Monetary Policy, Inflation, and the Business Cycle

Chapter 6

A Model with Sticky Wages and Prices

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August 2007

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Throughout the previous chapters we have modelled the labor market as a perfectly competitive market, in which households and firms take the wage as given. In the present chapter we depart from that assumption by introducing some imperfections in the labor market and analyzing their consequences for monetary policy. In particular we assume that workers have some monopoly power, which allows them to set the wage for the differentiated labor services they supply. Furthermore, as we did with price-setting firms in chapter 3, we assume here that workers face Calvo-type constraints on the frequency with which they can adjust wages.

A key result emerges from the analysis of the model with sticky wages and prices: fully stabilizing price inflation is no longer optimal. Instead the central bank should be concerned about both price and wage stability, since fluctuations in both price and wage inflation, as well as in the output gap, are a source of inefficiencies in the allocation of resources that result in welfare losses for households. Accordingly, the optimal policy seeks to strike a balance between three different objectives, with the relative weights attached to them being a function of the underlying parameter values.

The chapter is organized as follows. In Section 1 we describe a benchmark model in which both sticky wages and sticky prices coexist. In Section 2 we derive the model's log-linearized equilibrium conditions. Section 3 discusses the relevant central bank's objective function. Section 4 derives and characterizes the optimal monetary policy, while Section 5 studies the performance of alternative simple rules, and their merits as an approximation to the optimal policy. Section 6 concludes with some bibliographical notes.

1 A Model with Staggered Wage and Price Setting

In the present section we lay out a model of an economy in which nominal wages, as well as prices, are sticky. Following Erceg, Henderson, and Levin (2000), wage stickiness is introduced in a way analogous to price stickiness, as modelled in chapter 3. In particular, we assume a continuum of differentiated labor services, all of which are used by each firm. Each household is specialized in one type of labor, which it supplies monopolistically.[1] Each

[1] Equivalently, one can think of a continuum of unions, each of which represents a set of households/workers specialized in a given labor service, and sets the wage on their behalf.

period only a (constant) fraction of household/labor types, drawn randomly from the population, can adjust their posted nominal wage. As a result, the aggregate nominal wage responds sluggishly to shocks, generating inefficient variations in the wage markup. In addition, wage inflation, combined with the staggering of wage adjustments, brings about relative wage distortions and an inefficient allocation of labor, in a way symmetric to the relative price distortions generated by price inflation in the presence of staggered price-setting.

Next we describe the problem facing firms and households in this environment.

1.1 Firms

As in chapter 3, we assume a continuum of firms, indexed by $i \in [0, 1]$, each of which produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t \; N_t(i)^{1-\alpha} \tag{1}$$

where $Y_t(i)$ denotes the output of good i, A_t is an exogenous technology parameter common to all firms, and $N_t(i)$ is an index of labor input used by firm i and defined by

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} \; dj \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{2}$$

where $N_t(i,j)$ denotes the quantity of type-j labor employed by firm i in period t. Note that parameter ϵ_w represents the elasticity of substitution among labor varieties. Note also that we assume a continuum of labor types, indexed by $j \in [0, 1]$.

Let $W_t(j)$ denote the nominal wage for type-j labor effective in period t, for all $j \in [0, 1]$. As discussed below, wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm i and labor type j, given the firm's total employment $N_t(i)$:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} \; N_t(i) \tag{3}$$

for all $i, j \in [0,1]$, where

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} \, dj \right]^{\frac{1}{1-\epsilon_w}} \tag{4}$$

is an aggregate wage index. Substituting (3) into the definition of $N_t(i)$, one can obtain the convenient aggregation result

$$\int_0^1 W_t(j) N_t(i,j) \, dj = W_t \; N_t(i)$$

i.e. the wage bill of any given firm can be expressed as the product of the wage index, W_t , and that firm's employment index, $N_t(i)$.

Hence, and conditional on an optimal allocation of the wage bill among the different types of labor implied by (3), a firm adjusting its price in period t will solve the following problem, which is identical to the one analyzed in chapter 3:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k \; E_t \left\{ Q_{t,t+k} \; \left(P_t^* \; Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} \; C_{t+k}$$

for $k = 0, 1, 2, ...$where $Q_{t,t+k} \equiv \beta^k \; (C_{t+k}/C_t)^{-\sigma} \; (P_t/P_{t+k})$ is the stochastic discount factor for nominal payoffs, $\Psi_{t+k}(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t. Notice that we have now added a subscript p to parameters θ and ϵ, for symmetry with their labor market counterparts.

As shown in chapter 3, the aggregation of the resulting price setting rules yields, to a first order approximation and in a neighborhood of the zero inflation steady state, the following equation for price inflation π_t^p :

$$\pi_t^p = \beta \; E_t\{\pi_{t+1}^p\} - \lambda_p \; \widehat{\mu}_t^p \tag{5}$$

where $\widehat{\mu}_t^p \equiv \mu_t^p - \mu^p = -\widehat{mc}_t$ and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$. Note that, for the sake of symmetry with the wage inflation equation derived below, we choose to write the inflation equation as a function of the (log) deviation of the average price markup from its desired (or steady state) value, instead of the (log) marginal cost. Hence, and as discussed in chapter 3, the presence (or anticipation) of average price markups below their desired levels leads firms adjusting prices to raise the latter, thus generating positive inflation.

1.2 Households

We assume a continuum of households indexed by $j \in [0,1]$. As in the basic model of chapter 3, a typical household seeks to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \, U(C_t(j), N_t(j)) \right\}$$

subject to a sequence of budget contraints (to be specified below), and where $N_t(j)$ is the quantity of labor supplied, and

$$C_t(j) \equiv \left(\int_0^1 C_t(i,j)^{1-\frac{1}{\epsilon_p}} \, di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{6}$$

is a consumption index analogous to the one used in chapter 3, where $i \in [0,1]$ indexes the type of good. The main difference relative to the baseline model of chapter 3 is that now each household is assumed to specialize in the supply of a different type of labor, also indexed by $j \in [0,1]$. Furthermore, each household has some monopoly power in the labor market, and posts the (nominal) wage at which it is willing to supply specialized labor services to firms that demand them. Alternatively, we can think of many households specializing in the same type of labor (with their joint mass remaining infinitesimal), and delegating their wage decision to a trade union which acts in their interest.

In a way analogous to our assumptions on the price setting constraints facing firms, we assume that each period only a fraction $1 - \theta_w$ of households/unions, drawn randomly from the population, reoptimize their posted nominal wage. Under the assumption of full consumption risk sharing across households, all households/unions resetting their wage in any given period will choose the same wage, since they face an identical problem.[2] Next we

[2] We will be thus assuming the existence of complete set of securities markets, which will guarantee that in equilibrium the marginal utility of consumption is equalized across households at all times (assumining identical initial conditions).

formalize the problem facing households and proceed to solve it.

1.2.1 Optimal Wage Setting

Let us first consider how households choose the wage for their labor type, when allowed to reoptimize that wage. Consider a household resetting its wage in period t, and let W_t^* denote the newly set wage. The household will choose W_t^* in order to maximize

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \; U(C_{t+k|t}, N_{t+k|t}) \right\} \tag{7}$$

where $C_{t+k|t}$ and $N_{t+k|t}$ respectively denote the consumption and labor supply in period $t+k$ of a household that last reset its wage in period t. Thus, expression (7) can be interpreted as the expected discounted sum of utilities generated over the (uncertain) period during which the wage remains unchanged at the level W_t^* set in the current period. Note that the utility generated under any other wage set in the future is irrelevant from the point of view of the optimal setting of the current wage, and can thus be ignored in (7).

Maximization of (7) is subject to the sequence of labor demand schedules and flow budget constraints that are effective while W_t^* remains in place, i.e.

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} \; N_{t+k}$$

$$P_{t+k} \; C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k+1} D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^* \; N_{t+k|t} - T_{t+k}$$

for $k = 0, 1, 2, ...$where $N_{t+k} \equiv \int_0^1 N_{t+k}(i) \; di$ denotes aggregate employment in period $t+k$, $D_{t+k|t}$ is the market value in period $t+k$ of the portfolio of securities held at the beginning of that period by households that last reoptimized their wage in period t, while $E_{t+k}\{Q_{t+k,t+k+1} D_{t+k+1|t}\}$ is the corresponding market value as of period $t+k$ of the porfolio purchased in that period, and which yields a random payoff $D_{t+k+1|t}$. The remaining variables are defined as in chapter 3.

The first order condition associated with the problem above is given by:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t \left\{ N_{t+k|t} \; U_c(C_{t+k|t}, N_{t+k|t}) \; \frac{W_t^*}{P_{t+k}} + \mathcal{M}_w \; U_n(C_{t+k|t}, N_{t+k|t}) \right\} = 0$$

where $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

Letting $MRS_{t+k|t} \equiv -\frac{U_n(C_{t+k|t}, N_{t+k|t})}{U_c(C_{t+k|t}, N_{t+k|t})}$ denote the marginal rate of substitution between consumption and hours in period $t+k$ for the household resetting the wage in period t, we can rewrite the optimality condition above as

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t \left\{ N_{t+k|t} \; U_c(C_{t+k|t}, N_{t+k|t}) \; \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w \; MRS_{t+k|t} \right) \right\} = 0 \tag{8}$$

Note that in the limiting case of full wage flexibility ($\theta_w = 0$), we have

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w \; MRS_{t|t}$$

for all t. Thus, $\mathcal{M}_w$ is the wedge between the real wage and the marginal rate of substitution that prevails in the absence of wage rigidities, i.e. the *desired* gross wage markup.

