MARGINAL VERSUS AVERAGE TAX RATES: AN APPLICATION TO FDI

1 Introduction

In most of the literature on capital income taxation, the marginal and average tax rates are the same (e.g., Chamley (1986), Judd (1987)) and therefore the distinction between them is not relevant. However, in the context of income taxation and redistribution, it is typically the marginal tax rate which affects efficiency, whereas it is the average tax rate which affects the progression of the income tax and consequently its redistributive feature [e.g., Mirrlees (1971), Sadka (1976) and Jakobsson (1976)].

Similarly, when discrete choices are involved (such as whether or not to produce at all, whether to choose one location or another by firms and individuals, or one profession or another by individuals), it is usually the average tax rate which is relevant; see, for instance, Devereux and Hubbard (2000) for a recent analysis of location choice by multinationals.

In this chapter we develop a stylized model of FDI in which marginal tax rates and average tax rates diverge. In this framework the level and the location of investment are analyzed in relation to the effects of the average and marginal tax rates. The organization of this chapter is as follows. Section 2 develops the analytical framework. Section 3 considers the case of equity-finance and section 4 analogously analyzes the case of debt finance. Section 5 concludes.

2 Analytical Framework

Consider a two-period economy with a single, all-purpose good. In the first period there exists a continuum of N firms which differ from each other by a productivity index ε. We denote a firm which has a productivity index of ε by an $\varepsilon-$firm. The cumulative distribution function of ε is denoted by $G(\cdot)$. For the sake of simplicity, we normalize the number of firms to one; $N = 1$.

We denote the initial net capital stock of each firm by $(1-\delta)K_0$. This consists of the net initial stock, K_0, of the preceding period, multiplied by one minus the depreciation rate, δ. If an $\varepsilon-$firm invests I in the first period, it augments its capital stock to $K = (1-\delta)K_0+I$ and its gross output in the second period will be $F(K)(1+\varepsilon)$. Naturally $\varepsilon \geq -1$ so that $G(-1) = 0$. We assume that there exists is a fixed setup cost of investment, C, independent of ε.

Initially, all firms are owned by domestic residents. Foreign direct investors (henceforth, FDIors) may purchase the domestic firms in the first period. We assume that the FDIors require a rate of return, r^*, which is lower than the rate potential domestic buyers require. This differential may be explained, for instance, by a corresponding country-risk differential in the loan and bond market, or some synergism between the FDIors' line of business in their source country and the firms in the domestic economy, etc. Thus, FDIors will outbid potential domestic buyers so that they acquire control over the domestic firms.[1]

2.1 Equity Finance

Suppose new investment is equity-financed by the FDIors.[2] If an $\varepsilon-$firm invests $I = K-(1-\delta)K_0$ in the first period, it will generate a cash flow of:

$$[F(K)(1+\varepsilon)-\delta K]\,(1-\theta)+K+\theta C$$

in the second period. We assume that the physical rate of deprreciation, δ, is also the rate allowed for tax purposes. There is a corporate tax at the rate θ. We assume, as is the case in practice, that the setup cost (C) incurred in the first period is deductible and carried forward as a tax loss to the second period.

The objective of an $\varepsilon-$firm is to maximize the value of the firm to its shareholders. That is:

[1] This simple specification may, of course, be generalized to include other sectors in which FDIors do not acquire control over all firms.

[2] An equivalent specification is to let the original owners invest and then sell the firms to the FDIors.

$$\underset{K}{Max}\left\{\frac{(1-\theta)(1-t-t^*)[F(K)(1+\varepsilon)-\delta K-C]+C+K}{1+r^*}-[K-(1-\delta)K_0+C]\right\}, \quad (1)$$

where:

t - tax paid on dividends in the host country;[3]

t^* - additional (if any) tax on dividends paid by the FDIors in their source country;[4]

r^* - **net** rate of return required by FDIors.

The first-order condition for the maximization of (1) yields the optimal K as a function of ε, denoted by $K(\varepsilon)$:

$$F'[K(\varepsilon)](1+\varepsilon)-\delta=\frac{r^*}{(1-\theta)(1-t-t^*)}. \quad (2)$$

Note, however, that the firm always has the option to stick to its old capital stock [namely, $(1-\delta)K_0$] and avoid the setup cost (C) of a new investment. Therefore, whether an $\varepsilon-$firm will indeed carry the new investment prescribed by equation (2) depends on whether its productivity is high enough so as to more than offset the fixed setup cost required for new investments

$$\frac{(1-\theta)(1-t-t^*)\left\{F\left[K(\varepsilon)\right](1+\varepsilon)-\delta K(\varepsilon)-C\right\}+K(\varepsilon)+C}{1+r^*}-$$

$$K(\varepsilon)-(1-\delta)K_0+C\geq$$

$$\frac{(1-\theta)(1-t-t^*)\{F\left[(1-\delta)K_0\right](1+\varepsilon)-\delta(1-\delta)K_0\}+(1-\delta)K_0)}{1+r^*}.$$

[3]We assume that all profits in the second period are distributed as dividends and not retained by the firm so as to generate capital gains.

