1. Check that the CHSH inequality is not violated by the correlation function for the LHV model discussed at class.

2. The state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0,0,0\rangle - |1,1,1\rangle)$$

(known in the literature as the "GHZ state"), can be used to formulate a condition on LHV "without inequalities" as follows. Let this state be prepared and shared between three spatially separated laboratories A, B and C ($|000\rangle$ stands for $|0\rangle_A \otimes |0\rangle_B \otimes |0\rangle_C$ etc.. We suppressed above the trivial spatial part of the wavefunction, assuming that each qubit is localized in a separate laboratory.)

Now define the operators:

$$O_1 = \sigma_x^A \sigma_y^B \sigma_y^C \quad O_2 = \sigma_y^A \sigma_x^B \sigma_y^C \quad O_3 = \sigma_y^A \sigma_y^B \sigma_x^C \quad O_4 = \sigma_y^A \sigma_y^B \sigma_y^C$$

2.1. Prove that

$$O_i |\Psi\rangle = s_i |\Psi\rangle$$

with $s_1 = s_2 = s_3 = 1$ and $s_4 = -1$.

2.2. Prove the operator identity

$$O_1 O_2 O_3 = -O_4$$

2.3. The minus sign on the right hand side of 2.2 turns out to be inconsistent with LHV. Let’s assume that $\sigma_x$ and $\sigma_y$ take a well defined value prior to the measurement at each lab, say the value of $\sigma_x$, $V(\sigma_x) = \pm 1$. Since $\Psi$ is an eigenstate of $O_1$ we can conclude (using 2.1) that $V(O_1) = V(\sigma_x^A) V(\sigma_y^B) V(\sigma_y^C) = 1$, and find 3 more such relations. Show however that this is inconsistent with the identity 2.2 above.

3. In the process of a quantum teleportation, Alice sends an unknown state to Bob. Show that :

3.1 The probabilities for the 4 results of the Bell-basis measurement are equal.

3.2 The reduced density matrix of Bob’s qubit after Alice performed the Bell measurement, but before he learns the outcome, is given by a unit matrix.

3.3 It is possible to distinguish between the Bell states by performing first a CNOT gate between the two qubits and then measuring the spin of each qubit separately.

4. Find the Schmidt decomposition of the state

$$\frac{1}{\sqrt{2}} \left( |00\rangle + \frac{1}{\sqrt{2}} (|0\rangle(|0\rangle + |1\rangle) \right)$$