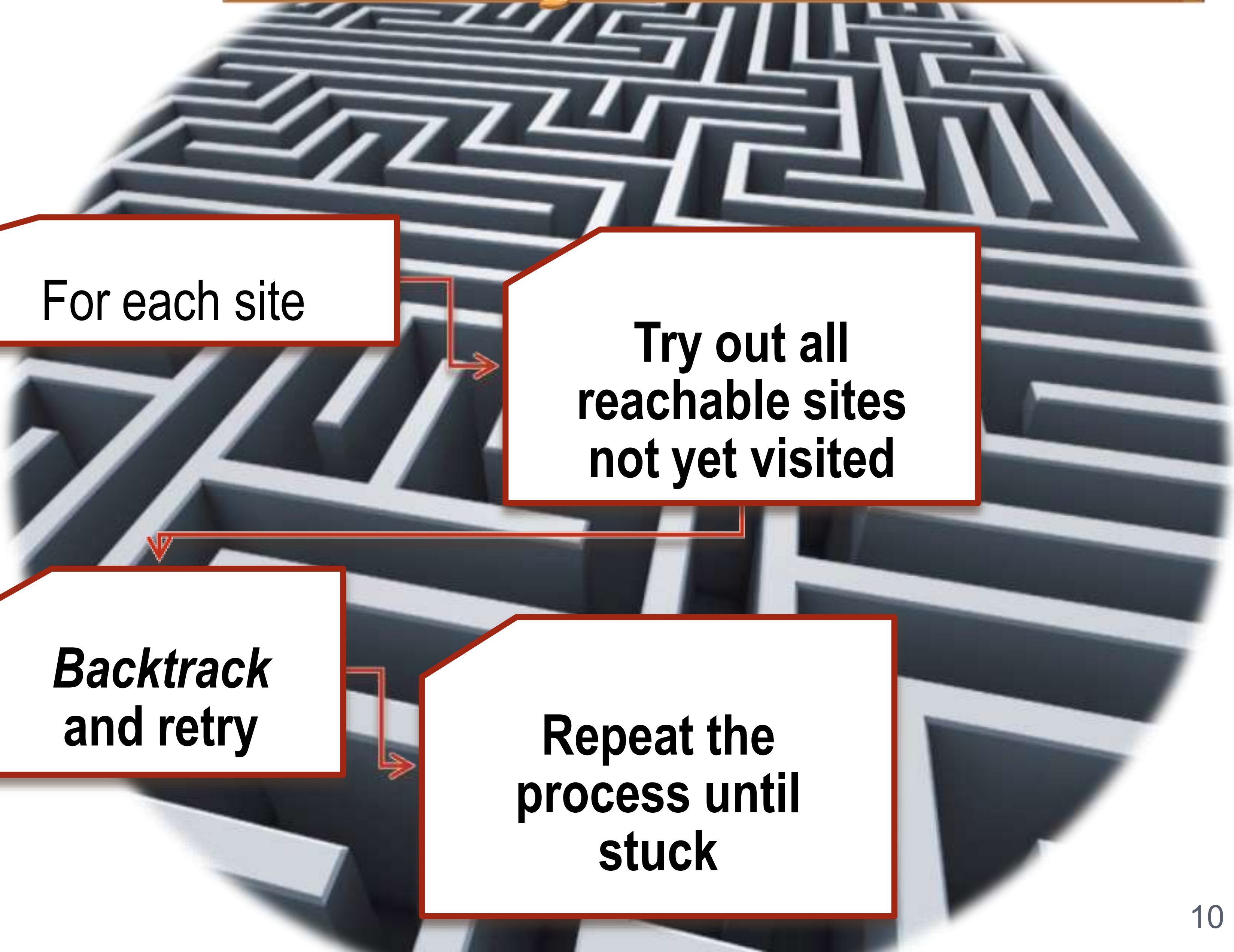
Can you do better?

Tour Problem

- Plan a trip that visits every location exactly once.



Naive Algorithm (Backtracking)



For each site

Try out all reachable sites not yet visited

***Backtrack* and retry**

Repeat the process until stuck

How Much Time?

Sites	Steps
N	N!
5	120
15	1307674368000
100	$\approx 9 \cdot 10^{157}$

On a computer that can check 10,000 options per second, this will still take 4 years!

