1) Assume a country with initial external debt, $B_0$. The fiscal authorities plan to have a surplus in the trade balance, from now until the indefinite future, which is a constant share, $\theta$, of the interest accumulated through the current period. Thus, the trade balance in period $s$, $TB_s$, is equal to:

$$TB_s = -\theta r B_s, \theta > 0$$

where $r$ is the world rate of interest.

a) Describe the time path for $B$.

b) Show that the economy’s intertemporal resource constraint is met with such policy.

c) Assume that the policy changes to maintaining a trade balance surplus $-\theta (1 + r) B_s$, where $B_0 < 0$. Is this policy consistent with the economy’s intertemporal resource constraint?

2) Assume an economy smoothing path. That is $B = (1 + r)$, where $B$ is the subjective discount factor. Define the expected value of the annuity permanent value of the stochastic macro variable $X$ by,

$$E_t X_t^- = r / (1 + r) \sum_{s=t}^{\infty} (1/1 + r)^{s-t} E_t X_s$$

Assume that investment, and government spending are zero.

a) Show that the current account surplus is equal to:

$$CA_t = Y_t - E_t Y_t^-$$

where $Y$ is output.
b) Show that \( CA_t \) is a forecast of declines in future outputs, such that

\[
CA_t = - \sum_{s=t+1}^{\infty} \frac{1}{1+r} E_t \Delta Y_s
\]

where \( \Delta Y_s = Y_s - Y_{s-1} \)

3) Assume a cost-of-adjustment investment technology,

\[
Z_t = I_t \left(1 + \frac{g}{2} \frac{I_t}{K_t}\right), \quad g > 0
\]

\[
I_t = K_{t-1} - (1 - \delta) K_t
\]

where \( K, I, Z \) and \( \delta \), are the capital stock, the net increase in in the capital stock over the period, investment and depreciation rate, respectively.

Assume that \( A \) follows a first order autoregressive stochastic process

\[
A_t - A^- = \rho(A_{t-1} - A^-) + \epsilon_t, \quad 0 \leq \rho \leq 1
\]

a) Derive the first-order condition for:

\[
Max E_t \sum_{s=t}^{\infty} \frac{1}{1+r} A_s K_s^\alpha
\]

where \( A_s K_s^\alpha \) is a Cobb-Douglas production function. Show that the optimal investment rule amounts to a stochastic second-order difference equation in \( K \).

b) Linearize the difference equation around a steady state \( A_s = A^- \) and \( K \) is solved from \( \delta + r = \alpha A^- (K^-)^{\alpha-1} \).

c) Apply the forward-backward solution technique from Sargent’s Macreconomics, and derive the solution for \( K_t \).