7

Trade Balance Dynamics

The past decade has witnessed the development of a large theoretical literature on the dynamic-optimizing (or intertemporal) approach to the current account. The models developed have typically emphasized the effects on the current-account balance of real factors such as productivity, the terms of trade, and government spending and taxes, which operate through intertemporal substitution in consumption, production, and investment. But how important is the role of intertemporal substitution? Might this micro-based theory indeed be wrong? We can answer this question by deriving the empirical implications of the theory and by proving or disproving the importance of the role played by intertemporal substitution. Although the following discussion does not engage in formal statistical testing, the numbers it presents and analyzes shed light on the validity of the intertemporal theories' key testable hypotheses.

The Mundell-Fleming approach to the macroeconomic modeling of an open economy (as Chapters 3 and 4) treats the trade balance as a side show, important only for its effect on current output. This is perhaps because it pays little attention to capital and debt accumulation. At center stage are the exchange rate, output, and employment. Recall that, under a flexible exchange rate, a current transitory fiscal expansion, which does not alter expectations about the future value of the exchange rate, induces a rightward shift of the IS schedule, raising the level of output (under the Keynesian assumption of price rigidity) and raising the domestic interest rate.

To maintain interest parity, the rise in the interest rate must result in the appreciation of the domestic currency. The current account must deteriorate, because output has risen and the domestic currency has appreciated. Under a fixed exchange rate, interest arbitrage ensures equality between the domestic and foreign interest rates. Consequently, a fiscal expansion that induces a rightward shift of the IS schedule gains full potency in raising the level of output, because there is no currency appreciation to offset it. The current account must deteriorate in this case too. Yet the links between the fiscal deficit and the trade deficit on the one hand, and between the trade deficit and the value of the domestic currency on the other, are empirically weak (see e.g., Kotlikoff, (1992, chap. 3)).

In contrast with this standard static model, the modern intertemporal optimizing approach provides a framework suitable for positive and normative analyses of current-account dynamics. The predictive content of the model is enhanced by taking explicit account of the intertemporal budget constraint and of optimization by individual households and firms.

The key factors governing the nature of the macroeconomic equilibrium differ drastically across the two models. In the static income-expenditure model, the nature of the macroeconomic equilibrium reflects the relative magnitudes of parameters measuring the effects of changes in income on spending and the demand for money. In the intertemporal model, by contrast, the nature of the equilibrium reflects parameters measuring the effects of intertemporal substitution and the debt-income position. What might we learn about certain recent episodes by following the intertemporal approach rather than the less rigorous Mundell-Fleming approach? Income-expenditure models of the Mundell-Fleming sort suggest a simple relation between the government budget and economic activity: a cut in the government deficit depresses consumption and output. In many countries, however, large cuts in government spending carried out as part of stabilization programs have led to expansions rather than contractions in economic activity and have resulted in improvements in the current-account balance (see, e.g., Giavazzi and Pagano (1990); Bertola and Drazen, (1993), and Razin and Sadka (1995)). In Denmark in the early 1980s and in Ireland in the late 1980s, the government deficit was large relative to GDP, and public debt was growing rapidly. Giavazzi and Pagano (1990) show, however, that the consumption-to-GDP ratio rose and the current account improved in the aftermath of stabilization programs that made large budget cuts. The distinct feature of the mid-1985 disinflation program in Israel was a major and severe fiscal and monetary restraints. The public sector domestic deficit fell to about zero to two percent of GNP from about 12% prior to the stabilization. Similarly, Razin and Sadka (1995) show that the fiscal consolidation resulted surprisingly in consumption and output booms. These results are inconsistent with the predictions of income-expenditure models, but are quite consistent with the predictions of intertemporal models.

A basic assumption that characterizes all intertemporal models is capital mobility. If there is no such mobility, a country cannot engage in intertemporal substitution, and there can be no intertemporal approach. It is suggestive to think in terms of a dichotomy between perfect and imperfect capital mobility. Perfect capital mobility seems to prevail, more or less, between developed countries, whereas imperfect capital mobility seems to prevail between developed and less developed countries. To the extent that this observation is true, we should expect the intertemporal model to perform better in explaining current-account fluctuations among developed countries (that is, those belonging to the OECD) than among less developed countries.

7.1 Current-Account Theory

The intertemporal approach, like the income-expenditure approach, begins with the national-income identity. Unlike earlier approaches, however, it models investment and consumption (saving) in ways that focus on intertemporal optimization and the differing effects of various shocks. It distinguishes, in particular, among four types of shocks: those that are transitory in duration, those that are persistent, those that are country specific, and those that are common across countries. Each type of shock has distinct effects on the dynamics of a country's saving-investment balance. Thus, its current account balance is driven by different shocks in distinctly different ways.

The benchmark model we use to illustrate the intertemporal approach assumes the existence of riskless assets that are traded freely, a single representative agent, and perfect competition in the goods market. Nevertheless, the main findings about the different effects of the various shocks carry over also to intertemporal models with risky assets, heterogeneous populations, and imperfect competition. The conclusions depends importantly, however, on the implicit assumption, maintained throughout, that only non-contingent borrowing is possible, because that assumption rules out diversification against country-specific shocks (see Obstfeld (1995), who examines the theory and evidence on diversification).

By the national account identity, the current-account balance is given by

$$CA_t = Y_t - Z_t - C_t + (R-1)F_{t-1},$$

where CA_t, Y_t, Z_t, C_t, R_t, and F_{t-1}, stand for period t current-account surplus, output, investment, consumption, (gross) interest rate (one plus the world rate of interest), and lagged foreign assets respectively. (Note that F is the negative of foreign (one period) debt B in equation (5.6) in Chapter 5.) We look first at the modeling of investment and then at the modeling of consumption (saving).

Investment

Consider a small open economy, producing a single aggregate tradable good (see Leiderrnan and Razin (1991); Mendoza (1991); and Glick and Rogoff (1992). The production function for that good, *Y*, is Cobb-Douglas:

$$Y_t = AK_t^{\alpha}, \tag{7.1}$$

where A, α, and K denote the productivity level, the distributive share of capital, and the capital stock, respectively. We assume that productivity shocks follow a first-order autoregressive stochastic process:

$$A_t = \rho A_{t-1} + \epsilon_t, \; 0 \le \rho \le 1, \tag{7.2}$$

where ρ and ϵ denote the persistence parameter and an i.i.d. term, with mean $\bar{\epsilon}$ respectively.