Is a Problem Tractable?

Yes

- and here's an efficient algorithm that solves it

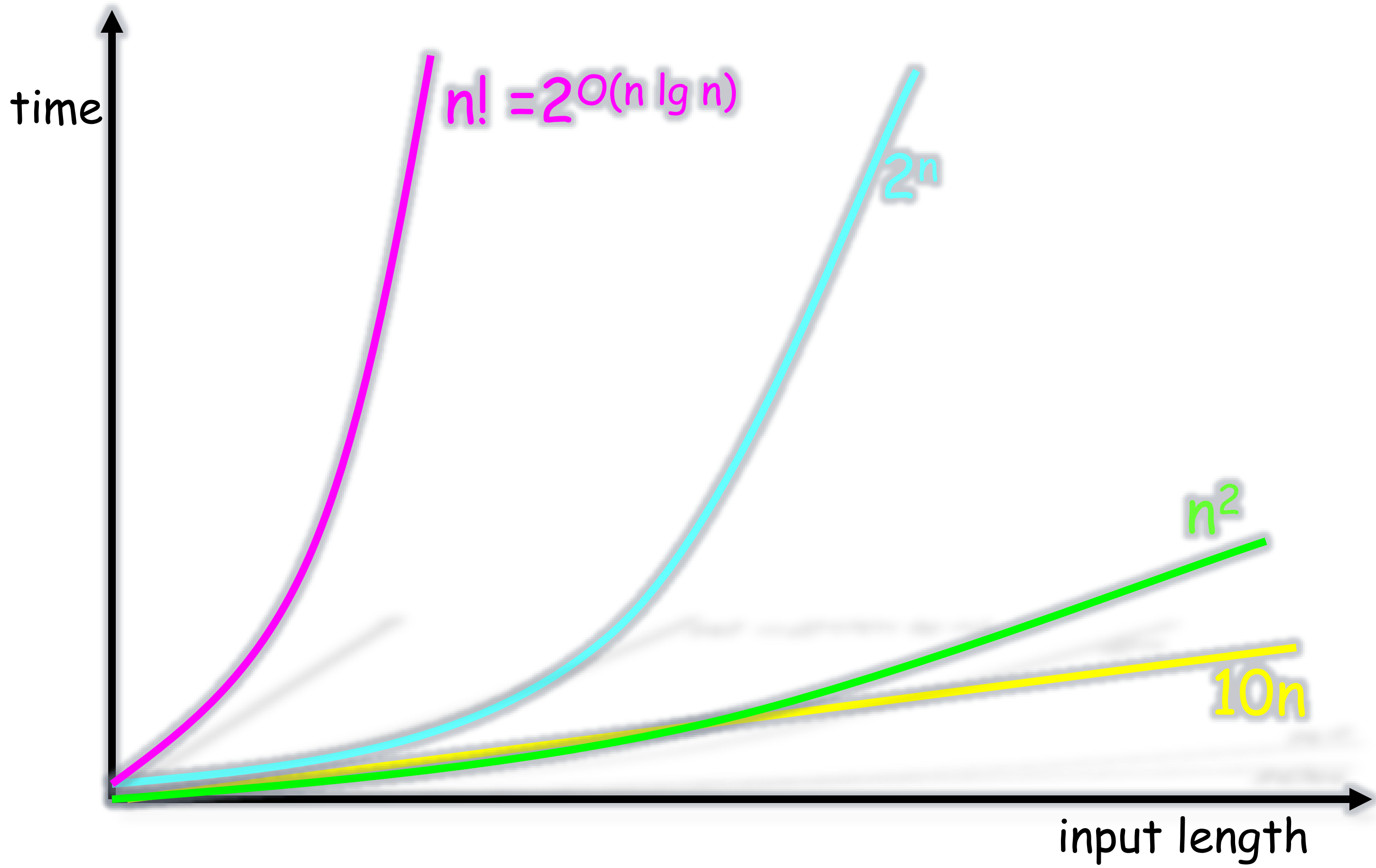
