Economics 762
Homework # 4

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1) Assume a country with initial external debt, B_0.The fiscal authorities plan to have a surplus in the trade balance, from now until the indefinite future, which is a constant share, θ, of the interest accumulated through the current period. Thus, the trade balance in period s, TB_s, is equal to:

$$TB_s = -\theta r B_s \, , \theta > 0$$

where r is the world rate of interest.

a) Describe the time path for B.

b) Show that the economy's intertemporal resource constaint is met with such policy.

c) Assume that the policy changes to maintaining a trade balance surplus $-\theta(1+r)B_s$, where $B_0 < 0$. Is this policy consistent with the economy's intertemporal resource constraint?

2) Assume an economy smoothing path.That is $B = (1/1+r)$, where B is the subjective discount factor. Define the expected value of the annuity permanent value of the stochastic macro variable X by,

$$E_t \tilde{X_t} = r/(1+r)[\sum_{s=t}^{\infty}(1/1+r)^{s-t} E_t X_s$$

Assume that investment,and government spending are zero.

a) Show that the current account surplus is equal to:

$$CA_t = Y_t - E_t \tilde{Y_t}$$

where Y is output.

b) Show that CA_t is a forecast of declines in future outputs, such that

$$CA_t = -\sum_{s=t+1}^{\infty} (1/(1+r)E_t \Delta Y_s$$

where $\Delta Y_s = Y_s - Y_{s-1}$

3) Assume a cost-of-adjustment investment technology,

$$\begin{aligned} Z_t &= I_t(1 + (g/2)I_t/K_t)\ ,\ g > 0 \\ I_t &= K_{t-1} - (1-\delta)K_t \end{aligned}$$

where K, I, Z and δ, are the capital stock,the net increase in in the capital stock over the period, investment and deprciation rate, respectively.

Assume that A follows a first order autoregressive stochastic process

$$A_t - A^- = \rho(A_{t-1} - A^-) + \epsilon_t,\ 0 \leq \rho \leq 1$$

a) Derive the first-order condition for:

$$Max\ E_t \sum_{s=t}^{\infty} (1/1+r)^{s-t} A_s K_s^{\alpha}$$

where $A_s K_s^{\alpha}$ is a Cobb-Douglas production function.Show that the optimal investment rule amounts to a stochastic second-order difference equation in K.

b) Linearize the difference equation around a steady state $A_s = A^-$ and K is solved from $\delta + r = \alpha A^-(K^-)^{\alpha-1}$.

c) Apply the forward-backward solution technique from Sargent's Macreconomics, and derive the solution for K_t.