

Exercise No. 13: The BCS State

1. (a) Prove that if $|G\rangle$ is the Bogoliubov ground state, the true vacuum of the system is given by

$$|0\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} \alpha_{-\mathbf{k}}^{\dagger} \alpha_{\mathbf{k}}^{\dagger} \right) |G\rangle .$$

- (b) Show that the state $|G\rangle$ is normalized.
2. Explicitly perform the Bogoliubov transformation and normal-ordering in order to obtain $K = H - \mu \hat{N}$ neglecting normal-ordered four-operator interaction terms.
3. Compute the condensation energy, i.e. the energy difference between the BCS and the normal ground states, neglecting terms of order G .