Firms maximize the expected value of the discounted sum of profits subject to the available production technology and to a cost-of-adjustment investment technology. According to the latter, gross investment (Z) is specified as

$$Z_t = I_t\left(1 + \frac{g}{2}\frac{I_t}{K_t}\right), \tag{7.3}$$

where $I_t = K_{t+1} - K_t$, and g denote net capital formation (assuming zero depreciation) and the cost-of-adjustment coefficient, respectively. Thus, in the presence of costs of adjustment, gross investment typically exceeds net capital formation, because of the costs of the reorganization and retraining associated with the installation of new capital equipment (similar to the specification (5.1) in Chapter 5).

The optimal-investment rule implies that the cost of investing an additional unit of capital in the current period must be equal to the expected present value of the next period's marginal productivity of capital *plus* the next period's induced fall in the adjustment cost of investment resulting from the enlarged stock of capital (that is, the derivative of (7.3) with respect to K) plus the residual value in the next period of the capital remaining for the entire future:

$$E_t R^{-1}\left[\alpha A_{t+1} K_{t+1}^{\alpha-1} + \frac{g}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + q_{t+1}\right] = q_t, \tag{7.4}$$

where E_t is the expectation operator based on period t information, $q_t = 1 + g(I_t/K_t)$ is the firm's market value per unit of capital (the Tobin q measure), and R is the gross interest rate. Observe that $I_t = (q_t - 1)K_t/g$, therefore $I_t \geq 0$ as $q_t \geq 1$. Thus q_t is the (marginal) Tobin q measure. Note also that $q_t Kt$ represents value of the firm, so that q is also the average Tobin q measure (see Hayashi (1982)).

At the deterministic steady state, $I_t = 0$, and the investment rule reduces to an equality between the rate of interest and the marginal productivity of capital:

$$R - 1 = \alpha \bar{A}(\bar{K})^{\alpha-1}, \tag{7.5}$$

where $\bar{A}$ and $\bar{W}$ are the steady-state levels of productivity and the stock of capital, respectively.

Linearizing (7.4) around the steady state[1] yields

$$k_t + a_0 E_t k_{t+1} + a_1 E_t k_{t+2} = b_0 - b E_t A_{t+1}, \tag{7.6}$$

where $k = K - \bar{K}$ denotes the deviation of the capital stock from its steady state level,

$$a_0 = \frac{(R-1)(\alpha-1) - g(1+R)}{R_g};$$

$$a_1 = \frac{1}{R};$$

$$b_0 = \frac{\bar{K}}{g}\left(\frac{R-1}{R}\right); \ and$$

$$b = \frac{\alpha \bar{K}^{\alpha}}{kg}.$$

The solution for k_t (see Appendix A) is given by

$$k_t = \lambda_1 k_{t-1} + \lambda_1 b \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2} \right)^i E_t A_{t+1+i} + \frac{b_0}{1 - \lambda_2} \tag{7.7}$$

where $\lambda_1 < 1$ and $\lambda_2 > 1$ are the roots of the quadratic equation $1 + a_0\lambda + a_1\lambda^2 = 0$, with $\lambda_1 + \lambda_2 = a_0$ and $\lambda_1\lambda_2 = a_1$.[2] Lagging (7.7) by one period and subtracting it from the period t equation yields the corresponding solution for the desired investment flow:

$$Z_t \cong I_t = \lambda_1 I_{t-1} + \lambda_1 b \sum_{i=1}^{\infty} \left(\frac{1}{\lambda^2} \right)^{i-1} [E_t A_{t+i} - E_{t-1} A_{t+i-1}]. \tag{7.8}$$

The first term on the right-hand side of (7.8) captures the effects on period t investment of lagged productivity shocks, and the second term captures the revisions of expectations about future productivity shocks (revisions based on the change in information from period t - 1 to period t). Because such shocks are persistent, realizations convey new information about future shocks.

Substituting (7.2) into (7.8) yields

$$I_t = \lambda_1 I_{t-1} + \left(\frac{\lambda_1 \lambda_2 b \rho}{\lambda_2 - \rho} \right) \Delta A_t \tag{7.8'}$$

where $\Delta A_t = A_t - A_{t-1}$. If the shocks are *country specific* and *permanent*, ρ in equation (7.2) is equal to 1, and we have a random walk. Substituting ρ = 1 into (7.8) yields

$$I_t = \lambda_1 I_{t-1} + \left(b \frac{\lambda_1 \lambda_2}{\lambda_2 - 1} \right) \Delta A_t. \tag{7.9}$$

Subtracting I_{t-1} from both sides yields

$$\Delta Z_t \cong \Delta I_t = (\lambda_1 - 1)\Delta I_{t-1} + \left(b\frac{\lambda_1 \lambda_2}{\lambda_2 - 1} \right) \Delta A_t . \qquad (7.10)$$

Thus, current investment is shown to be positively correlated with a permanent country-specific productivity shock.

If, instead, ρ = 0 in (7.8), the country-specific shocks are only *transitory*. Recomputing the change in investment yields

$$\Delta Z_t \cong \Delta I_t = (\lambda_1 - 1)\Delta I_{t-1} . \qquad (7.11)$$

Hence, a transitory productivity shock has no impact whatsoever on current investment.

Consider now what happens if productivity shocks are *common* to all countries. The shock will raise the world rate of interest, R - 1, whether or not the shock is persistent. If it is persistent, it will tend to raise current investment by raising future productivity, but the rise in the cost of capital will outweigh the expected rise in future productivity, thereby weakening the effect on current investment. If it is not persistent, it will affect current investment only marginally through its impact on world saving and thereby on the world rate of interest.