Note also that in a perfect foresight, zero inflation steady state we have

$$\frac{W^*}{P} = \frac{W}{P} = \mathcal{M}_w \; MRS$$

Log-linearizing (8) around that steady state yields, after some algebraic manipulation, the following approximate wage setting rule:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \tag{9}$$

where $\mu^w \equiv \log \mathcal{M}_w$.

The intuition behind wage setting rule (9) is straightforward. First, w_t^* is increasing in expected future prices, since households care about the purchasing power of their nominal wage. Second, w_t^* is increasing in the expected average marginal disutilities of labor (in terms of goods) over the life of the wage, since households want to adjust their expected average real wage accordingly, given expected future prices.

As in previous chapters, we specialize the utility function to be of the form:

$$U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$

The assumed separability between consumption and hours, combined with the assumption of complete asset markets implies that consumption is independent of the wage history of a household, i.e. $C_{t+k|t} = C_{t+k}$ for $k = 0, 1, 2, ...$ a result that we invoke in what follows. Thus we can write the (log) marginal rate of substitution in period $t + k$ for a household that last reset its wage in period t as $mrs_{t+k|t} = \sigma c_{t+k} + \varphi n_{t+k|t}$.

Letting $mrs_{t+k} \equiv \sigma c_{t+k} + \varphi n_{t+k}$ define the economy's *average* marginal rate of substitution, we have

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi \left(n_{t+k|t} - n_{t+k} \right) \\ &= mrs_{t+k} - \epsilon_w \varphi \left(w_t^* - w_{t+k} \right) \end{aligned}$$

Hence we can rewrite (9) as

$$\begin{aligned} w_t^* &= \frac{1 - \beta\theta_w}{1 + \epsilon_w \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \ E_t \left\{ \mu_w + mrs_{t+k} + \epsilon_w \varphi \ w_{t+k} + p_{t+k} \right\} \\ &= \frac{1 - \beta\theta_w}{1 + \epsilon_w \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \ E_t \left\{ (1 + \epsilon_w \varphi) \ w_{t+k} - \widehat{\mu}_{t+k}^w \right\} \\ &= \beta\theta_w \ E_t\{w_{t+1}^*\} + (1 - \beta\theta_w) \ \left(w_t - (1 + \epsilon_w \varphi)^{-1} \ \widehat{\mu}_t^w \right) \end{aligned} \tag{10}$$

where $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denotes the deviations of the economy's (log) average wage markup $\mu_t^w \equiv (w_t - p_t) - mrs_t$ from its steady state level μ^w.

1.2.2 Wage Inflation Dynamics

Given the assumed wage setting structure, the evolution of the aggregate wage index (4) is given by

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1 - \theta_w)(W_t^*)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}}$$

The previous equation can be log-linearized around the zero (wage) inflation steady state to yield:

$$w_t = \theta_w \ w_{t-1} + (1 - \theta_w) \ w_t^* \tag{11}$$

Combining (10) and (11), and letting $\pi_t^w = w_t - w_{t-1}$ denote wage inflation, we can obtain, after some manipulation, our baseline wage inflation equation:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \ \widehat{\mu}_t^w \tag{12}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Note that this wage inflation equation has a form analogous to (5), the equation describing the dynamics of price inflation. The intuition behind it is identical: when the average wage in the economy is below the level consistent with maintaining (on average) the desired markup, households readjusting their nominal wage will tend to increase the latter, thus generating positive wage inflation.

In the present model wage inflation equation (12) replaces condition $w_t - p_t = mrs_t$, one of the optimality conditions associated with the households' problem used extensively in previous chapters. The imperfect adjustment of nominal wages will generally drive a wedge between the real wage and the marginal rate of substitution of each household and, as a result, between the average real wage and the average marginal rate of substitution, leading to variations in the average wage markup and, given (12), also in wage inflation.

1.2.3 Other Optimality Conditions

In addition to the optimal wage setting condition (8), the solution to the above household's problem yields also a conventional Euler equation as an optimality condition, as derived in chapter 2 using a simple variational argument:

$$\frac{Q_t}{P_t} \, U_c(C_t, N_{t|t-k}) = \beta \, E_t \left\{ \frac{U_c(C_{t+1}, N_{t+1|t-k})}{P_{t+1}} \right\}$$

where, as in previous chapters Q_t is the price in period t of a one-period riskless discount bond paying one unit of currency in $t+1$. The left-hand side of the above equation represents the loss in utility resulting from the reduction in consumption required to purchase one such bond (for a household that last reset its wage in period $t-k$), while the right hand side reflects the expected utility gains from consuming the associated one-period ahead payoff.

Under the assumption on utility made above, we can log-linearize that optimality condition to yield

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi^p_{t+1}\} - \rho\right) \tag{13}$$

where $i_t \equiv -\log Q_t$ is the nominal yield one the one-period bond. Note that the previous Euler equation takes the same form as those used in earlier chapters, being thus independent of the presence or not of wage rigidities.

2 Equilibrium

We start our analysis of the model's equilibrium by deriving a version of the equations for price and wage inflation in terms of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$. Importantly, the concept of natural output y_t^n used in the present chapter is to be understood as referring to the equilibrium level of output in the absence of *both* price and wage rigidities. We also introduce a new variable, the *real wage gap,* denoted by $\widetilde{\omega}_t$, and formally defined as

$$\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$$

where $\omega_t \equiv w_t - p_t$ denotes the real wage, and where ω_t^n is the *natural real wage*, i.e. the real wage that would prevail in the absence of nominal rigidities, and which is given by

$$\begin{aligned} \omega_t^n &= \log(1-\alpha) + (y_t^n - n_t^n) - \mu^p \\ &= \log(1-\alpha) + \psi_{wa}^n \, a_t - \mu^p \end{aligned}$$

where $\psi_{wa}^n \equiv \frac{1-\alpha\psi_{ya}^n}{1-\alpha} > 0$ and $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ (with the latter as derived in chapter 3).

First, we relate the average price markup to the output and real wage gaps. Using the fact that $\mu_t^p = mpn_t - \omega_t$, we have

$$\begin{aligned} \widehat{\mu}_t^p &= (mpn_t - \omega_t) - \mu^p \\ &= (\widetilde{y}_t - \widetilde{n}_t) - \widetilde{\omega}_t \\ &= -\frac{\alpha}{1-\alpha} \, \widetilde{y}_t - \widetilde{\omega}_t \end{aligned} \tag{14}$$

Hence, combining (5) and (14) we obtain the following equation for price inflation as a function of the output and real wage gaps:

$$\pi_t^p = \beta \; E_t\{\pi_{t+1}^p\} + \kappa_p \; \widetilde{y}_t + \lambda_p \; \widetilde{\omega}_t \tag{15}$$

bwhere $\kappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Similarly, we have

$$\begin{aligned} \widehat{\mu}_t^w &= \omega_t - mrs_t - \mu^w \\ &= \widetilde{\omega}_t - (\sigma \; \widetilde{y}_t + \varphi \; \widetilde{n}_t) \\ &= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right) \widetilde{y}_t \end{aligned} \tag{16}$$

Combining (12) and (16) we obtain an analogous version of the wage inflation equation in terms of the output and real wage gaps:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \, \widetilde{y}_t - \lambda_w \, \widetilde{\omega}_t \tag{17}$$

where $\kappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)$.

In addition, we have an identity relating the changes in the wage gap to price inflation, wage inflation, and the natural wage:

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \tag{18}$$

In order to complete the non-policy block of the model, equilibrium conditions (15), (17), and (18) must be supplemented with a dynamic IS equation familiar from earlier chapters, and which can be derived by combining the goods market clearing condition $y_t = c_t$ with Euler equation (13), and rewriting the resulting expression in terms of the output gap, as follows:

$$\widetilde{y}_t = -\frac{1}{\sigma} \, (i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{19}$$

where the natural interest rate $r_t^n \equiv \rho + \sigma E_t\{\Delta y_t^n\}$ should now be understood as the one prevailing in an equilibrium with flexible wages and prices.

Finally, and in order to close the model we need to specify how the interest rate is determined. We do so by postulating an interest rate rule of the form

$$i_t = \rho + \phi_p \, \pi_t^p + \phi_w \, \pi_t^w + \phi_y \, \widetilde{y}_t + v_t \tag{20}$$

where v_t is an exogenous component, possibly a function of r_t^n and $\Delta\omega_t^n$.(or their leads and lags), and normalized to have zero mean.

