

FIXED EXCHANGE RATES
and Foreign Exchange Intervention

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Central Bank Balance Sheet

Assets

- (1) Foreign Assets
- (2) Domestic Assets

Liabilities

- (1) Deposits held by
Private Banks
- (2) Currency in circulation

H = Base Money

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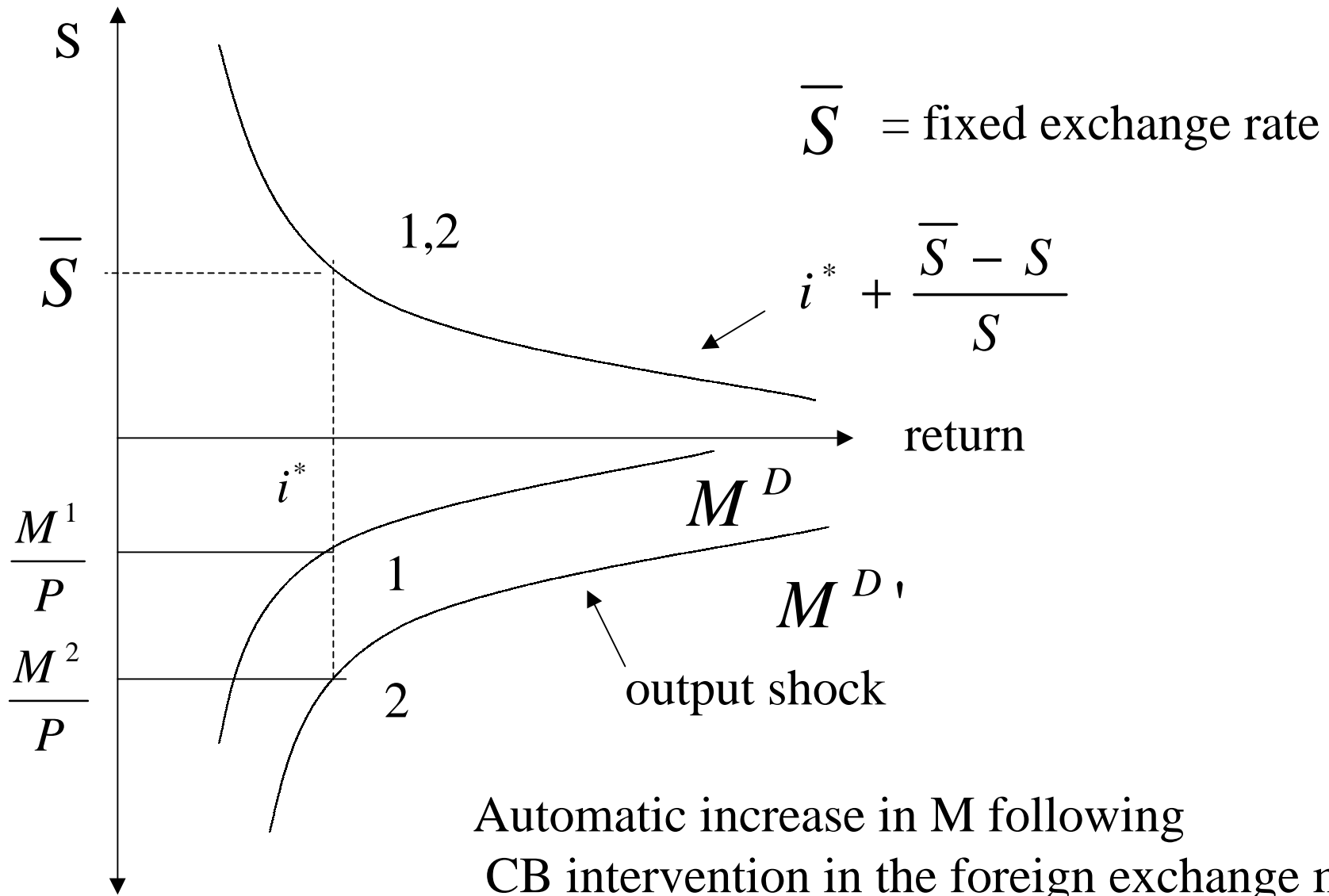
Foreign Assets Sale —————>