Consumption

We now turn to the modeling of consumption (saving). Consider the key elements of consumption behavior, based on the familiar permanent-income hypothesis (which holds only when the representative consumer has full access to world capital markets). The representative agent chooses a consumption path so as to maximize his/her lifetime utility. Using a simple functional form,

$$E_t \sum_{i=0}^{\infty} \delta^i u(C_{t+i}), \qquad u(C) = hC - \frac{1}{2}C^2, \tag{7.12}$$

subject to the constraint

$$C_t + F_t = Y_t + RF_{t-1}, \tag{7.13}$$

where δ and F denote the subjective discount factor and the stock of foreign assets, respectively. Net output, Y, accounts for the resources used up in investment (that is, Z_t has been subtracted from Y_t). Assuming, for simplicity, consumption tilting ($\delta R = 1$), the solution to the consumer's optimization problem is given by

$$C_t = \beta W_t, \qquad \beta = \frac{R-1}{R}, \tag{7.14}$$

where W_t denotes wealth, so that $W(R - 1)/R$ represents the corresponding permanent income flow. Wealth consists of the expected discounted flow of domestic income *plus* income from the initial stock of foreign assets:

$$W_t = E_t \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i Y_{t+i} + RF_{t-1}. \tag{7.15}$$

The general-equilibrium aspect of our framework is reflected by the fact that the representative agent's wealth depends on the economy-wide output stream, which is in turn determined by investment behavior. Accordingly, the realized sequence of current and future productivity shocks (and the induced investment path) are the driving forces behind consumption spending. Specifically, the linear approximation of the production function around the steady state yields

$$Y_t = d_0 + d_K K_t + d_A A_t. \tag{7.16}$$

where $d_0 = \bar{Y} = \bar{A}\bar{K}^{\alpha}; \bar{A} = \bar{\epsilon}/1 - \rho$; $d_K = k - 1$; and $d_A = \bar{K}^{\alpha}$. Substituting (7.16), together with (7.2) and (7.7), into the wealth term in (7.14) and (7.15) yields the closed-form solution for current consumption spending as a function of the observable (current and past) productivity levels and of foreign asset holdings as follows (see Appendix B):

$$\begin{aligned} C_t = {} & Y_{t-1} + \frac{R}{R-\lambda_1} d_K I_{t-1} + \left(\frac{R-1}{R-\rho} d_A + \frac{d_K}{k-\lambda_1} \frac{\lambda_1 \lambda_2 b \rho}{\lambda_2 - \rho} \right) \Delta A_t \\ & + \frac{\rho - 1}{k - \rho} d_A (A_{t-1} - \bar{A}) + (R-1) F_{t-1} \end{aligned} \tag{7.14$'$}$$

Consider specifically the effects on consumption of *persistent* country-specific productivity shocks, representing persistence by the extreme case $\rho = 1$. Writing (7.16) in first-difference form and substituting (7.2) and (7.9) into the resulting expression yields

$$\Delta Y_t = (\lambda_1 - 1) d_K \Delta I_{t-1} + \left(\frac{\lambda_1 \lambda_2}{\lambda_2 - 1} b d_K + d_A \right) \Delta A_t. \tag{7.17}$$

Writing (7.14)' in first-difference form for $\rho = 1$ yields

$$\begin{aligned} \Delta C_t = {} & \left\{ \frac{\lambda_1 \lambda_2}{\lambda_2 - 1} b d_K \left(1 + \frac{\lambda_1}{R} \frac{(R-1)}{(R-\lambda_1)} \right) + d_A \right\} \Delta A_t \\ & + (R-1) \Delta F_{t-1} \end{aligned} \tag{7.18}$$

Observe that the coefficient of ΔA_t in (7.18) is larger than the corresponding coefficient in (7.17). The economic intuition is straightforward. The effect of a productivity change (ΔA_t) on current consumption is subject to two reinforcing influences: first, if investment is held constant in response to the shock, current income and current consumption should rise by equal amounts; this effect is captured by the term $\lambda_1\lambda_2 b d_K/(\lambda_2 - 1) + d_A$ in (7.17) and (7.18); second, the productivity shock (ΔA_t), however, raises the entire expected future investment path and thus leads to a larger future capital stock and larger future income. Consequently, permanent income (and, along with it, current consumption) should rise by more than current income. This effect is captured by the term $\{\lambda_1(R - 1)/[R(R - \lambda_1)]\}[\lambda_1\lambda_2 b d_K/(\lambda_2 - 1)]$ in (7.18).

Consider, instead, a *transitory* productivity shock ($\rho = 0$). It follows from (7.11) that investment is not affected at all, and the change in wealth must therefore equal the transitory increment to current income with no change in future expected income. Indeed, substituting $\rho = 0$ into (7.14)' yields

$$\Delta C_t = (R-1)\left(\Delta F_{t-1} + \frac{d_A}{R}\Delta A_t\right). \tag{7.19}$$

Now, comparing (7.18) and (7.19), it is evident that transitory shocks have relatively weak effects on current consumption. This is also in line with standard consumption theory.

It is noteworthy that disturbances other than productivity shocks, such as changes in government spending, can be incorporated by making only slight modification in the framework. Recall that, even under Ricardian assumptions, government spending can have real effects in an intertemporal framework. Under Ricardian equivalence, an increase in government spending that is fully anticipated reduces a household's wealth and consumption and thus affects the current account. Its effects are weaker in the absence of Ricardian equivalence or when the increase is not fully anticipated (see also Backus, Kehoe, and Kydland [1992], which looks at the effects of temporary and permanent changes in government spending.)

Current Account

Substituting the solutions for Y_t, C_t, and Z_t into the definition of the current-account balance; we get

$$CA_t = -\lambda_1\left(\frac{d_k}{R-\lambda_1}-1\right)I_{t-1} - \left[\frac{\rho-1}{R-\rho}d_A + \frac{\lambda_1\lambda_2 b\rho}{\lambda_2-\rho}\left(\frac{d_K}{R-\lambda_1}+1\right)\right]\Delta A_t \\ - \frac{\rho-1}{R-\rho}d_A(A_{t-1}-\bar{A}) \tag{7.20}$$

With ρ = 1, the effects on the current account of persistent country-specific shocks are given by:

$$\Delta CA_t = \left\{-\frac{\lambda_1\lambda_2}{\lambda_2-1}\, b d_K\, \frac{\lambda_1}{R}\, \frac{(R-1)}{(R-\lambda_1)} - \frac{\lambda_1\lambda_2}{\lambda_2-1}\, b\right\}\Delta A_t \\ + (R-1)CA_{t-1} - \lambda_1\left(\frac{d_K}{R-\lambda_1} - 1\right)\Delta I_{t-1} \tag{7.21}$$

The coefficient of ΔA_t in (7.21) is negative. Consequently, a permanent country-specific productivity-enhancing shock must, for two reasons, worsen the cur rent account. First, it causes investment spending to rise. Second, it causes current consumption spending to rise by more than the current rise in output. This means that the current account has to be negatively correlated with persistent country-specific productivity shocks.