Plugging (20) into (19) to eliminate the interest rate, and collecting the remaining conditions (15), (17), (18), (19) and (20) we can represent the equilibrium dynamics by means of a system of the form

$$\mathbf{A}_{w,0} \, \mathbf{x}_t = \mathbf{A}_{\mathbf{w},\mathbf{1}} \, E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_w \, \mathbf{z}_t \tag{21}$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \ \pi_t^p, \ \pi_t^w, \ \widetilde{\omega}_{t-1}]'$, $\mathbf{z}_t \equiv [\widehat{r}_t^n - v_t, \ \Delta\omega_t^n]'$,

$$\mathbf{A}_{w,0} \equiv \begin{bmatrix} \sigma + \phi_y & \phi_p & \phi_w & 0 \\ -\kappa_p & 1 & 0 & 0 \\ -\kappa_w & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}_{w,1} \equiv \begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & \beta & 0 & \lambda_p \\ 0 & 0 & \beta & -\lambda_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_w \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

and where $\{\mathbf{z}_t\}$ follows a given exogenous process.

An important property of (21) is worth emphasizing at this point: in general, the system does *not* have a solution satisfying $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ for all t, not even under the assumption that the intercept of the interest rate rule adjusts one-for-one to variations in the natural rate of interest ($v_t = \widehat{r}_t^n$, for all t). An implication of that result is that the allocation associated with the equilibrium with flexible prices and wages cannot be attained in the presence of nominal rigidities in both goods and labor markets. The intuition for the previous result rests on the idea that in order for the constraints on price and wage setting not to be binding (and hence not to distort the equilibrium allocation) all firms and workers should view their current prices and wages as the desired ones, making any adjustment unnecessary and leading to constant aggregate price and wage levels, i.e. zero inflation in both markets. Note, however, that such an outcome implies a constant real wage, which will generally be inconsistent with the flexible price/flexible wage allocation. Only when the natural wage is constant (so that $\Delta\omega_t^n = 0$ for all t) and as long as as the central bank adjusts the nominal rate one for one with changes in the natural rate (i.e. $v_t = \widehat{r}_t^n$, for all t) the outcome $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ for all t is a solution to (21) and, hence, is consistent with equilibrium.

A second question of interest relates to the conditions that the rule (20) must satisfy to guarantee a unique stationary equilibrium or, equivalently, a unique stationary solution to the system of difference equations (21). Given that vector $\mathbf{x}_t$ contains three non-predetermined and one predetermined variables, (local) uniqueness requires that three eigenvalues of $\mathbf{A}_w$ lie inside, and one outside, the unit circle.

Figure 6.1 displays the configurations of coefficients ϕ_p and ϕ_w associated with a unique equilibrium, as well as the region of indeterminacy, under the assumption that $\phi_y = 0$. As before we restrict our analysis to non-negative values for those coefficients. The condition for uniqueness implied by the numerical analysis underlying Figure 6.1 is given by

$$\phi_p + \phi_w > 1$$

or, what is equivalent, the central bank must adjust the nominal rate more than one-for-one in response to variations in any arbitrary weighted average of price and wage inflation. The previous condition can be viewed as extending the Taylor principle requirement discussed in earlier chapters to the case where we allow the central bank to respond to wage inflation, in addition to price inflation. Figure 6.2 shows how the region consistent with a determinate equilibrium in the (ϕ_p, ϕ_w) parameter space becomes larger as the coefficient on the output gap ϕ_y increases.

2.1 Dynamic Responses to a Monetary Policy Shock

Not surprisingly, the presence of staggered wage-setting influences the economy's equilibrium response to different shocks. Figure 6.3 illustrates this point by displaying the responses of output, price inflation, wage inflation and real wages to a monetary policy shock. Both the policy intervention (a persistent increase in the interest rate rule shifter, v_t) and the model's calibration are identical to the analogous exercise shown in chapter 3. In particular, we assume a simple policy rule of the form (20) with $\phi_p = 1.5$ and $\phi_y = \phi_w = 0$. The only difference is that here we allow for sticky wages, introduced as described above. In order to disentangle the role played by each type of rigidity, we show results for three alternative calibrations of θ_p and θ_w. The first calibration corresponds to an economy in which price and wage rigidities coexist. As in our baseline model of chapter 3 we assume $\theta_p = 2/3$. We set $\theta_w = 3/4$, which implies an average duration of wage spells of four quarters. The latter assumption seems to accord with the empirical evidence (e.g. Taylor (1999)). The second calibration assumes sticky prices and flexible wages ($\theta_p = 2/3$, $\theta_w = 0$) and, hence, corresponds to the basic model introduced in chapter 3. Finally, the third calibration corresponds to an economy with flexible prices and sticky wages ($\theta_p = 0$, $\theta_w = 3/4$). The intervention consists of an increase of 0.25 percentage points in the exogenous component of the interest rate rule. That change would lead, in the absence of an endogenous component in the interest rate rule, to an impact increase of 1 percentage point in the (annualized) nominal interest rate. As in the analogous exercise of chapter 3 we assume an autoregressive coefficient of 0.5 in the AR(1) process followed by the interest rate shifter.

In order to interpret the results shown in Figure 6.3, it is useful to take the responses under sticky prices and flexible wages-already discussed in chapter 3 and represented here by the dashed lines–as a benchmark. The presence of both sticky wages and prices (responses shown in solid line) generates, not surprisingly, a more muted response of wage inflation. The latter partly explains the sluggish response of the real wage, which in turn reduces the impact of the decline in activity on the real marginal cost and, hence, the limited size of the inflation response. As a result, there is only a moderate endogenous response of the monetary authority to the lower inflation, thus implying persistently higher interest rates, which in turn account for the larger decline in output. By contrast, in the flexible wage economy, the decline in activity leads to an (implausibly) large and persistent reduction in the real wage, which amplifies the size of the price inflation drop, and the endogenous reaction of the monetary authority, leading to an overall more muted response of output.

Consider next the consequences of assuming the presence of sticky wages and flexible prices (impulse responses represented by dotted lines). Again, the presence of sticky wages dampens the response of wage inflation to the contractionary monetary policy shock. But now, and given the absence of constraints on price adjustment, price inflation falls considerably in response to the decline in activity and the ensuing lower marginal costs. The large decline in prices in turn leads to a rise in the average real wage which, in turn, dampens (and eventually overturns) the effects of the activity decline on price inflation.

Neither the large negative response of wage inflation and the real wage in the sticky price/flexible wage model, nor the rapid fall in price inflation and the resulting large increase in the real wage in the sticky wage/flexible price model appear to be consistent with existing estimates of the dynamic effects of exogenous monetary policy shocks. The latter estimates, and in particular those of the response of real wages to a monetary olicy shock, are instead more in line with the predictions of the model with both sticky prices and wages.[3]

3 Monetary Policy Design with Sticky Wages and Prices

In the present section we explore some of the normative implications of the coexistence of sticky prices and sticky wages, as modelled in the framework

[3]See, e.g., Christiano, Eichenbaum and Evans (2005).

above, for the conduct of monetary policy. In so doing, and in order to keep the analysis as simple as possible, we make the necessary assumptions to guarantee that the *natural* allocation, i.e. the equilibrium allocation in the absence of nominal rigidities, is also the efficient allocation. Given the absence of mechanisms (e.g. capital accumulation) for the economy as a whole to transfer resources across periods, the efficient allocation corresponds to the solution of a sequence of static social planner problems of the form:

$$\max \int_0^1 U(C_t(j), N_t(j))\, dj$$

subject to (1), (2), (6), as well as the usual market clearing conditions. The optimality conditions for that problem are given by

$$C_t(i,j) = C_t, \;\; all \; i,j \in [0,1] \tag{22}$$

$$N_t(i,j) = N_t(j) = N_t(i) = N_t, \;\; all \; i,j \in [0,1] \tag{23}$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t \tag{24}$$

where $MPN_t \equiv (1-\alpha)A_t N_t^{1-\alpha}$. Note that, under our assumptions, if *all* firms and households reoptimize ther prices each period they will all choose the same prices and wages and, hence, (22) and (23) will be satisfied. On the other hand optimal price and wage setting implies

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}}\, \mathcal{M}_w$$

and

$$P_t = \mathcal{M}_p\, \frac{(1-\tau)W_t}{MPN_t}$$

where τ is an employment subsidy, funded through lump sum taxes. Note that by setting $\tau = 1 - \frac{1}{\mathcal{M}_p \mathcal{M}_w}$ condition (24) is also satisfied, thus guaranteeing the efficiency of the flexible price/flexible wage equilibrium allocation. The latter property is assumed to hold for the remainder of the chapter.

In the appendix we derive a second order approximation to the average welfare losses experienced by households in the economy with sticky wages and prices, resulting from fluctuations around a steady state with zero wage and price inflation. When the latter is efficient, as is the case under the optimal subsidy derived above, those welfare losses, expressed as a fraction of steady state consumption are given by:

$$\mathbb{W} = \frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} (\pi_t^w)^2 \right) + t.i.p. \tag{25}$$

where $t.i.p.$ collects various terms that are independent of policy. Thus, and ignoring the latter terms, we can write down the average period welfare loss as a linear combination of the variances of the output gap, price inflation and wage inflation given by

$$\mathbb{L} = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon_p}{\lambda_p}\, var(\pi_t^p) + \frac{\epsilon_w (1-\alpha)}{\lambda_w}\, var(\pi_t^w) \tag{26}$$

Note that the relative weight of each of the variances is a function of the underlying parameter values. The weights associated with output gap and price inflation fluctuations are identical to those derived and discussed in chapter 4 for a version of the model economy with sticky prices and flexible wages. The presence of sticky wages implies an additional source of welfare losses, associated with wage inflation fluctuations. The contribution of wage inflation volatility to the welfare losses is increasing in (i) the elasticity of substitution among labor types (ϵ_w), (ii) the elasticity of output with respect to labor input, $1-\alpha$, and (iii) the degree of wage stickiness θ_w (which is inversely related to λ_w). Note that (i) and (ii) amplify the negative effect on aggregate productivity of any given dispersion of wages across labor types, while (iii) raises the degree of wage dispersion resulting from any given rate of wage inflation different from zero.