Base Money contracts

Foreign Assets Purchase —————>

Base Money Expands

Fixed Exchange Rate



Result: Base Money is endogenous

The Sustainability of Fixed Exchange Rate Regime

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$$\frac{M}{P} = L(i) = e^{-\eta i} \quad \text{demand for money}$$

$$\log \frac{M}{P} = -\eta i$$

$$(1) \quad m_t - p_t = -\eta i_t$$

$$i_t = i^* + \frac{dS_t}{S_t dt} = i^* + \frac{d \log S_t}{dt} \cong i^* + \Delta s_t \quad \text{interest parity}$$

$$\Delta s_t = s_{t+1} - s_t$$

$$(2) \quad i_t = i_t^* + (s_{t+1} - s_t)$$

Purchasing Power Parity

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$$P_t = S_t P_t^*$$

$$\log P_t = \log S_t + \log P_t^*$$

$$(3) \quad p_t = s_t + p_t^*$$

Substitute (2) & (3) into (1):

$$(4) \quad m_t = s_t + p_t^* - \eta(s_{t+1} - s_t) - \eta i_t^*$$

$$P_t^* = P^*, \quad i_t^* = i^*$$

$$m_t = \text{constant} + s_t - \eta(s_{t+1} - s_t)$$

Fixed Exchange Rate

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$$m_t = \bar{s} + p_t^* \quad \text{money supply is totally endogenous}$$

$$\bar{m} = \bar{s} + p^* \quad \text{if } P_t^* = P^*$$

A Simple Model (Krugman 1979) $P^* = i^* = 0$

$$m_t = \bar{m} = \bar{s} \quad \text{fixed exchange rate}$$

$$m_t - s_t = -\eta (s_{t+1} - s_t) = -\eta\mu, \quad \text{flexible exchange rate}$$