When shocks of this sort are *not* persistent (ρ = 0), however, consumption responds only weakly, and investment does not respond at all. Substituting ρ = 0 into (7.20) yields

$$\Delta CA_t = \frac{d_A}{R}\Delta A_t + (R-1)CA_{t-1} - \lambda_1\left(\frac{d_K}{R-\lambda_1}-1\right)\Delta I_{t-1} \tag{7.22}$$

The positive coefficient of the productivity term implies that a positive transitory productivity shock tends to move the external balance into surplus. This means that the current account has to be positively correlated with nonpersistent country-specific shocks.

We have shown that an extreme transitory country specific shock ($\rho = 0$) generates a positive correlation between current account position and domestic output, whereas an extreme permanent shock ($\rho = 1$) leads to a negative correlation. In practice, we do not have either one of the two extreme degrees of persistence. In order to determine the condition under which such correlation is positive or negative, we calculate a critical value of ρ, ρ^*, which generates a zero correlation. From (7.20), define

$$F(\rho) = -\frac{\rho - 1}{R - \rho} - \frac{\rho\alpha}{(\lambda_2 - \rho)g}\left[\frac{R-1}{R-\lambda_1} + 1\right].$$

Observe that $F(0) > 0$ and $F(1) < 0$. Then, $F(\rho^*) = 0$. If $\rho > \rho^*$, the current account-output correlation is negative; while if $\rho < \rho^*$, the correlation is positive.

A *global* shock that affects all countries should have a significantly different impact on the external balance than would a country-specific shock. A persistent productivity-enhancing shock common to all countries will raise the world rate of interest. The rise in the interest rate should dampen the increases in current consumption and investment spending that would be produced by a comparable country-specific shock. Thus, the response of the current account to a persistent global shock must be smaller than the response to a country specific shock. In a world of identical countries, in fact, the ultimate change in the world rate of interest produced by a global productivity-enhancing shock must rule out any observable change in any country's current-account balance, because all countries cannot experience simultaneous improvements in their current accounts. A global nonpersistent positive shock generates excess world saving and thereby exerts a downward pressure on the world rate of interest, which, in turn, will stimulate current spending. Consequently, the response of the current account to a transitory global shock must be weaker than the response to a transitory country-specific shock. The compositional point holds here as well, for, if all countries are identical, the ultimate change in the world interest rate in response to a global shock must be just large enough to prevent any change in any country's current account.

7.2 Correlations between Saving and Investment

The typical impulse response of saving (that is, the difference between output and consumption), investment, and the trade balance to a positive, but not fully persistent, productivity shock is presented in Figure 7.1. This pattern of saving, investment and trade balance is consistent with the behavior of output (7.16), consumption (7.14)', investment (7.8)', and the current account balance (7.20). There is a positive impact effect on saving and a downward monotonic adjustment back to the initial equilibrium, reflecting the fact that consumption is smoothed relative to output. The impulse response of investment shown in the figure indicates a large positive impact effect, followed by a sharp drop and a monotonic convergence to the initial equilibrium, reflecting the intertemporal substitution in investment induced by the shock. The trade balance fluctuates, first deteriorating, then improving, and finally converging to the initial equilibrium, reflecting the alternating positions of saving and investment.

Insert Fig 7.1

These patterns explain why the covariance between saving and investment is typically *positive* under the assumption of perfect capital mobility (see Obstfeld (1986)). Recall that the covariance expression includes a quadratic term, the product of saving and investment. Therefore, observations involving large deviations from the initial equilibrium, such as the positive impact effects, take on large weights in the covariance formula, and the covariance becomes positive when the time spent at each point on the impulse function is the same. By implication, a positive covariance between saving and investment should not necessarily be interpreted as an indication of capital immobility, as was argued by Feldstein and Horioka (1980). In fact, the narrow differentials between interest rates on offshore and onshore assets denominated in the same currency indicate that capital mobility is more nearly perfect than zero among the developed countries. Furthermore, the observed positive covariance does not pose a challenge to the intertemporal approach, because it is in fact predicted by that approach.

7.3 Real-Exchange-Rate Theory

Up to this point, we have assumed that all goods are traded in world markets. In this sub-section, we introduce goods that are not traded; their relative prices are determined exclusively in the domestic economy. In this case, macroeconomic shocks have domestic effects additional to those discussed in the previous section because they affect the relative prices of non-traded goods (that is, the inverse of the real exchange rate).

The intertemporal approach provides important insights into the time-series properties of the real exchange rate, the relative price of tradable in terms of non-tradable. Following recent intertemporal models of the trade balance and the real exchange rate (see Razin, (1984); Mendoza, (1992); Rebelo, (1992a), and especially Rogoff (1992)), we assume a stylized two-sector model of a small open economy. Preferences over consumption of tradable, C^T, and non-tradables, C^N, are represented by a Cobb-Douglas intertemporal utility function (similar to the specifications in Chapter 6):

$$V(C^T, C^N) = (C^T)^{1-\gamma}(C^N)^{\gamma}. \qquad (7.23)$$

Equality between the marginal rate of substitution between tradables and non-tradables and their corresponding relative price implies,

$$P = \frac{(1-\gamma)C^N}{\gamma C^T}, \tag{7.24}$$

where P denotes the relative price of non-tradable in terms of tradables and is thus the real exchange rate.