In general, and as argued above, the lower bound of zero welfare losses that characterizes an allocation where $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ for all t is not attainable. The optimal policy will thus have to strike a balance in stabilizing the three abovementioned variables.

In the limiting case of flexible wages we have $\lambda_w \longrightarrow +\infty$, and the term in the loss function associated with wage inflation volatility vanishes (i.e. wage inflation is no longer costly). Note that in that case the wage markup is constant and hence

$$\begin{aligned}\widetilde{\omega}_t &= \sigma\, \widetilde{c}_t + \varphi\, \widetilde{n}_t \\ &= \left(\sigma + \frac{\varphi}{1-\alpha}\right) \widetilde{y}_t\end{aligned}$$

which, substituted into (17), yields a new Keynesian Phillips curve identical to that derived in chapter 3, namely,

$$\pi_t^p = \beta\ E_t\{\pi_{t+1}^p\} + \kappa_p\ \widetilde{y}_t$$

where $\kappa_p \equiv \lambda_p \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Accordingly, and as shown in chapter 3, there is no longer a trade-off between stabilization of price inflation and stabilization of the output gap, with the optimal policy requiring that $\pi_t^p = \widetilde{y}_t = 0$ for all t.

Similarly, in the limiting case of flexible prices (but sticky wages), we have $\lambda_p \longrightarrow +\infty$ so that only the terms associated with fluctuations in the output gap and wage inflation remain a source of welfare losses. In that case, and using the fact that price markups will be constant, we can write

$$\begin{aligned}\widetilde{\omega}_t &= \widetilde{y}_t - \widetilde{n}_t \\ &= -\frac{\alpha}{1-\alpha}\ \widetilde{y}_t\end{aligned}$$

which substituted into (17) yields the wage inflation equation

$$\pi_t^w = \beta\ E_t\{\pi_{t+1}^w\} + \kappa_w\ \widetilde{y}_t$$

where $\kappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. In that case, the optimal policy will attain the zero lower bound for the welfare losses, by fully stabilizing the output gap and wage inflation, i.e. $\pi_t^w = \widetilde{y}_t = 0$ for all t.

Thus, we see that with the exception of the limiting case of full wage flexibility, a policy that seeks to stabilize price inflation completely (i.e. a strict price inflation targeting policy) will be suboptimal. The same is true for a strict wage inflation targeting policy, with the exception of an economy with fully flexible prices.

4 Optimal Monetary Policy

Next we characterize the optimal monetary policy in the economy in which both prices and wages are sticky. For concreteness we restrict ourselves to the case of full commitment. The central bank will seek to maximize (25) subject to (15), (17), and (18) for $t = 0, 1, 2, ...$ Let $\{\xi_{1,t}\}$, $\{\xi_{2,t}\}$ and $\{\xi_{3,t}\}$ denote the sequence Lagrange multipliers associated with the previous constraints, respectively. The optimality conditions for the optimal policy problem are thus given by

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \ \widetilde{y}_t + \kappa_p \ \xi_{1,t} + \kappa_w \ \xi_{2,t} = 0 \tag{27}$$

$$\frac{\epsilon_p}{\lambda_p} \ \pi_t^p - \Delta\xi_{1,t} + \xi_{3,t} = 0 \tag{28}$$

$$\frac{\epsilon_w(1-\alpha)}{\lambda_w} \ \pi_t^w - \Delta\xi_{2,t} - \xi_{3,t} = 0 \tag{29}$$

$$\lambda_p \ \xi_{1,t} - \lambda_w \ \xi_{2,t} + \xi_{3,t} - \beta E_t\{\xi_{3,t+1}\} = 0 \tag{30}$$

for $t = 0, 1, 2, ...$which, together with the constraints (15), (17), and (18), and given $\xi_{1,-1} = \xi_{2,-1} = 0$ and an initial condition for $\widetilde{\omega}_{-1}$ characterize the solution to the optimal policy problem. We can represent in a compact way the equilibrium under the optimal policy as the stationary solution to the dynamical system

$$\mathbf{A}_0^* \ \mathbf{x}_t = \mathbf{A}_\mathbf{1}^* \ E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}^* \ \Delta a_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \ \pi_t^p, \ \pi_t^w, \widetilde{\omega}_{t-1}, \ \xi_{1,t-1}, \ \xi_{2,t-1}, \ \xi_{3,t}]'$ and where $\mathbf{A}_0^*$, $\mathbf{A}_\mathbf{1}^*$ and $\mathbf{B}^*$ are defined in the Appendix.

Figure 6.4 displays the responses of the output gap, price and wage inflation, and the real wage to a positive technology shock, under the optimal policy and for the three parameter calibrations considered earlier. Note that, as shown in chapter 4, when only prices are sticky (dashed line) the optimal policy implies full stabilization of the price level and no effect on inflation. Since that policy replicates the flexible price/flexible wage equilibrium allocation the responses of both output and the real wage correspond to their natural counterparts, with the necessary adjustment of the real wage attained through large and persistent wage inflation which, given the assumed flexibility of wages, causes no distortions.

When only wages are sticky (dotted line), and in a way consistent with the discussion in the previous section, the natural allocation can also be attained, though now it requires full stabilization of nominal wages and, hence, zero wage inflation. The latter requirement in turn implies that the adjustment in the real wage be achieved through negative price inflation which, given the assumption of flexible prices, is no longer costly in terms of welfare.

When both prices and wages are sticky the natural allocation can no longer be attained. In that case the optimal policy strikes a balance between attaining the output and real wage adjustments warranted by the rise in productivity and, on the other hand, keeping wage and price inflation close to zero to avoid the distortions associated with nominal instability. As a result, and in response to a positive technology shock, the real wage rises but not as much as the natural wage (note that the latter coincides with the response under the two previous calibrations). Given the convexity of welfare losses in price and wage inflation, it is optimal to raise the real wage smoothly, through a mix of negative price inflation and positive wage inflation. The implied sluggishness of the real wage, combined with the improvement in technology, accounts for the observed overshooting of output, which rises above its natural level, generating a positive output gap.

Next we examine a particular configuration of parameter values for which the optimal policy takes a simple form, which can be characterized analytically.

4.1 A Special Case with an Analytical Solution

Let us assume $\kappa_p = \kappa_w$ and $\epsilon_p = \epsilon_w(1-\alpha) \equiv \epsilon$. Note that in that case optimality conditions (27)-(29) in the monetary policy problem simplify to a single condition relating price and wage inflation to the output gap and given by

$$\lambda_w \ \pi_t^p + \lambda_p \ \pi_t^w = -\frac{\lambda_p}{\epsilon} \ \Delta \widetilde{y}_t$$

for $t = 1, 2, 3, ...$, and $\lambda_w \ \pi_0^p + \lambda_p \ \pi_0^w = -\frac{\lambda_p}{\epsilon} \ \widetilde{y}_0$ for period 0. Let us define the following weighted average of price and wage inflation

$$\pi_t \equiv (1-\vartheta) \ \pi_t^p + \vartheta \ \pi_t^w \tag{31}$$

where $\vartheta \equiv \frac{\lambda_p}{\lambda_p + \lambda_w} \in [0,1]$ is increasing (decreasing) in the degree of wage (price) rigidities.

Thus we can write the above optimality condition in terms of the composite inflation measure:

$$\pi_t = -\frac{\vartheta}{\epsilon} \ \Delta \widetilde{y}_t$$

for $t = 1, 2, 3, ...$, and $\pi_0 = -\frac{\vartheta}{\epsilon}\ \widetilde{y}_0$ in period 0. Equivalently, the optimal policy must meet the following target criterion for $t = 0, 1, 2, ...$

$$\widehat{q}_t = -\frac{\vartheta}{\epsilon}\ \widetilde{y}_t \tag{32}$$

where $\widehat{q}_t \equiv q_t - q_{-1}$, and $q_t \equiv (1-\vartheta)\ p_t + \vartheta\ w_t$ is a weighted average of the (log) price and wage levels.

Note that, independently of parameter values, one can always combine the wage and price inflation equations (15) and (17) to obtain the following version of the new Keynesian Phillips curve in terms of composite inflation:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t \tag{33}$$

where now $\kappa \equiv \frac{\lambda_p \lambda_w}{\lambda_p + \lambda_w}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Thus, (33) implies that there is no tradeoff between stabilization of the output gap and stabilization of the particular composite measure of inflation introduced above.