No

- and I can *prove* it

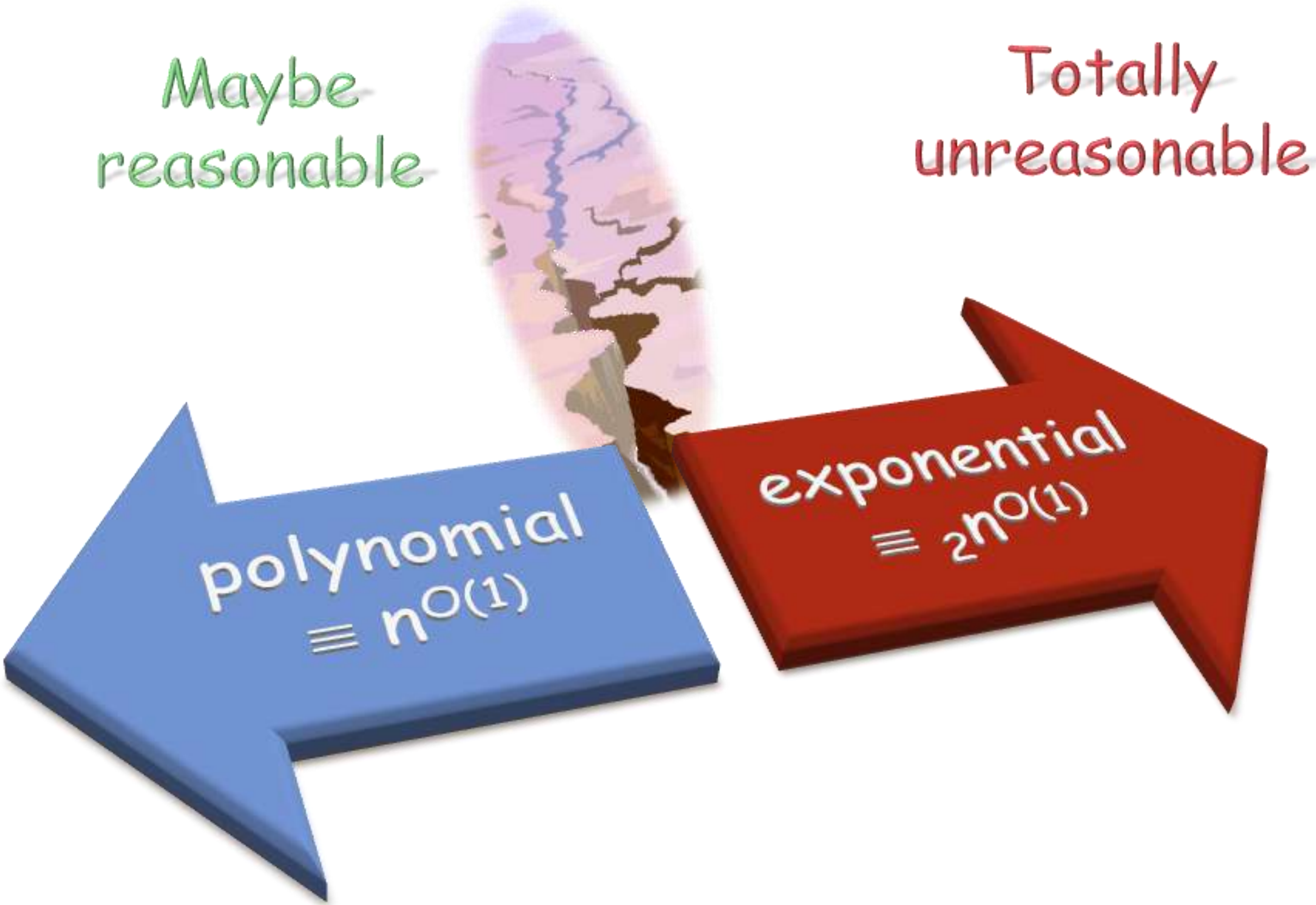
?

- and if neither is the case?