The representative agent is infinitely lived and seeks to maximize

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \left(V_t^{1-\sigma} - 1 \right) \right]. \tag{7.25}$$

Sectoral outputs are represented by Cobb-Douglas production functions:

$$Y^T = A^T (K^T)^{1-\alpha} (L^T)^{\alpha}, \tag{7.26}$$

$$Y^N = A^N (K^N)^{1-\nu} (L^N)^{\nu}, \tag{7.27}$$

Intersectoral Factor Mobility

The classic model of the real exchange rate, which was developed by Balassa (1964) and Samuelson (1964), assumes that capital and labor can move freely between sectors and capital is internationally mobile. The model thus represents an economy in a long-run equilibrium, with free capital flows. Given the common wage and rental rates in the two sectors and the fact that the rental rate of capital in the tradable sector is nailed down by the world rate of interest, the standard profit-maximization conditions imply

$$dp = \left(\frac{\nu}{\alpha} \right) da^T - da^N, \qquad (7.28)$$

where a lower-case letter denotes the logarithm of a variable indicated by the corresponding upper-case letter. (For derivation of (7.28), see Appendix C.) This equation asserts that the path of the logarithm of the real exchange rate is completely determined by the productivity shocks da^T and da^N, regardless of the aggregate-demand conditions. Thus, a productivity increase in the tradable sector leads to a real appreciation. Under purchasing-power parity holding for tradable goods, the domestic inflation rate is driven exclusively by shocks to the outputs of tradables and non-tradables, as indicated by (7.27). Therefore, intersectoral factor mobility implies that the real exchange rate is highly sensitive to shocks to the output of the traded good, and, to the extent that these shocks are transitory, the real exchange rate will display a relatively low degree of persistence.

Sector-Specific Factors

The polar opposite to the above-mentioned case is that in which factors are intersectorally immobile. That case can be viewed as describing an economy in short-run equilibrium and thus explaining month-to-month fluctuations of the real exchange rate. As has been emphasized by Rogoff (1992), the equilibrium real exchange rate responds in the short run mainly to aggregate demand shocks in a way that is akin to the behavior of consumption, which smooths out transitory shocks to income.

The intertemporal smoothing of expected marginal utility implies that

$$(x_t)^{1-\gamma}(V_t)^{-\sigma} = \beta R E_t (x_{t+1})^{1-\gamma}(V_{t+1})^{-\sigma}, \qquad x = \frac{C^N}{C_T}. \qquad (7.29)$$

Setting aside shocks to the supply of nontradable goods (so that equilibrium C^N is constant) and assuming no consumption tilting (so that $\delta R = 1$), we can substitute (7.24) into (7.29) to get

$$P_t^{[1-\gamma(1-\sigma)]} = E_t P_{t+1}^{[1-\gamma(1-\sigma)]}. \qquad (7.30)$$

Approximating the exponential term P^x for any parameter x by the linear term $(1 + xp)$, where p denotes the logarithm of P, we can rewrite (7.30) as

$$p_t = E_t p_{t+1}.$$

Thus, the logarithm of the real exchange rate will follow a random walk, regardless of the underlying shocks to the traded-goods sector.[3] Intersectoral factor mobility implies, therefore, that the time series of the real exchange rate will display a relatively high degree of persistence.

7.4 Evidence on Persistence and the Commonality of Shocks

Having set out the theory to highlight the relevant issues, we shall proceed in this sub-section to look at some evidence. We shall be concerned with two types of empirical work, which identify the nature of shocks and the testable implications of the dynamic-optimizing (intertemporal) approach.

Drawing on Razin and Rose (1992), we provide some evidence on the time series nature of the shocks that operate on output, consumption and investment (for similar work, focused on segregating global and country-specific shocks, see Glick and Rogoff (1992)). The data set comprises 138 countries and spans the period from 1950 to 1988. It is taken from the Penn World Tables, documented in Summers and Heston (1991).

PERSISTENCE

To address the issue of persistence, Razin and Rose (1992) computed simple Dickey-Fuller tests for (the logarithms of) each of our variables. At conventional levels of statistical significance, the data typically do not reject the hypothesis that a single unit root exists in the univariate representations of output, consumption, and investment. Razin and Rose ran separate tests for consumption, output, and investment and for each of the 138 countries; of these, eighteen tests (4.5 percent) rejected the null hypothesis of a unit root at the 5 percent significance level, and five tests (1.3 percent) rejected the null hypothesis at the 1 percent significance level. These results are quite close to what would be expected under the null hypothesis, implying that the data are consistent with the existence of unit roots in the autoregressive representations of the variables.

It is well known that such tests have low power against stationary alternatives and that there are serious problems in interpreting the test results as demonstrating a high degree of persistence. Thus, we should view the findings as being consistent with a high degree of persistence in shocks but by no means as definitive.

COMMONALITY OF SHOCKS

The models developed earlier in this chapter indicate that the dynamics of the saving-investment balance should depend critically on whether shocks are country specific or common across countries. Accordingly, Razin and Rose (1992) tested for the nature of the shocks using standard factor-analytic techniques. The factor analysis was performed across countries on the detrended measures of output, consumption, and investment. The results are given in Table 7.1. Because the national-accounts data in the Penn World Tables are sometimes unavailable for the entire 1950-88 period, Table 7.1 provides results for two sets of countries: those with at least 20 annual observations and those with at least 35 observations; results for the different sets of countries are quite comparable.

Insert Table 7.1.

Factor-analysis results depend critically on the method of detrending. When the variables are detrended by using the standard linear trend (TS) method, four factors (those corresponding to the largest four eigenvalues) typically account for around three-quarters of the variation in all three series; the first factor alone accounts for over one-third of the total variation. This finding may indicate that only a small number of important shocks have been common across countries. The fractions fall by approximately one-half, however, when the first-differencing (DS) method of detrending is employed (a method that implicitly adopts a random-walk model of trend).

To summarize, the evidence indicates that many business-cycle shocks are both persistent and common to many countries.

Volatility, Persistence, and Correlations

Intertemporal models predict that the degree of capital-market integration and the nature of shocks are key determinants of the volatility of consumption (saving), investment, and the current account. In this subsection, we provide time-series evidence on current-account dynamics so as to shed some light on the empirical importance of the effects identified by the theoretical models discussed in Sections 7.1 through 7.3.

Volatility measures for the current account (as a percentage of GDP) and for the logarithms of per capita GDP are exhibited in Figure 7.2 for a sample of fifty-eight countries. The data pertain to the period from 1967 to 1990; as before, they come from the Penn World Tables (Mark 5). To measure volatility, we use the standard deviation of the (first-difference) detrended variable. Each country is identified by the first two or three letters of its name.

Insert Figure 7.2

There is a cluster of mainly developed countries and fastest-growing less developed countries that show relatively low current-account and output volatility; this group includes countries such as Japan and Indonesia. The group with high current-account volatility and low output volatility includes countries such as Venezuela and Iran, which are major oil producers.