Using (32) to substitute $\widetilde{y}_t$ out in (33) and rewriting the latter in terms of levels (using $\pi_t \equiv \widehat{q}_t - \widehat{q}_{t-1}$) we obtain the following second order difference for the composite price level:

$$\widehat{q}_t = a\ \widehat{q}_{t-1} + a\beta\ E_t\{\widehat{q}_{t+1}\} = 0$$

for $t = 0, 1, 2, ...$where $a \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa\epsilon}$. The only stationary solution to the previous difference equation must satisfy $\widehat{q}_t = \delta\ \widehat{q}_{t-1}$ where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$ for $t = 0, 1, 2, ...$Given that $\widehat{q}_{-1} = 0$, it follows that the optimal policy requires stabilizing the composite price level at its inherited value, or equivalently,

$$\pi_t = 0$$

and, as a result,

$$\widetilde{y}_t = 0$$

for $t = 0, 1, 2, ...$

Thus, in the particular case considered here, the optimal policy takes a simple form: the central bank should focus uniquely on targeting and fully stabilizing a weighted average of price and wage inflation, with the weights determined by underlying parameters. In particular the relative weight of price (wage) inflation is increasing in the degree of price (wage) stickiness.

A nice feature of the optimal policy in the particular case analyzed above is that its implementation by the monetary authority does not rely on the output gap being observable: it suffices that the monetary authority keeps track of the composite inflation measure, and responds (aggressively) to any deviation from zero of that measure. Of course, and as seen above, for the general case the optimal policy does not have such a simple characterization, requiring instead that the central bank follow a much more complicated target rule satisfying (15), (17), and (27) through (30) simultaneously. In that context, it is of interest to know to what extent different simple monetary policy rules may be able to approximate the optimal policy, an issue to which we turn our attention in the following section.

5 Evaluation of Simple Rules under Sticky Wages and Prices

In this section we consider a number of simple monetary policy rules and provide a quantitative evaluation of their impact on welfare. Given a parameter calibration, that evaluation is based on the unconditional period losses implied by each simple rule, and given by (26). In the simulation underlying that exercise, variations in the technology parameter a_t are assumed to be the only source of fluctuations. That parameter follows an AR(1) process with an autoregressive coefficient $\rho_a = 0.9$ and a standard deviation for its innovation of 0.01. We set the remaining parameters (other than the stickiness parameters θ_p and θ_w) at their baseline values. For θ_p and θ_w we consider three alternative calibrations, as discussed below.

We analyze six different simple rules. The first rule, which we refer to as strict price inflation targeting, requires that price inflation be zero at all times ($\pi_t^p = 0$ all t). We also assume an analogous rule for wage inflation, i.e. a strict wage inflation targeting rule. Our third rule stabilizes the weighted average of price and wage inflation given by (31). We refer to that rule as strict composite inflation targeting rule. As shown in the previous section, that rule is optimal whenver some specific conditions on the model's parameters are satisfied (which is not the case under for baseline calibration). But even when those conditions are not satisfied, that rule has a special interest since, as implied by (33), it is equivalent to a rule that fully stabilizes the output gap.

The remaining three rules considered take the form of a simple interest rate rule

$$i_t = \rho + 1.5\ \pi_t$$

where π_t refers, respectively, to price inflation, wage inflation or composite inflation (31). We refer to these rules as *flexible* (price, wage or composite) inflation targeting rules.

Table 6.1 reports the main findings of that exercise. For each simple rule we report the implied standard deviation of (annualized) price inflation, (annualized) wage inflation and the output gap, as well as the corresponding average period welfare loss. In addition to the simple rules, the table also reports the corresponding statistics for the optimal policy, which provides a useful benchmark. The top panel reports statistics corresponding to the calibration of the wage and price stickiness parameters used earlier in this chapter, namely, $\theta_p = 2/3$ and $\theta_w = 3/4$. Relative to that benchmark, the second panel assumes a lower degree of wage rigidity ($\theta_w = 1/4$), while the third panel reports results for a lower degree of price rigidity ($\theta_p = 1/3$).

For our baseline calibration (top panel), the optimal policy implies near-constancy of the output gap, and a standard deviation of wage inflation which is one-third that of price inflation. The implied welfare losses (relative to the unattainable first-best allocation) are very small, less than 1/40 of a percent of steady state consumption. Among the simple rules, the one that targets composite inflation does, for practical purposes, as well as the optimal policy, generating a very similar pattern of volatilities of the three welfare-relevant variables. Given that wage inflation has a weight of 0.77 in composite inflation, it is perhaps not surprising that a strict wage inflation targeting ranks second among the simple rules considered, with implies losses only slightly above those of the optimal policy. Interestingly, under this baseline calibration, price inflation targeting rules are the worst, largely due to the large fluctuations in wage inflation and the output gap that result from following those rules.

When we consider the second calibration (with lower wage rigidity), the ranking among strict targeting policies is not affected, even though the relative losses from targeting price inflation now decline considerably and are almost identical to those resulting from strict wage inflation targeting. In fact, when we look at flexible targeting rules, price inflation targeting appears as slighty more desirable than wage inflation targeting, though still less so than targeting composite inflation.

Finally, under our third calibration (associated with a lower degree of price rigidity), the relative desirability of wage inflation targeting increases, even though targeting composite inflation is still the most desirable strict targeting policy. That relative ranking is reversed when we consider flexible targeting rules, with wage inflation targeting being now the most desirable, as it was the case under the baseline calibration. Finally, it is worth noting that the losses associated with price inflation targeting are again one order of magnitude above the ones resulting from the rules that seek to stabilize wage inflation.

Overall, we view the message conveyed by the exercise of this section as twofold. First, in the presence of sticky wages (coexisting with sticky prices) policies that focus exclusively on stabilizing price inflation are clearly suboptimal. Secondly, and in the absence of further imperfections, a policy that responds aggressively to an appropriate weighted average of price and wage inflation emerges as a most desirable one. Of course, choosing the appropriate weights remains a challenge. Our quantitative analysis, based on calibrations that are likely to span the range of plausible parameters, suggests that a policy that gives a dominant weight to wage inflation in the definition of that composite generates small additional losses relative to the optimal policy. Interestingly, that conclusion appears at odds with the practice of most central banks, which seem to attach little weight to wage inflation as a target variable, with the interest in that variable often limited to its ability to influence (and thus help predict) current and future price inflation developments.

6 Notes on the Literature

Early examples of non-optimizing rational expectations models with nominal wage rigidities can be found in the work of Fischer (1977) and Taylor (1980). Cooley and Cho (1995) and Bénassy (1995) were among the first papers that embedded the assumption of sticky nominal wages in a dynamic stochastic general equilibrium model, and examined its implications for the properties of a number of variables, in the presence of both real and monetary shocks.

Erceg, Henderson and Levin (2000) developed the new Keynesian model with both staggered price and staggered wage contracts à la Calvo that has become the framework of reference in the literature, and on which much of the present chapter builds. The focus of their paper was, like the present chapter, on the derivation of the implications for monetary policy. A similar focus, including a discussion of the special case in which targeting a weighted average of wage and price inflation is optimal, can be found in Woodford (2003, chapter 6) and Giannoni and Woodford (2003). Other work has focused instead on the impact of staggered wage setting on the persistence of the effects of monetary policy shocks. See e.g. Huang and Liu (2002) and, especially, Woodford (2003, chapter 3) for a detailed discussion of the role of wage stickiness in that regard.

Staggered wage setting is also a common feature of medium-scale models like those of Kim (2000), Smets and Wouters (2003), and Christiano, Eichenbaum and Evans (2005). An analysis of the optimal implementable rules in such a model can be found in Schmitt-Grohé and Uribe (2006), which also includes a numerical analysis of the requirements that the coefficients of the interest rate rule must satisfy to guarantee uniqueness of the equilibrium

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Appendix

A Second Order Approximation to Welfare Losses with Price and Wage Stickiness

Using a second order Taylor expansion to household j's period t utility around the steady state, combined with a goods market clearing condition, and integrating across households, yields:

$$\int_0^1 (U_t(j)-U)\,dj \simeq U_c C\left(\widehat{y}_t + \frac{1-\sigma}{2}\,\widehat{y}_t^2\right) - U_n N\left(\int_0^1 \widehat{n}_t(j)\,dj + \frac{1+\varphi}{2}\int_0^1 \widehat{n}_t(j)^2\,dj\right)$$

where $\sigma \equiv -\frac{U_{cc}}{U_c}C$ and $\varphi \equiv \frac{U_{nn}}{U_n}N$, and where we have made use of the market clearing condition $\widehat{c}_t = \widehat{y}_t$.

Define aggregate employment as $N_t \equiv \int_0^1 N_t(j)\,dj$, or, in terms of log deviations from steady state and up to a second order approximation:

$$\widehat{n}_t + \frac{1}{2}\,\widehat{n}_t^2 \simeq \int_0^1 \widehat{n}_t(j)\,dj + \frac{1}{2}\int_0^1 \widehat{n}_t(j)^2\,dj$$

Note also that

$$\begin{aligned}
\int_0^1 \widehat{n}_t(j)^2 dj &= \int_0^1 (\widehat{n}_t(j) - \widehat{n}_t + \widehat{n}_t)^2 dj \\
&= \widehat{n}_t^2 - 2\widehat{n}_t\epsilon_w \int_0^1 \widehat{w}_t(j)\,dj + \epsilon_w^2 \int_0^1 \widehat{w}_t(j)^2\,dj \\
&\simeq \widehat{n}_t^2 + \epsilon_w^2\; var_j\{w_t(j)\}
\end{aligned}$$

where we have made use of the labor demand equation $\widehat{n}_t(j) - \widehat{n}_t = -\epsilon_w \widehat{w}_t(j)$ and the fact that $\int_0^1 \widehat{w}_t(j)\,dj = \frac{(\epsilon_w - 1)}{2}\; var_i\{w_t(i)\}$ is of second order, a result analogous to that obtained for prices.

