

Exercise No. 1: Classical Field Theory

1. Using the Lagrangian of the electromagnetic field

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} ,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and taking $c = 1$,

- (a) Compute the energy density in the absence of any charges or currents in terms of \mathbf{E} and \mathbf{B} . Bring the expression to the known form.

Hint: remember that the Hamiltonian density can be shifted by a total divergence.

- (b) The Lagrangian in the presence of charges and currents is

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + j_\mu A^\mu .$$

Find the equations of motion in the relativistic notation and then write them in terms of \mathbf{E} and \mathbf{B} .

2. Find the energy spectrum and the eigenstates of a non-relativistic spinless electron in a constant magnetic field.
3. A spin- $\frac{1}{2}$ particle is placed in a rotating magnetic field

$$\mathbf{B} = B_1 (\hat{\mathbf{x}} \cos \omega t + \hat{\mathbf{y}} \sin \omega t) + B_0 \hat{\mathbf{z}} .$$

- (a) What are the energy levels and the eigenstates of the system at $t = 0$?
- (b) Calculate the exact time evolution of a spin pointing initially up in the z -direction.