[4]This is an **additional** tax paid, whether under a foreign tax credit system, or under foreign tax deduction system.

Therefore, there exists a cutoff level of ε, denoted by ε^0, so that an $\varepsilon-$firm will invest if and only if $\varepsilon \geqq \varepsilon^0$. The cutoff level of ε is defined by:

$$\begin{aligned} \pi(\varepsilon^0) &\equiv \{F[K(\varepsilon^0)] - F[(1-\delta)K_0]\}(1+\varepsilon^0) - \delta[K(\varepsilon^0) - (1-\delta)K_0] \\ &= \frac{r^*[I(\varepsilon^0)+C]}{(1-\theta)(1-t-t^*)} + C. \end{aligned} \tag{3}$$

where $\pi(\varepsilon^0)$ is the incremental pre-tax profit generated by the new investment $I(\varepsilon) \equiv K(\varepsilon) - (1-\delta)K_0$, which requires a setup cost of C.

While the marginal productivity condition (2) determines the level of investment that each firm will undertake (if at all), condition (3) can be viewed as determining the location of investment. Firms with a productivity index larger than ε^0 would indeed attract new foreign investment. Firms with a productivity index below ε^0 will attract no new foreign investment, and some of them may relocate in the FDIors' source country, depending on the difference between the corporate and the individual tax rates prevailing in the source country and those prevailing in the home country.

Another way of describing how the location of investment is determined is obtained by substituting (2) into (3) to get:

$$\begin{aligned} &\{F[K(\varepsilon^0)] - F[(1-\delta)K_0]\}(1+\varepsilon^0) - \\ &\{F'[K(\varepsilon^0)](1+\varepsilon^0)\}[K(\varepsilon^0) - (1-\delta)K_0] = [1 + \frac{r^*}{(1-t-t^*)(1-\theta)}]C. \end{aligned} \tag{3'}$$

Condition (3′) thus states that the **infra-marginal incremental** output, generated by the new investment [the left-hand-side of (3′)] must equal (the tax-adjusted future value of) the setup cost; see Figure 1.

Strictly speaking, the above framework is a two-period, static model. Nevertheless, it could also be interpreted, after some straightforward modifications, as a two-period snapshot of a multi-period, dynamic economy; see the appendix.

3 Marginal Tax Rate versus Average Tax Rate

One can now show that the marginal-productivity condition (2) that determines the level of investment depends on the **marginal** tax rate applied to the capital income of FDIors, whereas the location-of-investment condition (3) depends on the **average** tax rate.

To see this, note that the marginal tax rate applicable to the FDIors, at both the corporate and the individual level combined, is:

$$t_M \equiv 1-(1-\theta)(1-t-t^*), \tag{4}$$

because, at the margin, a pre-tax income of one dollar yields a post-tax income of just $(1-\theta)(1-t-t^*)$ dollars. To see this, define the marginal tax rate as the pre-tax marginal product of capital (net of depreciation), minus the post-tax marginal rate of return accruing to the FDIor, divided by the pre-tax marginal product of capital. The pre-tax marginal product of capital is $F'-\delta$; the post-tax marginal rate of return accruing to the FDIor is r^*. Thus, using equation (2), the marginal tax rate is:

$$\begin{aligned} t_M &= \left(\frac{r^*}{(1-\theta)(1-t-t^*)}-r^*\right) \Big/ \left(\frac{r^*}{(1-\theta)(1-t-t^*)}\right) \\ &= 1-(1-\theta)(1-t-t^*), \end{aligned}$$

which proves equation (4). Note that t_M is the same for all firms (that is, independent of ε). Note also that equation (2) can be restated as:

$$(1-t_M)\{F'[K(\varepsilon)](1+\varepsilon)-\delta\}=r^*. \tag{2'}$$

Similarly, the average tax rate (t_A) levied on the income generated by the new investment made by an ε−firm can be defined as:

$$t_A(\varepsilon)=\left[\frac{\pi(\varepsilon)}{I(\varepsilon)+C}-\frac{(1-t_M)\pi(\varepsilon)+t_MC}{I(\varepsilon)+C}\right] \Big/ \left[\frac{\pi(\varepsilon)}{I(\varepsilon)+C}\right]. \tag{5}$$

(Recall that t_MC is the tax saved in the second period as a result of the setup cost C incurred in the first period.) Naturally, the average tax rate is firm-specific, that is, $t_A = t_A(\varepsilon)$. Upon some rewriting, we can also define the average tax rate as:

$$t_A(\varepsilon) = t_M \left[1 - C/\pi(\varepsilon)\right]. \tag{5'}$$

Note that the average tax rate is always below the marginal tax rate. Also, a higher average tax rate is levied on a more productive firm.