$$\text{if } s_{t+1} - s_t = \mu$$

International Reserves

$$B_{H,t} + \bar{S} B_{F,t} = \bar{M}$$

$$B_{H,t+1} + \bar{S} B_{F,t+1} = \bar{M}$$

$$B_{H,t+1} - B_{H,t} = -\bar{S} [B_{F,t+1} - B_{F,t}]$$

$$B_{F,t+1} - B_{F,t} = -\frac{1}{\bar{S}} [B_{H,t+1} - B_{H,t}]$$

$$B_{F,t+1} - B_{F,t} = -\frac{1}{\bar{S}} \mu B_{H,t}$$

Central Bank Balance Sheet

$$M_t = B_{H,t} + \bar{S} B_{F,t}$$

Domestic Credit Expands Indefinitely

$$\frac{B_{H,t+1} - B_{H,t}}{B_{H,t}} = \mu \quad \text{rate of expansion}$$

$$b_{H,t+1} - b_{H,t} \cong \mu$$

“Shadow” Exchange Rate

$$\tilde{s}_t = b_{H,t} + \eta\mu$$

Logarithmic Approximation

$$B_{H,t+1} = (1 + \mu)B_{H,t}$$

$$b_{H,t+1} = \log(1 + \mu) + b_{H,t}$$

$$f(x) = f'(x^0)(x - x^0)$$

$$\log(1 + \mu) = 1(1 + \mu - 1) = \mu$$

$$x = 1 + \mu, \quad x^0 = 1$$

$$b_{H,t+1} = b_{H,t} + \mu$$

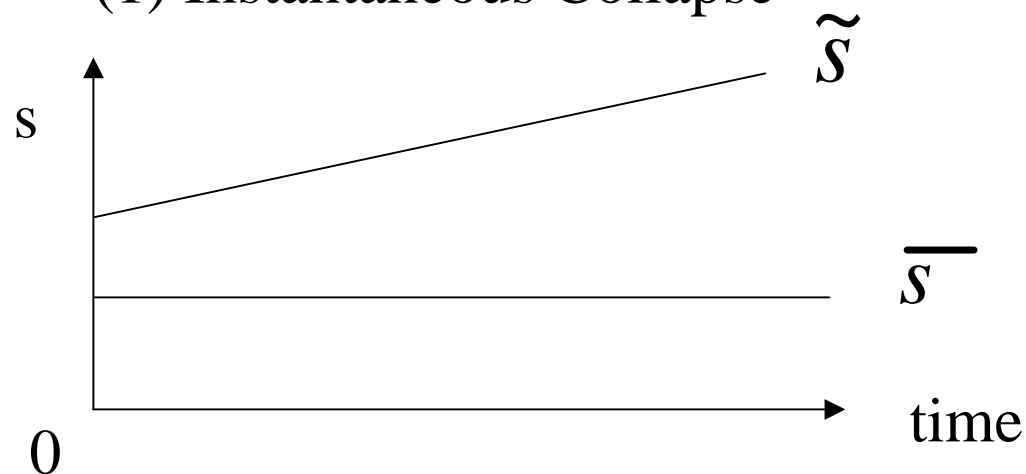
The “Shadow” exchange rate is:

a market-based exchange rate when the central bank has no international reserves:

$$\tilde{s}_t = b_{H,t} + \eta\mu$$

Implications:

(1) Instantaneous Collapse

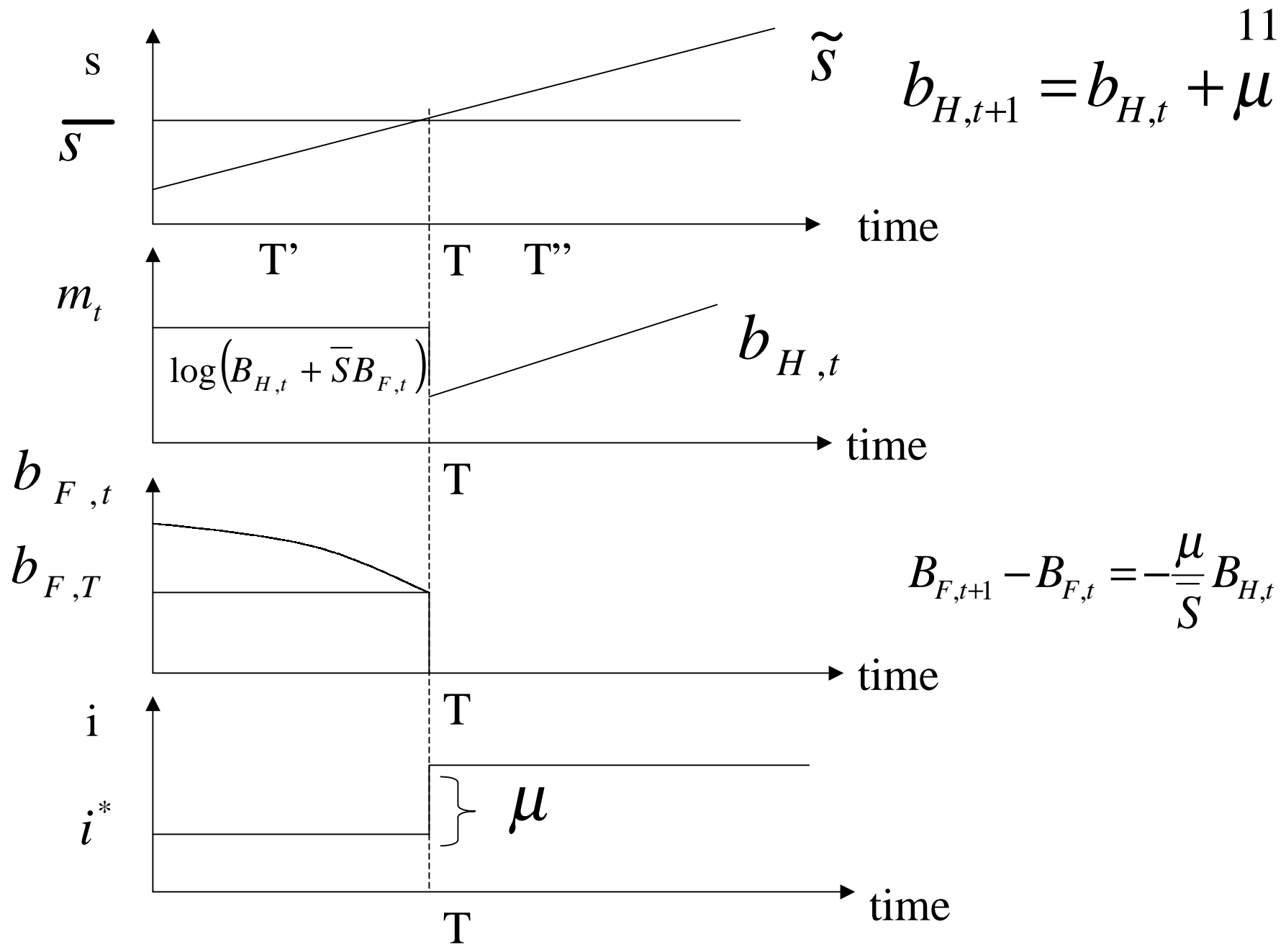


(2) Calculations:

$$\tilde{s}_T = \bar{s} \Rightarrow b_{H,T} + \eta\mu = \bar{s} \Rightarrow b_{H,T}$$

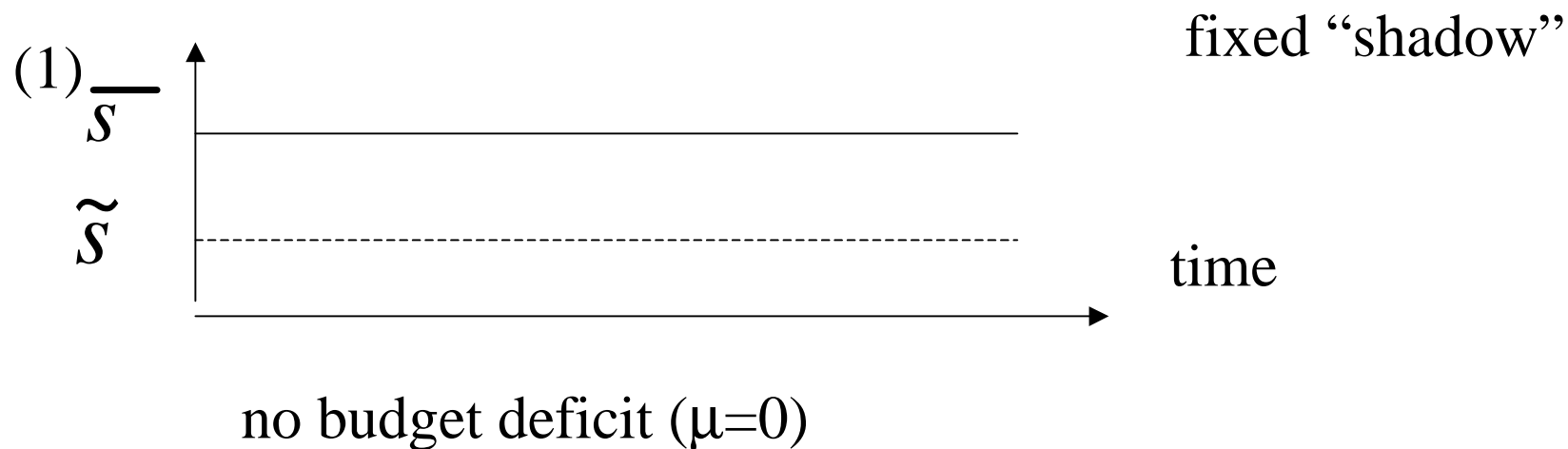
$$b_{H,t+1} = b_{H,t} + \mu \Rightarrow$$

$$b_{H,T} = b_{H,T-1} + \mu = b_{H,T-2} + 2\mu = T\mu b_{H,0} \Rightarrow T$$



Sustainability of Fixed Exchange Rate

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(2) imperfect asset substitutability

(a) regulating capital inflows

(b) risk premium

$$i_t = i_t^* + \frac{ES_{t+1} - S_t}{S_t} + \rho$$

ρ is a function of external debt

if ρ is a function of external debt (B) minus domestic assets (A)
a sterilized intervention which keeps M constant switches reserves
(negative external debt) for domestic assets would change the risk
premium, and change domestic interest rate. Sales of reserves
accompanied by purchase of domestic bonds will raise ρ and i .