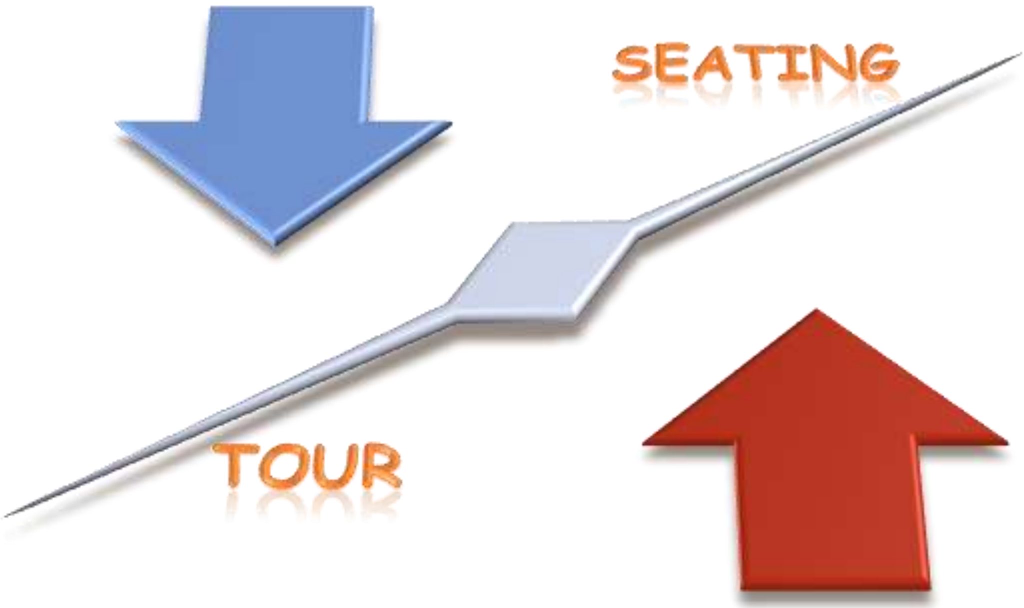
Growth Rate: rough classification



Basic split in time-complexity



Which is Harder?



Relations Between Problems

If

- assuming an efficient procedure for *B* there is an efficient procedure for *A*

an efficient procedure for *A*
using

an efficient procedure for *B*

Then

- *A* cannot be radically harder than *B*

Reductions

an efficient procedure for *A* using

an efficient procedure for *B*

A cannot be radically harder than *B*

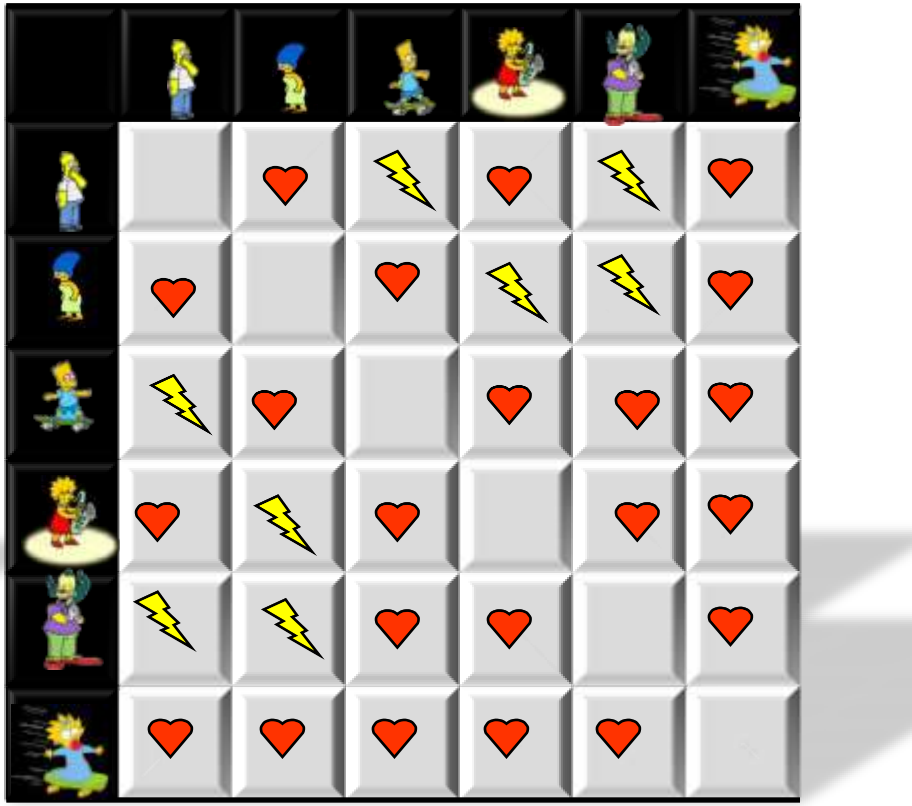
- In other words:

B is at least as hard as *A*

Notation

A ≤p B

Reduce Tour to Seating



Find someone who can seat next to everyone

Reduce Tour to Seating

Completeness:

- If there's a tour, there's a way to seat all the guests around the table.