Two major conclusions can be gleaned from Figure 7.2. The less-developed countries show more current-account and output volatility than do the developed countries, and the ratio of current-account volatility to output volatility (measured by the slope of a ray from the origin that fits the cluster of observations) is not markedly different for less-developed and developed countries.

Table 7.2 provides a set of statistics describing the time-series properties of the trade balance, output, the terms of trade, the real effective exchange rate, and the interest rate for each of the seven largest developed countries and for a sample of less-developed countries. It reports measures of volatility and persistence and the correlations between pairs of variables (see also Mendoza, (1992)). Observe that relative price changes (such as changes in the terms of trade, the real exchange rate, and the rate of interest) cause income effects for the country akin to shifts in output, in addition to the direct substitution effects. Thus, for example, because a deterioration in the terms of trade means that, with the same quantity of exports, the country is able to import reduced amounts of goods and services from abroad, real income falls. The distinction between temporary and permanent changes is as relevant here as for the case of output shocks. The temporary versus permanent distinction is also relevant for the intertemporal substitution effect (as, e.g. in Svensson and Razin in (1983) and Obstfeld (1982).

Insert Table 7.2

The main regularities shown in Table 7.2 can be summarized as follows.

(1) There is a significant degree of persistence in output, the terms of trade, and the real exchange rate, a finding similar to our earlier conclusion based on the Penn World Tables.

(2) The trade balance is in most cases more volatile than the terms of trade or output.

(3) The trade balance and the terms of trade are positively correlated for most of the countries, in line with the Harberger-Laursen-Metzler effect. Recall that this proposition predicted that a deterioration of the terms of trade would reduce saving. According to the intertemporal approach, a temporary deterioration of the terms of trade will induce substitution from current to future consumption (that is, will increase saving), but a permanent deterioration will not.

(4) Looking across countries, one cannot detect the link predicted by the theory between the persistence of output or terms-of-trade shocks and the correlation between the trade balance on the one hand and the terms of trade or output on the other. It seems that a more structural, econometric approach is needed to test the validity of this implication of the intertemporal approach. It should be noted, however, that Mendoza (1992) reproduced the expected relationship by a different method. He constructed two benchmark economies to characterize a "typical" developed country and a "typical" less-developed country. Conditioning them with empirically based parameters pertaining to terms-of trade shocks, he was able to simulate the Harberger-Laursen-Metzler effect whereby the persistence parameter of the terms-of-trade shocks is positively associated with the correlation between the trade balance and the terms of trade.

(5) The real rate of interest and terms of trade are more volatile for less developed than for developed countries, and the volatility of the trade balance is also significantly larger for less-developed than for developed countries.

(6) The correlation between the rate of interest and the trade balance is positive for most countries. This is consistent with the presence of intertemporal substitution; the current-account balance will improve if a rise in the interest rate reduces current spending on consumption and investment and augments future spending.

(7) The real exchange rate is only weakly correlated with the trade balance. In contrast to the Mundell-Fleming model, however, the intertemporal model does not make a clear prediction concerning this correlation.

(8) The real exchange rate shows a high degree of persistence and a relatively low correlation with the terms-of-trade shocks. This may support the validity of the consumption-smoothing model of the real exchange rate discussed in the second part of Section 7.3.

(9) Finally and most importantly, the trade balance is in most cases negatively correlated with output. Recall that a permanent country-specific shock worsens the trade balance for two reasons. First, it raises investment; second, it causes current consumption to rise by more than the current rise in output. This finding is therefore in line with the predictions of the intertemporal model in Sections 7.1 and 7.2.

Sachs (1981) investigated nonstructural regressions describing the behavior of the current account for both developed and less-developed countries. He emphasized that most of the explanatory power of his regressions was the result of an investment surge that led to current-account deficits; saving rates changed little. Further developments in theory and methodology have facilitated structural testing.

Structural Testing

A full-blown optimizing model is difficult to estimate because it is often impossible to reduce it to a small number of tractable equations. There have been, however, a few attempts at empirical implementation.

The intertemporal model predicts that shocks that are persistent and common to all countries (that is, formed by a GNP-weighted average of the individual productivity measures) have no effect on the trade balance. To test this proposition, Glick and Rogoff (1992) computed the Solow residuals for each country and broke them down into country-specific and global shocks and into transitory and persistent shocks. They found that the various shocks enter current-account regressions with the predicted signs. The hypothesis stood up to the annual data of eight developed countries for the period from 1960 to 1990. In particular, Glick and Rogoff found that the coefficient of the productivity variable in their trade-balance equation was, as predicted, larger than the corresponding coefficient in their investment equation. They, however, did not rigorously incorporate the cross-equation restrictions implied by the theory, and the fit of their regression equations was weak in several cases.

Leiderman and Razin (1991) estimated an intertemporal model for Israel using monthly data for the 1980s. They found strong evidence in favor of consumption smoothing (indicated by an offsetting response of private saving to changes in government saving and only a small proportion of liquidity constrained consumers), as well as a strong response of investment to country-specific productivity shocks.

Mendoza (1991, 1992) provides recursive simulations based on a calibrated model with empirically based parameters that lend support to the emphasis that the intertemporal approach attaches to the persistence of shocks and to consumption smoothing.

Finally, Razin and Rose (1992) provide indirect tests of the intertemporal approach. The approach predicts that capital-market integration will lower consumption volatility while raising investment volatility to the extent that productivity shocks are idiosyncratic and non-persistent. They use a unique panel data set (ranging from the 1950s to the late 1980s and covering developed as well as less-developed countries); it includes indicators of barriers to trade in goods and (financial) capital. The results of their study are inconclusive, for they did not find a strong link between business-cycle volatility and openness. Countries with greater capital mobility (that is, fewer barriers to trade in financial assets), for instance, do not appear to have systematically smoother consumption streams or more volatile investment behavior.

7.5 Conclusion

In recent years, we have seen large, unsynchronized changes in national fiscal policies, and these have resulted in substantial budgetary imbalances, volatile real rates of interest and real exchange rates, and large current-account imbalances. The intertemporal approach provides a framework for analyzing these fiscal (and productivity) shocks and offers a coherent theory that can potentially account for the observed diversity of current-account balances. This chapter has illustrated the use of this approach in analyzing current-account dynamics and has reviewed the evidence supporting it.

