Exercise No. 14: The Dirac Equation

1. Show the following relations where $\gamma^0 = \beta$ and $\gamma^i = \beta\alpha_i$:

   (a) $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$,

   (b) $\gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu$,

   (c) $\gamma^\mu\gamma^\nu\gamma^\lambda\gamma_\mu = 4g^{\nu\lambda}$,

   (d) $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$,

   (e) $(a_\mu\gamma^\mu)(b_\nu\gamma^\nu) = a_\mu b^\mu - i\sigma^{\mu\nu}a_\mu b_\nu$, where a_μ and b_μ are four-vectors.

2. Find the transformation properties of the objects bellow under Lorentz transformations:

   (a) $\bar{\psi}\psi$,

   (b) $\bar{\psi}\gamma_5\psi$,

   (c) $\bar{\psi}\gamma^\mu\gamma_5\psi$,

   (d) $\bar{\psi}\sigma^{\mu\nu}\psi$.

3. Write the Maxwell equations in the absence of charges and currents in Dirac form in terms of a six-component field amplitude. What are the matrices corresponding to α and β?