Soundness:

- If there's a seating, we can easily find a tour path (no tour, no seating).

QED

- *seating is at least as hard as tour*

So Far

COULD NOT:

- find an *efficient algorithm* for problems

NOR

- prove they are *intractable*

DID MANAGE:

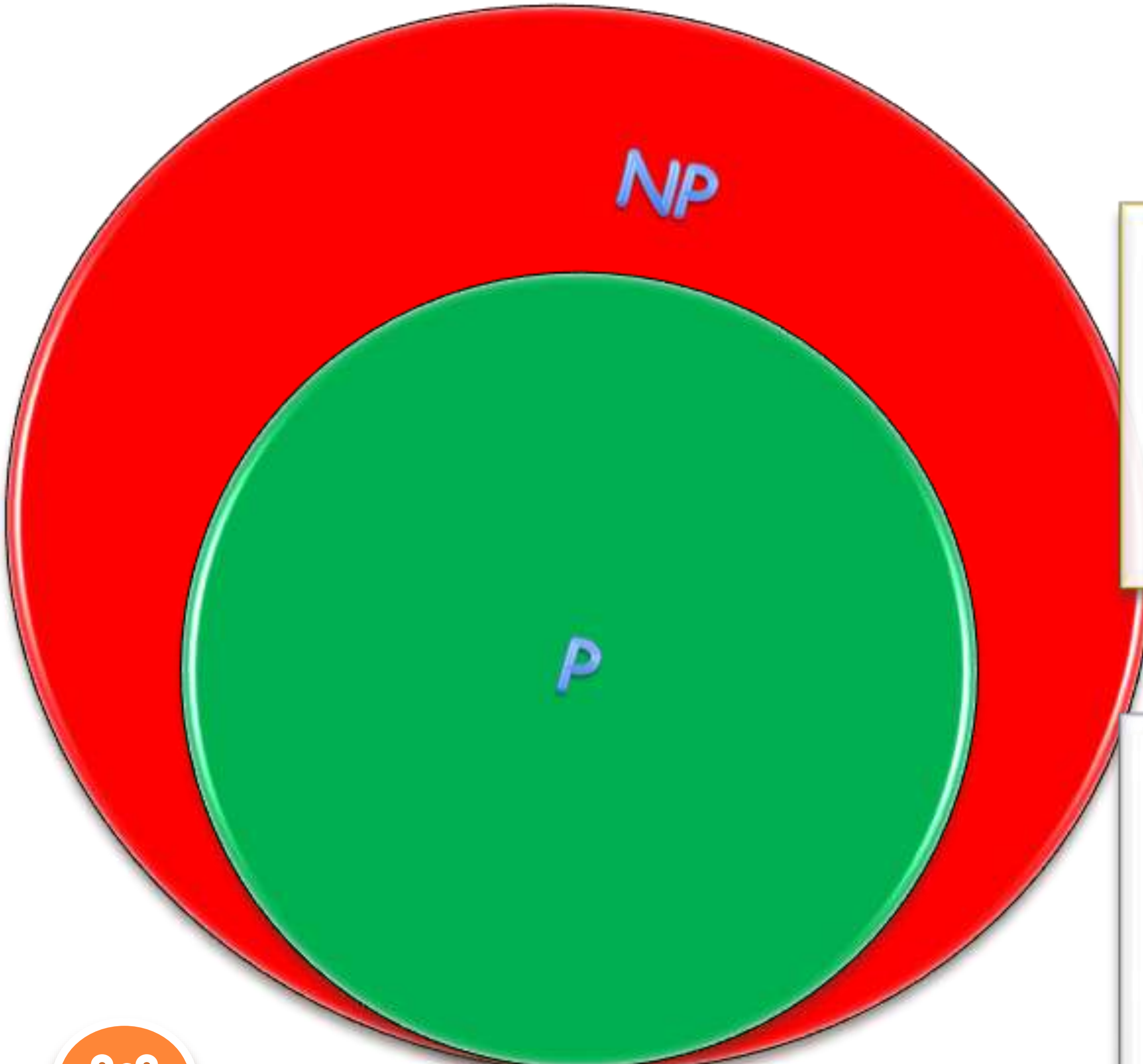
- to show a very *strong correlation* between *their complexity*