The intertemporal approach begins with the national-income identity and with detailed descriptions of the intratemporal and intertemporal budget constraints faced by the decisionmaking units. It models investment and consumption (saving) in ways that emphasize intertemporal optimization and the differing effects of various shocks and shows the importance of distinguishing among four types of shocks. These can be transitory or persistent in duration, country-specific or common across countries. Because different shocks have different effects on the saving-investment balance, they have different effects on the trend and volatility of the current-account balance.

Are there easier ways to explain current-account behavior? Can one take shortcuts that are simpler to implement than the rigorous modem approach? A popular method of applied analysis is to regress the current-account balance on such "price" variables as the real exchange rate and interest rates and on such "income" variables as output, government spending, tax-burden indicators, government debt, and money creation. The typical regression uses mostly current variables, except that lagged output is added to function jointly with current output as a proxy for permanent income. Most applied work, however, still emphasizes income and price elasticities of demand for exports and imports, a practice that can be rationalized only by invoking a one-period partial-equilibrium model.

Traditional studies test debt neutrality by asking whether regression coefficients on taxes and debt are significantly different from zero. Similarly, they test whether the exchange rate is effective in improving the trade deficit by the sign and statistical significance of the coefficient of the real exchange rate, allowing possibly for simultaneous-equations bias by the use of instrumental variables. This sort of reduced-form analysis, however, omits all of the variables suggested by the intertemporal model. It also fails to distinguish between the different types of shocks or between types of taxation (that is, taxes on capital income, labor income, or consumption). Accordingly, reduced form regression analyses of the trade balance are not likely to provide relevant information on the validity of debt neutrality, the sensitivity of the current account to exogenous or policy-induced changes in the exchange rate or the rate of interest, or on a host of other policy-related issues. That is because they ignore an important possibility. If current taxes are a good predictor of future government spending, a tax coefficient significantly different from zero will be consistent with the neutrality proposition and contrary to the traditional interpretation. Furthermore, a large positive current-output coefficient may indicate the presence of persistent productivity shocks, which play no role in the traditional approach.

The empirical implementation of the intertemporal approach has not been widespread, because intertemporal models are inherently intractable and demand much data. Nevertheless, there have been recent attempts to test some of the key hypotheses of this approach, and, as indicated in this chapter, the results are quite encouraging.

A drawback of other existing approaches is their inability to account for changes in the fiscal or monetary regime. An increase in the stock of government bonds, for example, may signal a future increase in taxes, because an increase will be needed to service the new debt. But the increase in debt may also signal a future fall in government spending or forthcoming monetary accommodation and inflation. Current econometric methods cannot distinguish between different types of regime change, with different implications for the debt-neutrality question and other important hypotheses. Innovations in the theory of endogenous policy should prove useful for this purpose.

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Appendix A: Solution to Second Order Stochastic Difference Equations

This appendix reproduces the backward and forward expansion solution methods for second order stochastic difference equations (based on Sargent (1987)). These methods are useful for solving a variety of stochastic dynamic problems. They are applied in this chapter as well as in Chapter 7.

A typical second order stochastic difference equation (SDE) takes the following form:

$$E_tY_{t+1} - a_1Y_t - a_2Y_{t-1} = b_1 + b_2E_tX_{t+1}, \qquad \text{(SDE)}$$

Define the lag (or backward shift) operator by $L^nZ_t = Z_{t-n}$ for n = 1, 2, 3, ... Then the above equation can be rewritten as:

$$(1 - a_1L - a_2L^2)E_tY_{t+1} = b_1 + b_2E_tX_{t+1}. \qquad \text{(A.1)}$$

Factorizing the polynomial expression in parenthesis on the left hand side yields

$$1 - a_1L - a_2L^2 = (1-\lambda_1L)(1-\lambda_2L),$$

where λ_1 and λ_2 are the roots of the polynomial, with $\lambda_1 + \lambda_2 = a_1$ and $-\lambda_1\lambda_2 = a_2$. If these are distinct roots with $\lambda_1 < 1$ (stable) and $\lambda_2 > 1$ (unstable), we can apply the factorization to (A.1) by dividing both sides by $1-\lambda_2L$ to get

$$(1 - \lambda_1L)E_tY_{t+1} = \frac{1}{1 - \lambda_2L}(b_1 + b_2E_tX_{t+1}). \qquad \text{(A.2)}$$

Notice that, formally,

$$\frac{1}{1-\lambda_2 L} = \frac{-(\lambda_2 L)^{-1}}{1-(\lambda_2 L)^{-1}} = \frac{-1}{\lambda_2 L}\left[1+\left(\frac{1}{\lambda_2}\right)L^{-1}+\left(\frac{1}{\lambda_2}\right)^2 L^{-2}+\ldots\right]$$

$$= -\left(\frac{1}{\lambda_2}\right)L^{-1}-\left(\frac{1}{\lambda_2}\right)^2 L^{-2}-\left(\frac{1}{\lambda_2}\right)^3 L^{-3}-\ldots$$

Applying this expansion to the right hand side of (A.2) yields

$$(1-\lambda_1 L)E_t Y_{t+1} = \frac{b_1}{1-\lambda_2} - b_2 E_t\left[\sum_{i=0}^{\infty} \lambda_2^{-i} X_{t+1+i}\right] + c\lambda_2^{t+1}. \qquad \text{(A.3)}$$

Imposing the transversality condition, the constant c has to be set to zero. Applying the backward shift, using the stable root λ_1 on the left hand side of (A.3), and moving this term to the right hand side yields the final solution to (SDE) as follows:

$$E_t Y_{t+1} = \lambda_1 Y_t + \frac{b_1}{1-\lambda_2} - b_2 E_t\left[\sum_{i=0}^{\infty} \lambda_2^{-i} X_{t+1+i}\right]. \qquad \text{(SSDE)}$$

Appendix B: Derivation of the Optimal Consumption Rule (7.14)'

In this appendix, we derive the optimal consumption rule (7.14)'. Rewrite the Y_{t+i} term in the wealth equation (7.15) as

$$Y_{t+i} = Y_t + \sum_{j=1}^{i} \Delta Y_{t+j}.$$

Substituting this into (7.15) (ignoring the foreign asset term), we get

$$W_t = \frac{R}{R-1}\left(Y_t + E_t\sum_{j=1}^{\infty} \frac{\Delta Y_{t+j}}{R^j} \right). \qquad \text{(B.1)}$$