Thus we can write

$$\int_0^1 (U_t(j)-U)\,dj \simeq U_c C\left(\widehat{y}_t + \frac{1-\sigma}{2}\,\widehat{y}_t^2\right) - U_n N\left(\widehat{n}_t + \frac{1+\varphi}{2}\,\widehat{n}_t^2 + \frac{\epsilon_w^2 \varphi}{2}\; var_j\{w_t(j)\}\right)$$

Next we derive a relationship between aggregate employment and output:

$$\begin{aligned}
N_t &= \int_0^1 \int_0^1 N_t(i,j)\ dj\ di \\
&= \int_0^1 N_t(i) \int_0^1 \frac{N_t(i,j)}{N_t(i)}\ dj\ di \\
&= \Delta_{w,t} \int_0^1 N_t(i)\ di \\
&= \Delta_{w,t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t}\right)^{\frac{1}{1-\alpha}}\ di \\
&= \Delta_{w,t}\ \Delta_{p,t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}
\end{aligned}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

Thus, the following second order approximation to the relation between (log) aggregate output and (log) aggregate employment holds

$$(1-\alpha)\ \widehat{n}_t = \widehat{y}_t - a_t + d_{w,t}\ + d_{p,t}$$

where $d_{w,t} \equiv (1-\alpha) \log \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $d_{p,t} \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

As shown in the appendix of chapter 4, $d_{p,t} \simeq \frac{\epsilon_p}{2\Theta}\ var_i\{p_t(i)\}$. Using an analogous derivation, one can show $d_{w,t} \simeq \frac{(1-\alpha)\epsilon_w}{2}\ var_j\{w_t(j)\}$.

Hence we can rewrite aggregate welfare as

$$\begin{aligned}
\int_0^1 (U_t(j) - U)\ dj \ &\simeq\ U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2}\ \widehat{y}_t^2\right) \\
&\quad - \frac{U_n N}{(1-\alpha)} \left(\widehat{y}_t + \frac{\epsilon_p}{2\Theta}\ var_i\{p_t(i)\} + \frac{\Upsilon}{2}\ var_j\{w_t(j)\} + \frac{1+\varphi}{2(1-\alpha)} \int_0^1 (\widehat{y}_t - a_t)^2
\end{aligned}$$

where $\Upsilon \equiv \epsilon_w(1-\alpha)(1+\epsilon_w\varphi)$ and where $t.i.p.$ stands for "terms independent of policy".

Let Φ denote the size of the steady state distortion, implicitly defined by $-\frac{U_n}{U_c} = MPN\ (1-\Phi)$. Using the fact that $MPN = (1-\alpha)(Y/N)$ we have

$$\int_0^1 \frac{U_t(j)-U}{U_c C}\, dj = \widehat{y}_t + \frac{1-\sigma}{2}\, \widehat{y}_t^2$$
$$-(1-\Phi)\ \left(\widehat{y}_t + \frac{\epsilon_p}{2\Theta}\ var_i\{p_t(i)\} + \frac{\Upsilon}{2}\ var_j\{w_t(j)\} + \frac{1+\varphi}{2(1-\alpha)}\ (\widehat{y}_t - a_t)^2\right) +$$

Under the "small distortion" assumption (so that the product of Φ with a second order term can be ignored as negligible) and igonoring the tips terms we can write:

$$\begin{aligned}
\int_0^1 \frac{U_t(j)-U}{U_c C}\, dj &= \Phi\ \widehat{y}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta}\ var_i\{p_t(i)\} + \Upsilon var_j\{w_t(j)\} - (1-\sigma)\ \widehat{y}_t^2 + \frac{1+\varphi}{1-\alpha}\ (\widehat{y}_t - a_t)^2\right) \\
&= \Phi\ \widetilde{y}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta}\ var_i\{p_t(i)\} + \Upsilon var_j\{w_t(j)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ \widehat{y}_t^2 - 2\left(\frac{1+\varphi}{1-\alpha}\right) \right. \\
&= \Phi\ \widetilde{y}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta}\ var_i\{p_t(i)\} + \Upsilon var_j\{w_t(j)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ (\widehat{y}_t^2 - 2\widehat{y}_t\ \widehat{y}_t^e)\right) \\
&= \Phi\ \widehat{x}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta}\ var_i\{p_t(i)\} + \Upsilon var_j\{w_t(j)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ \widehat{x}_t^2\right)
\end{aligned}$$

where $\widehat{y}_t^e \equiv y_t^e - y^e$, and where we have used the fact that $\widehat{y}_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$ .and $\widehat{y}_t - \widehat{y}_t^e = x_t - (y - y^e) = x_t - x \equiv \widehat{x}_t$.

Accordingly, we can write a second order approximation to the consumer's discounted (up to additive terms independent of policy), and expressed as a fraction of steady state consumption, as:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t\ \left(\frac{U_t - U}{U_c C}\right) = E_0 \sum_{t=0}^{\infty} \beta^t\ \left[\Phi\ \widehat{x}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta}\ var_i\{p_t(i)\} + \Upsilon var_j\{w_t(j)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\right.\right.$$

Using Lemma 2 in the Appendix of chapter 4, we can rewrite the welfare losses as

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t\ \left[\Phi\ \widehat{x}_t - \frac{1}{2}\left(\frac{\epsilon_p}{\lambda_p}\ (\pi_t^p)^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\ (\pi_t^w)^2 + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ \widehat{x}_t^2\right)\right]$$

Note that in the particular case of an efficient steady state we have $\Phi = 0$ and $\widehat{x}_t = x_t$. Moreover, if the optimal subsidy discussed in the text is in place, the steady state is efficient and we have $\Phi = 0$ and $\widehat{x}_t = x_t$. In addition, the model satisfies $y_t^n = y_t^e$ for all t , thus we have $\widehat{x}_t = x_t = \widetilde{y}_t$.

Definition of $\mathbf{A}_0^*$, $\mathbf{A_1^*}$ and $\mathbf{B}^*$

$$\mathbf{A}_0^* \equiv \begin{bmatrix} -\frac{\alpha\lambda_p}{1-\alpha} & 1 & 0 & 0 & 0 & 0 & 0 \\ -\lambda_w(\sigma + \frac{\varphi}{1-\alpha}) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ (\sigma + \frac{\varphi+\alpha}{1-\alpha}) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\epsilon_p}{\lambda_p} & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & \frac{\epsilon_w(1-\alpha)}{\lambda_w} & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_1^* \equiv \begin{bmatrix} 0 & \beta & 0 & \lambda_p & 0 & 0 & 0 \\ 0 & 0 & \beta & -\lambda_w & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\alpha\lambda_p}{1-\alpha} & -\lambda_w(\sigma + \frac{\varphi}{1-\alpha}) & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_p & \lambda_w & \beta \end{bmatrix}$$

$$\mathbf{B}^* \equiv [0, 0, 1, 0, 0, 0, 0]'$$

Exercises

1. Optimal Monetary Policy in a Sticky Wage Economy

We assume a representative firm that is perfectly competitive and has access to a technology described by:

$$y_t = a_t + n_t$$

where y_t, n_t, and a_t denote the logs of output, employment, and productivity, respectively. Prices are flexible. Assume

$$a_t = \rho \ a_{t-1} + \varepsilon_t$$

The representative household's optimal labor supply is given by:

$$w_t - p_t = \varphi \ n_t$$

where w_t and p_t denote the log of the wage and price levels, respectively.

(a) Derive the equilibrium behavior of employment and output under the assumption of flexible wages and prices.

(b) Next we introduce sticky wages. Each period half the workers set the (log) nominal wage, which remains constant for two periods, according to:

$$w_t^* = \frac{1}{2} \ (p_t + E_t\{p_{t+1}\}) + \ \frac{\varphi}{2} \ (n_t + E_t\{n_{t+1}\})$$

The average effective (log) wage paid by the firm in period t is thus

$$w_t = \frac{1}{2} \ (w_t^* + w_{t-1}^*)$$

Show that inflation evolves according to:

$$\pi_t = E_t\{\pi_{t+1}\} + \varphi \ \widetilde{n}_t + u_t$$

where $\widetilde{n}_t \equiv n_{t-1} + E_{t-1}\{n_t\} + n_t + E_t\{n_{t+1}\}$ and $u_t \equiv -4a_t - (p_t - E_{t-1}\{p_t\})$.

(c) Suppose that aggregate demand is given by by the IS equation:

$$y_t = -\frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{y_{t+1}\}$$

and assume that the optimal policy requires that the flexible wage allocation be replicated. Describe the equilibrium behavior of the interest rate, wage inflation, and price inflation under the optimal policy.