Interestingly,
we can also reduce the **seating** problem to the **tour** problem.

Can you?

Furthermore,
there is a whole class of problems, which can be **pair-wise efficiently reduced** to each other.

NP and P



P

- Efficiently computable

NP

- Solution efficiently verifiable

- Is P=NP?

NPC

Contains thousands of distinct problems

each reducible to all others

exponential-time algorithms

efficient algorithms

How can Complexity make you a Millionaire?

The "P vs. NP" question is the **most fundamental of CS**

Resolving it would bring you great honor…

… as well as significant fortune… www.claymath.org/

Philosophically: if P=NP

- Human ingenuity is redundant!
- So would mathematicians be!!

Is nature nondeterministic?

What's Ahead?

Next:

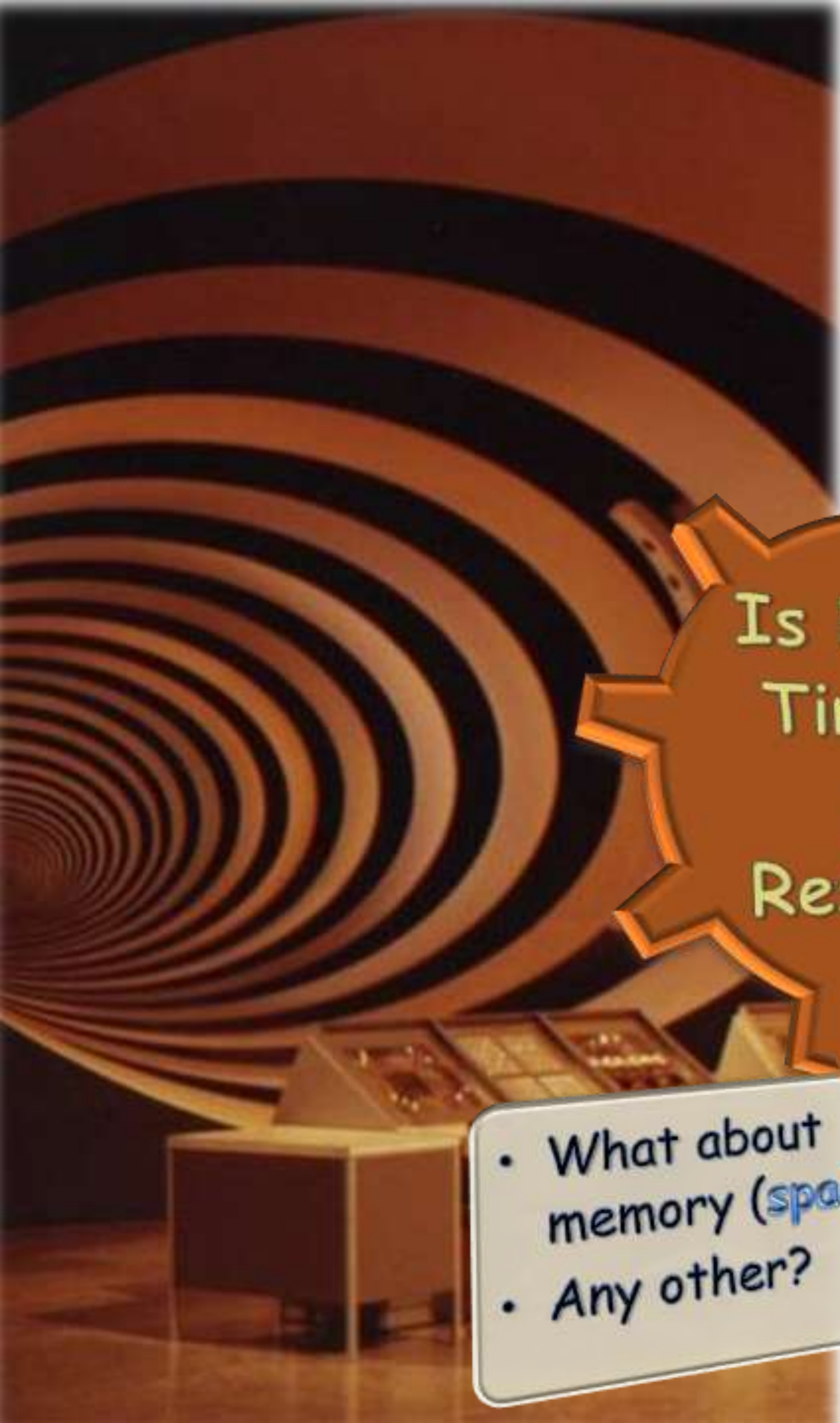
- we'll review basic questions explored through the course.

