

**QI2006 EX. 6**

1) Find an explicit unitary realization for a POVM that has the effect of concentrating the entanglement of a single copy of non-maximal state to a maximal entangled state. Show that the final average entanglement-entropy is reduced by this process.

2) Calculate the full entropy, the reduced entropy, the conditional entropy and mutual information for a non-maximal entangled pure state  $\alpha|00\rangle + \beta|11\rangle$ , and for the Werner (mixed) state  $\frac{1-x}{4}1 + x|EPR\rangle\langle EPR|$ . Plot the Venn diagram.

3) A pure state that is shared by 3 parties

$$|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}} \left( |0_A, 0_B, 0_C\rangle + |1_A, 1_B, 1_C\rangle \right)$$

or

$$|W\rangle_{ABC} = \frac{1}{\sqrt{3}} \left( |0_A, 0_B, 1_C\rangle + |0_A, 1_B, 0_C\rangle + |1_A, 0_B, 0_C\rangle \right)$$

3.1 What is the bi-partite entanglement entropy between A and B,C.

3.2 If the state of C is not available to A and B, is the reduced density matrix  $\rho_{AB}$  entangled?

3.3 C performs a projective measurement on his qubit and sends the result to A and B: 3.4 Can he destroy the entanglement between A and B?

3.5 By choosing an appropriate measurement can he help A and B to get a maximally entangled.

3.6 What if C performs the measurements above and does not send his results to A and B?

4) Show that entanglement-Entropy measure is additive:  $E(|\psi_{A_1, B_1}\rangle \otimes |\phi_{A_2, B_2}\rangle) = E(|\psi_{A_1, B_1}\rangle) + E(|\phi_{A_2, B_2}\rangle)$ .

5) Explain why the entanglement-entropy cannot be a good measure for the entanglement of mixed states.

6) It is possible to define an observable  $W$  that detects certain types of entanglement. An operator  $W$  acting on  $\mathcal{H}_{AB}$  is called an entanglement witness if it is a non-positive operator (some eigenvalues are negative) but for every non-entangled states  $W$  is positive  $\Psi_{AB} = \psi_A \otimes \phi_B$ ,  $\langle \Psi_{AB} | W | \Psi_{AB} \rangle \geq 0$ .

6.1 Prove that if a mixed state  $\rho_{AB}$  is separable then  $\text{Tr}(W\rho) \geq 0$  for all entanglement witnesses. This is a necessary condition for separability.

The usefulness of this result is that give  $\rho_{AB}$ , if we can find an entanglement witness such that  $\text{Tr}(W\rho) < 0$  then we automatically know that the state is entangled.

6.2 For two qubits show that  $W = 1 - 2|\phi^+\rangle\langle\phi^+|$  is an entanglement witness.