2. Optimal Monetary Policy with Wages Set in Advance

The representative firm is perfectly competitive and has access to a technology described by:

$$y_t = a_t + n_t$$

where y, n, a denote the logs of output, employment, and productivity, respectively. Prices are flexible. We assume

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a$$

The optimal labor supply satisfies:

$$w_t - p_t = \varphi \; n_t$$

where w and p denote the log of the (nominal) wage and price levels, respectively.

Aggregate demand is given by the dynamic IS equation:

$$y_t = -\frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{y_{t+1}\}$$

where i_t denotes the nominal interest rate and $\pi_t \equiv p_t - p_{t-1}$ is the inflation rate.

(a) Derive the equilibrium behavior of employment, output, and the real interest rate under the assumption of flexible wages and prices. Can one determine the corresponding equilibrium values for the nominal rate and inflation? Explain why.

(b) Next we introduce wage stickiness by assuming that nominal wages are set in advance (i.e. at the end of the previous period), according to the rule

$$w_t = E_{t-1}\{p_t\} + \; \varphi \; E_{t-1}\{n_t\}$$

Characterize the equilibrium behavior of output, employment, inflation, and the real wage under the assumption that the central bank follows the simple rule

$$i_t = \rho + \phi_\pi \, \pi_t$$

(d) Characterize the optimal policy and its associated equilibrium in the presence of sticky wages, and suggest an interest rate rule that would implement it (note: we assume efficiency of the equilibrium allocation in the absence of sticky wages).

3. Labor Market Institutions as a source of Long Run Money Non-Neutrality (based on Blanchard and Summers ())

A perfectly competitive representative firm maximizes profits each period

$$P_t Y_t - W_t N_t$$

subject to a technology $Y_t = N_t^{1-\alpha}$. Assume that the desired labor supply is inelastic and equal to one. Equilibrium in the goods market is given by

$$Y_t = \frac{M_t}{P_t}$$

with the nominal money supply following an AR(1) process (in logs):

$$m_t = \rho_m \, m_{t-1} + \varepsilon_t$$

Derive the equilibrium process for (the log) of output y_t, employment n_t, prices p_t, and real wages $w_t - p_t$ under each of the alternative assumptions on the wage setting process:

a) nominal wages are fully flexible and determined competitively.

b) nominal wages are set in advance, so that the labor market clears in expectation (i.e., $E_{t-1}\{n_t\} = 0$).

c) nominal wages are set in advance by a union, so that in expectation only currently employed workers are employed (i.e., $E_{t-1}\{n_t\} = n_{t-1}$)

d) Discuss the empirical relevance of the three scenarios in light of their implied properties (comovements, persistence) for real wages, employment and output.

4. Monetary Policy and Real Wage Rigidities

Assume that the representative household's utility is given $E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t)$ with $U(C_t, N_t) = C_t - \frac{1}{2} N_t^2$, where C_t denotes consumption and N_t denotes hours worked. Let firms' technology be given by the production function $Y_t = A_t N_t$, where Y_t denotes output and A_t is an exogenous technology parameter. All output is consumed.

Firms set prices in a staggered fashion à la Calvo, which results in the inflation dynamics equation:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t$$

where $\widehat{mc}_t \equiv mc_t - mc$ represent the log deviations of real marginal cost from its level in the zero inflation steady state.

(a). Derive an expression for the (log) of the *efficient* level of output (which we will denote by y_t^*) as a function of (log) productivity a_t (i.e., the level of employment that a benevolent social planner would choose, given preferences and constraints).

(b). Assume that the (log) nominal wage w_t is set each period according to the schedule $w_t = p_t + \frac{1}{1+\delta} n_t$, where $\delta > 0$ (the same assumption is maintained for parts (c), (d) and (e) below). Compare the behavior of the equilibrium real wage under that schedule with the one that would be observed under competitive labor markets. In what sense the condition $\delta > 0$ can be interpreted as a "real rigidity"?.

(c) Derive the implied (log) *natural* level of output (denoted by y_t^n), defined as the equilibrium level of output under flexible prices (when all firms keep a constant (log) markup μ).

(d) Derive an expression for the real marginal cost $\widehat{mc}_t$ as a function of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$.

(e) Derive the inflation equation in terms of the welfare-relevant output gap $y_t - y_t^*$. Show how the presence of real wage rigidities ($\delta > 0$) generates a trade-off between stabilization of inflation and stabilization of the welfare-relevant employment gap.

(f) Supose that the monetary authority has a loss function given by $E_0 \sum_{t=0}^{\infty} \beta^t \; [\alpha(y_t - y_t^*)^2 + \pi_t^2]$. Solve for the equilibrium process for inflation and output under the optimal monetary policy under discretion (time consistent solution), under the assumption of an i.i.d. technology process a_t. Explain the difference with the case of perfect competition in the labor market. (note: for simplicity you can assume that the frictionless markup μ is infinitesimally small when answering this question).

Table 6.1: Evaluation of Simple Rules

	Optimal Policy	*Strict Rules* Price	Wage	Composite	*Flexible Rules* Price	Wage	Composite
$\theta_p = \frac{2}{3}$ $\theta_w = \frac{3}{4}$							
$\sigma(\pi^p)$	0.64	0	0.82	0.66	1.50	1.08	1.12
$\sigma(\pi^w)$	0.22	0.98	0	0.19	1.05	0.30	0.42
$\sigma(\widetilde{y})$	0.04	2.38	0.52	0	0.75	1.16	0.01
$\mathbb{L}$	0.023	0.184	0.034	0.023	0.221	0.081	0.089
$\theta_p = \frac{2}{3}$ $\theta_w = \frac{1}{4}$							
$\sigma(\pi^p)$	0.29	0	0.82	0.21	1.40	1.45	1.30
$\sigma(\pi^w)$	1.24	2.91	0	1.63	1.49	0.98	1.25
$\sigma(\widetilde{y})$	0.19	0.61	0.52	0	0.29	0.68	0.32
$\mathbb{L}$	0.010	0.038	0.034	0.012	0.097	0.104	0.083
$\theta_p = \frac{1}{3}$ $\theta_w = \frac{3}{4}$							
$\sigma(\pi^p)$	1.64	0	1.91	1.75	2.58	2.10	2.10
$\sigma(\pi^w)$	0.11	0.98	0	0.06	1.47	0.07	0.10
$\sigma(\widetilde{y})$	0.17	2.38	0.27	0	0.87	0.60	0.58
$\mathbb{L}$	0.016	0.184	0.021	0.017	0.271	0.030	0.031

Figure 6.1

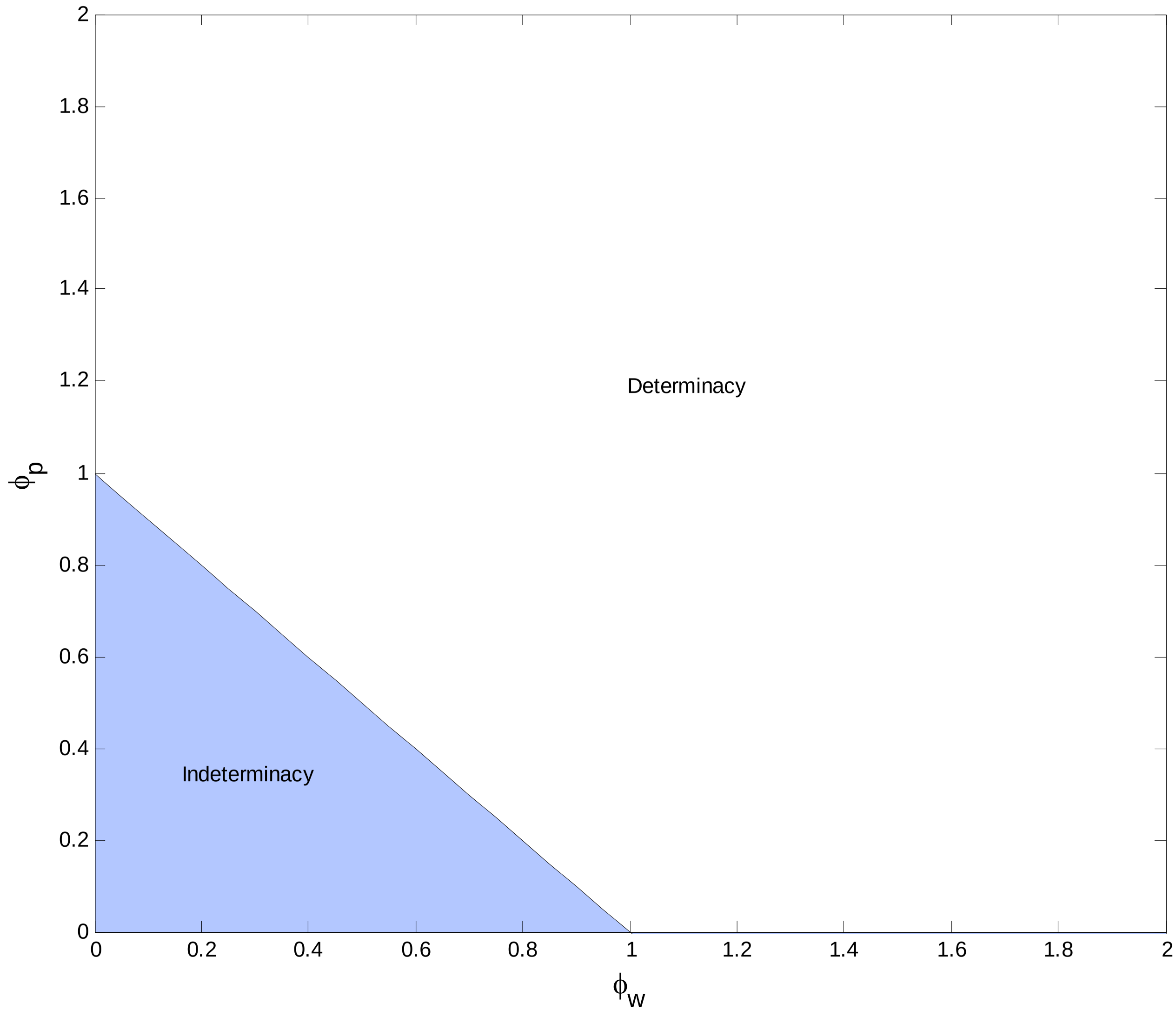
Determinacy
Indeterminacy
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
ϕ_w
ϕ_p