Generalized Tour Problem

- Each segment of the tour problem now has a **cost**
- find a **least-costly** tour

Approximate **the optimal tour?**

i.e. – find a tour that costs, say, no more than twice as much as the least costly.

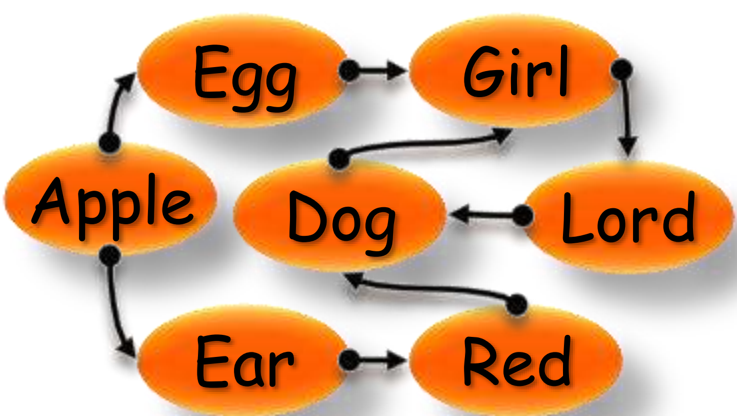


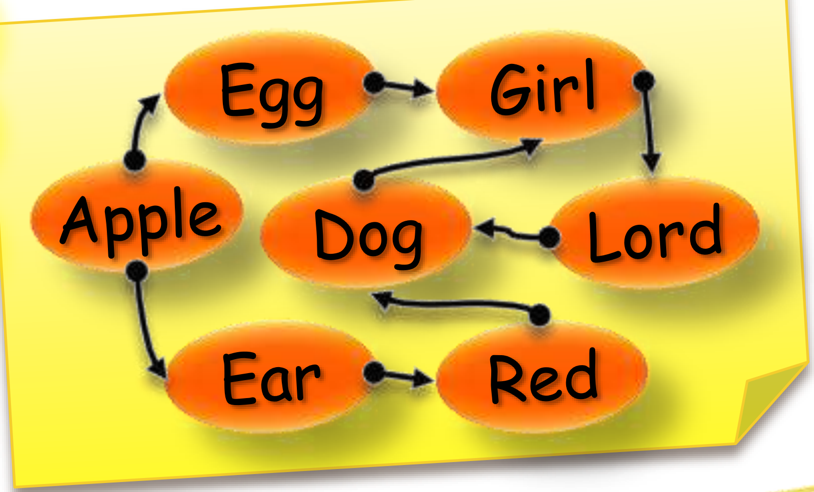
Is Running Time the only Resource?

- What about memory (space)?
- Any other?

