Exercise No. 9: Scattering of Light

1. A right circularly polarized beam of light with wavelength $\lambda = 7 \times 10^{-5}$cm and intensity of 1W/cm^2 illuminates a (low density) gas of 10^{23} electrons. Assume that the velocity of the electrons is negligibly small. Calculate the frequency and intensity of left circularly polarized light at angles $0°$, $90°$ and $180°$ with respect to the direction of the incident beam.

2. A hydrogen atom is in its ground state in a large box. Light of wave-vector $\mathbf{k}$ falls on the atom. Write down an explicit expression in terms of dipole matrix elements for the Raman scattering cross-section for the transition to a d state (ignore the recoil of the atom). Estimate the differential cross-section for the transition to a $3d$ state whose angular momentum projection on the z-direction is $+2$ for incident photons of an energy near $\frac{15}{16} \times 13.6$eV.
   You may use the following formula for the addition of spherical harmonics

$$Y_{l_1 m_1}(\Omega) Y_{l_2 m_2}(\Omega) = \sum_{l=|l_1-l_2|}^{l_1+l_2} \sum_{m=-l}^{l} \sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)}} \langle l_1, l_2; 0, 0 | l, 0 \rangle \times \\ \times \langle l_1, l_2; m_1, m_2 | l, m \rangle Y_{lm}(\Omega) \ ,$$

   where $\langle l_1, l_2; m_1, m_2 | l, m \rangle$ are the Clebsch-Gordan coefficients.

3. In scattering x-rays from a solid one does not usually analyze the frequencies of the scattered rays (why?), but rather measures the total scattered intensity as a function of scattering angle.

   (a) Show that the differential scattering cross-section for the frequency integrated intensity is given to within small corrections by

$$\frac{d\sigma}{d\Omega} = r_0^2 \frac{1 + \cos^2\theta}{2} S(\mathbf{k}_f - \mathbf{k}_i) \ ,$$

   where $\mathbf{k}_i$ is the wave-vector of the incident x-rays, $\mathbf{k}_f$ points in the direction of the detector from the solid, its magnitude equals k_i, θ is the angle between $\mathbf{k}_f$ and $\mathbf{k}_i$ and

$$S(\mathbf{q}) = \sum_f |\langle f | \rho_{-\mathbf{q}} | 0 \rangle|^2 \ ,$$

where ρ is the electron density operator. Assume the solid to be in its ground state initially.

(b) The term in $S(\mathbf{q})$ where $|f\rangle = |0\rangle$ gives the elastic scattering. Show that for a periodic electron distribution in the state $|0\rangle$ this term gives rise to peaks in the scattering cross-section in well-defined directions (Laue spots). From these one can determine the crystal structure of the solid.