Lagging (7.16) by one period and subtracting it from (7.16) yields

$$Y_t = Y_{t-1} + d_k I_{t-1} + d_A \Delta A_t. \qquad \text{(B.2)}$$

Substitute this into (B.1), we get

$$\frac{R-1}{R}W_t = Y_t + d_k E_t\sum_{i=1}^{\infty} \frac{I_{t+i-1}}{R^i} + d_A E_t\sum_{i=1}^{\infty} \frac{\Delta A_{t+i}}{R^i}. \qquad \text{(B.3)}$$

Decomposing δA_{t+j} as $(A_{t+j}-A)-(A_{t+j+1}-A)$ and subsituting it along with (7.8)' into (A2.3), we have

$$\frac{R-1}{R}W_t = Y_t + \frac{d_k}{R}\sum_{i=1}^{\infty} \frac{I_t\lambda_1^i}{R^i} + d_A E_t\left(\sum_{i=1}^{\infty} \frac{(A_{t+i}-\bar{A})\rho^i}{R^i} - \sum_{i=1}^{\infty} \frac{(A_{t+i-1}-\bar{A})\rho^i}{R^i} \right).$$

Expanding the geometric series and writing A_t-A as $(\delta A_t+A_{t-1}-A)$, we obtain

$$C_t = \frac{R-1}{R}W_t = Y_t + \frac{d_k}{R-\lambda_1}I_t + \frac{\rho-1}{R-\rho}d_A(\Delta A_t + A_{t-1} - \bar{A}).$$

Substituting (7.8)' and (B.2) into this equation yields the optimal consumption rule (7.14)' in the text.

Appendix C: Derivation of the Balassa-Samuelson Effect (7.28)

In this appendix, we derive the Balassa-Samuelson effect as revealed by (7.28). Denoting the wage and rental rates by W and R respectively, we have from the Cobb-Douglas production functions (7.26) and (7.27) and the profit-maximizing conditions

$$W^T = \alpha A^T \left(\frac{K^T}{L^T} \right)^{1-\alpha}. \tag{C.1}$$

$$R^T = (1-\alpha) A^T \left(\frac{K^T}{L^T} \right)^{-\alpha}. \tag{C.2}$$

$$W^N = \nu A^N \left(\frac{K^N}{L^N} \right)^{1-\nu}. \tag{C.3}$$

$$R^N = (1-\nu) A^N \left(\frac{K^N}{L^N} \right)^{-\nu}. \tag{C.4}$$

Intersectoral factor mobility implies

$$W^T = PW^N. \tag{C.5}$$

$$R^T = PR^N. \tag{C.6}$$

The international mobility of capital implies

$$R^T = R^*, \tag{C.7}$$

where R^* is the world rate of interest.

Substituting (C.2) into (C.7) to solve for K^T/L^T, we get

$$\frac{K^T}{L^T} = \left(\frac{(1-\alpha)A^T}{R^*}\right)^{\frac{1}{\alpha}}. \tag{C.8}$$

$$\alpha A^T\left(\frac{(1-\alpha)A^T}{R^*}\right)^{\frac{1-\alpha}{\alpha}} = P\nu A^N\left(\frac{(1-\nu)A^N P}{R^*}\right)^{\frac{1-\nu}{\nu}}. \tag{C.9}$$

Substituting (C.4) and (C.7) into (C.6) yields

Substituting (C.8) into (C.1) and (C.9) into (C.3) and the resulting expressions into (C.5), we have

$$\frac{K^N}{L^N} = \left(\frac{(1-\nu)A^N P}{R^*}\right)^{\frac{1}{\nu}}. \tag{C.10}$$

Taking logs and collecting terms, we get

$$p = \frac{\nu}{\alpha}a^T - a^N + \nu\left(\frac{1-\nu}{\nu} - \frac{1-\alpha}{\alpha}\right)r^* + constant. \tag{C.11}$$

Taking first differences while keeping the world rate of interest constant yields (7.28) in the text.

Problems

1. Consider the model in Section 7.1. Compute the correlation coefficient between savings ($Y_t - C_t$) and investment (Z_t) motivated by the Feldstein-Horioka puzzle. Based on the computed coefficient, discuss alternative tests which can discriminate between the Feldstein-Horioka segmented capital market hypothesis and the integrated capital market hypothesis underlying the model of Section 7.1.

2. Consider the model in Section 7.1. Recall that ρ^* is defined as the critical value of the persistence parameter which generates a zero correlation between the trade balance and output. Show how ρ^* depends on the deep parameters of the model: R, α, and g. Provide an economic interpretation.

3. Consider the model in Section 7.3 with intersectoral labor mobility. Derive the relation between real wages and the productivity shocks in the traded and non-traded goods sectors. Compare this relation to (7.28) and provide an interpretation.

4. Consider the real exchange rate smoothing property reflected in (7.30). Design an empirical test for this property. How are deviations from this property related to intersectoral wage differences?

Endnotes

1. To derive a linear approximation to equation (7.4), rewrite (7.4) using the definition of q_t (= $1 + g(I_t/K_t)$) and the definition of I_t (= K_{t+1} - K_t) as:

$$E_t R^{-1}\left[\alpha A_{t+1}K_{t+1}^{\alpha-1} + \frac{g}{2}\left(\frac{K_{t+2}-K_{t+1}}{K_{t+1}}\right)^2 + 1 + g\left(\frac{K_{t+2}-K_{t+1}}{K_{t+1}}\right)\right] = 1 + g\left(\frac{K_{t+1}-K_t}{K_t}\right).$$

In the deterministic steady state, $K_t = K_{t+1} = \bar{K}$, and R-1 = $\alpha A K^{\alpha-1}$. Applying the Taylor expansion (up to the first order) around the steady state using $k_t = K_t - \bar{K}$ to the above equation yields (7.6) in the text.

2. Define $h(\lambda) = 1 + a_0\lambda + a_1\lambda^2 = 0$. Note that $h(1) < 0$ and $h(0) > 0$. Therefore, $0 < \lambda_1 < 1 < \lambda_2$. This guarantees that the solution (7.7) is unique. Note also that, since $\lambda_1\lambda_2 = 1/R$, $0 < \lambda_1 < 1 < R < \lambda_2$.

3. If p_t follows a lognormal distribution, then (7.30) will imply that p_t must follow a random walk exactly.