Word Games:

Players take turns choose a word whose first letter matches other player's last

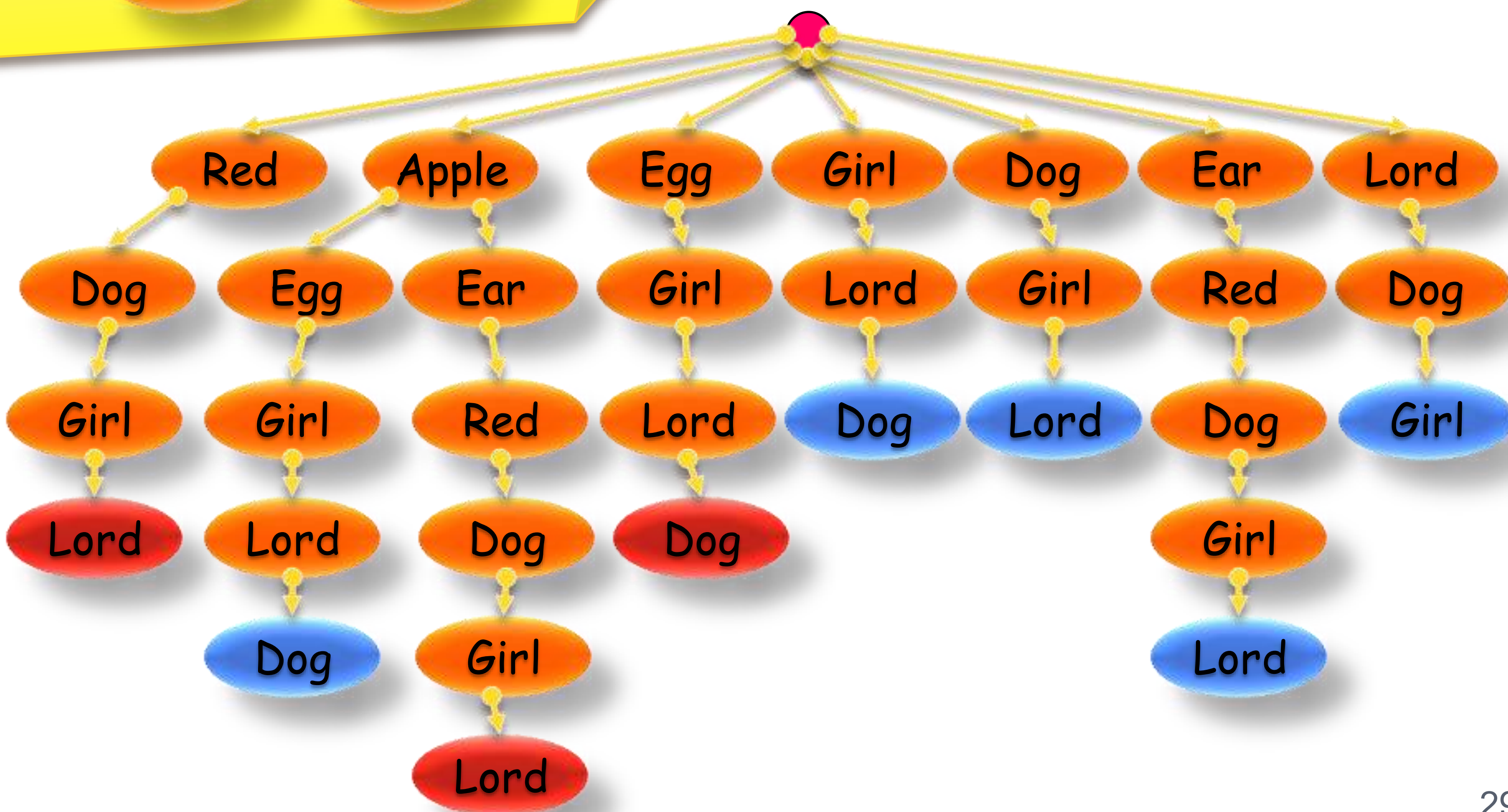




Can one compute a winning strategy?

How much time would it take?

How much space?



Summary

We have introduced two problems:

1. **Seating** ≡ HAMILTONIAN-CYCLE
2. **Tour** ≡ HAMILTONIAN-PATH

Unable to settle their **complexity** we, nevertheless, showed strong **correlations** between them

These problems are representatives of a large **class** of problems: NPC

Prognosis

Topics to be studied later:

- Approximation
- Space-bounded computations

WWindex

Complexity Theory	Computations	Completeness
Hamiltonian Path	Growth Rate	Completeness
Reducibility		Soundness
Complexity Classes	P	NP
NPC		
Exponential Time	www.claymath.org	Approximation