

## Exercise No. 8: Electromagnetic Interactions

1. (a) Prove that momentum is conserved in processes of photon emission and absorption in perturbation theory.  
(b) Show that a free electron cannot emit a single photon.
2. A hydrogen atom in its  $2p$  state is at rest, i.e., has zero momentum, in a large box. Calculate the details of the spontaneous emission from the atom. In your calculation, devise the appropriate form of the dipole approximation for this situation. Compute the life-time of the  $2p$  state.
3. A particle of mass  $m$  and charge  $e$ , whose unperturbed Hamiltonian is

$$H_0 = \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2 z^2 ,$$

is coupled the electromagnetic field.

- (a) Derive the selection rules for electric dipole transitions and state the possible energies and polarizations of the emitted or absorbed photon.
  - (b) What is the cross-section for such a particle, initially in its ground state, to absorb a single photon of energy  $n\hbar\omega$  and any polarization at an angle  $\theta$ ?
4. The quantized electromagnetic field is coupled to a classical current  $\mathbf{j}(\mathbf{r}, t)$  with no net charge. At  $t = 0$  the field is in the vacuum state. What is the state of the field at a later time  $t$ ? Compute the probability of finding  $n$  photons with a wave-vector  $\mathbf{k}$  and polarization  $\lambda$ .