Figure 6.2

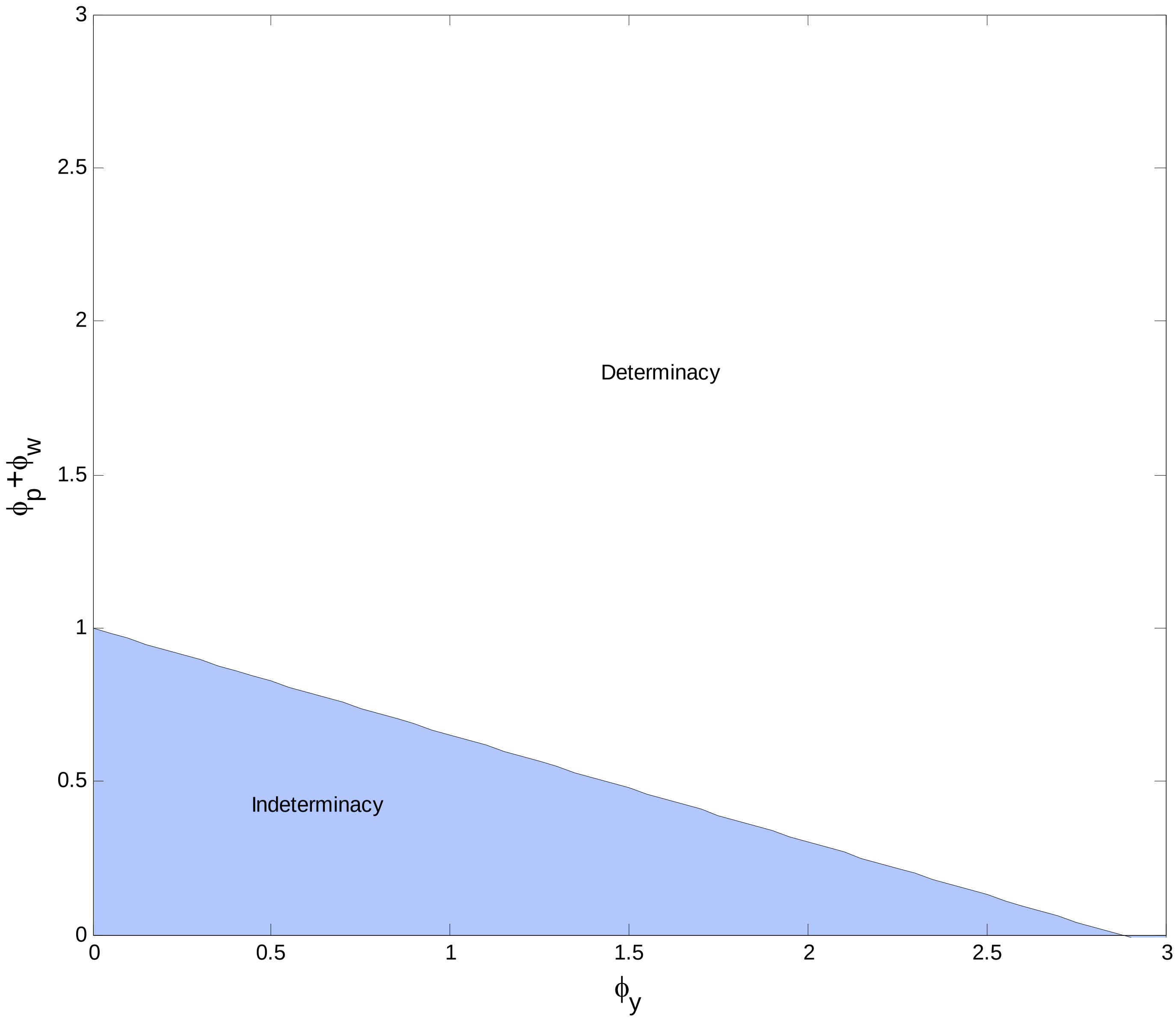

Figure 6.3: Sticky Wages and the Effects of a Monetary Policy Shock

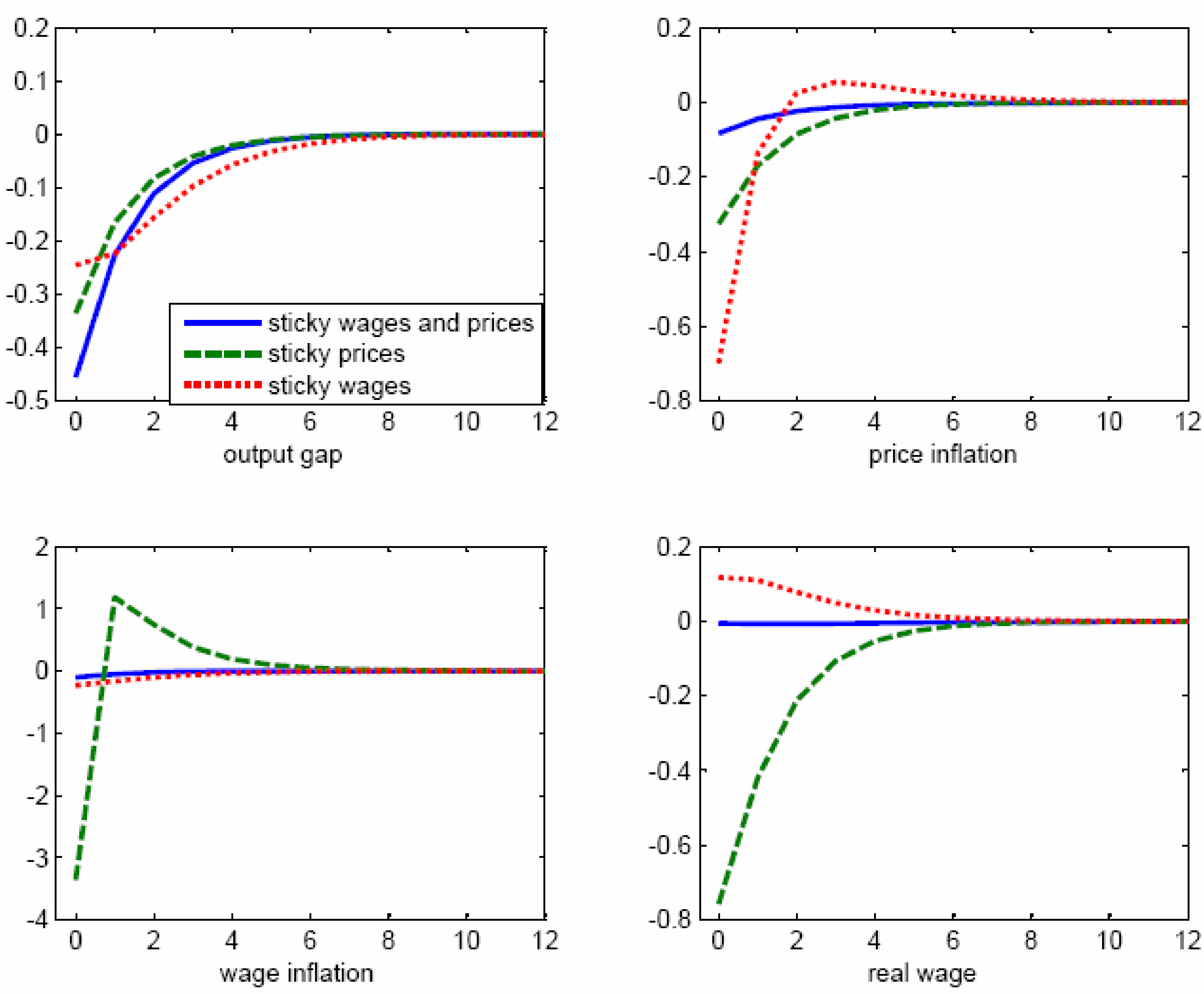

Figure 6.4: The Effects of a Technology Shock under the Optimal Policy