Substituting for the average tax rate levied on the ε^0−firm (that is, $t_A(\varepsilon^0)$) from equation (5′) into the location of investment condition (3), we can see that:

$$[1 - t_A(\varepsilon^0)]\pi(\varepsilon_0) = r^* I(\varepsilon^0) + (1 + r^*)C. \tag{3''}$$

Thus, it is the average tax rate, levied on the ε^0 rather than the marginal tax rate, which directly determines the location of investment. This does not mean to say that the marginal tax rate does not affect the location of investment at all. For $I(\varepsilon^0)$ which appears in condition (3″) is certainly affected by the marginal tax rate. What we mean is that the marginal tax rate does not directly enter condition (3″), that is: Once the level of investment function [namely, $I(\varepsilon) = K(\varepsilon) - (1 - \delta)K_0$] is determined, then it is only the average tax rate which determines the location of investment.

4 Debt Finance

So far we assumed that the FDIors equity-financed the firms' new investments. Let us now turn to the possibility of debt-finance. The amount of debt that is required in order to finance the new investment is $K - (1 - \delta)K_0 + C$. Suppose the firm borrows the money from the FDIors' source country. Thus, the net-of-tax cash flow of the firm in the second period is:

$$
\begin{aligned}
&F(K)(1+\varepsilon)+(1-\delta)K-(1+r)[K-(1-\delta)K_0+C] \\
&-\theta\{F(K)(1+\varepsilon)-\delta K-rK-(1-\delta)K_0+C]\}+\theta C= \\
&(1-\theta)\{F(K)(1+\varepsilon)-\delta K-r[K-(1-\delta)K_0+C]-C\}+(1-\delta)K_0,
\end{aligned}
$$

where r is the pre-tax rate of interest, prevailing at the FDIors' source country. The net (after all taxes, corporate and individual) cash flow to the FDIors is [when $K>(1-\delta)K_0$ and the setup cost C is incurred]:

$$
(1-t-t^*)(1-\theta)\{F(K)(1+\varepsilon)-\delta K-r[K-(1-\delta)K_0+C]-C\}+(1-\delta)K_0. \tag{6}
$$

The choice of K of an $\varepsilon-$firm is a solution to the maximization of equation (6), provided that $K>(1-\delta)K_0$. Denote the solution by $K(\varepsilon)$.

The first-order condition for the above maximization is:

$$
F'[K(\varepsilon)](1+\varepsilon)-\delta=r, \tag{7}
$$

as long as $K(\varepsilon)>(1-\delta)K_0$. Thus, the level of investment is not directly affected by taxes.

The cutoff level of ε in this debt-finance case is that level of ε which makes the firm indifferent between investing and providing its FDI-owners with a net cash flow in the second period equal to equation (6), and not investing at all and getting a net cash flow of:

$$
(1-t-t^*)(1-\theta)\left\{F\left[(1-\delta)K_0\right](1+\varepsilon)-\delta(1-\delta)K_0\right\}+(1-\delta)K_0.
$$

Therefore, the productivity cutoff level ε^0 is defined by:

$$
\begin{aligned}
(1-t-t^*)(1-\theta)\pi(\varepsilon^0) &= (1-t-t^*)(1-\theta)r[K(\varepsilon^0)-(1-\delta)K_0]+ \qquad (8') \\
(1-t-t^*)(1-\theta)(1+r)C &\equiv (1-t-t^*)(1-\theta)rI(\varepsilon^0)+(1-t-t^*)(1-\theta)(1+r)C,
\end{aligned}
$$

where $\pi(\cdot)$ is as defined in equation (3). Thus, the tax terms on all sides of equation (8') cancel out and we can define the cutoff productivity level of ε^0 by

$$\pi(\varepsilon^0) = rI(\varepsilon^0) + (1+r)C. \tag{8}$$

Thus, the cutoff level of ε is not affected by taxes either.

The marginal tax rate is defined as the pre-tax marginal product of capital (net of depreciation), minus the post-tax rate of return, divided by the pre-tax marginal product of capital. The pre-tax marginal product of capital is equal to r by equation (7). However, the post-tax rate of return to the investors in the FDIors' source country is only $r^* = (1-t-t^*)r$. Therefore, the marginal tax rate (t_M) is equal to:

$$t_M = t + t^*. \tag{9}$$

Because interest expenses are deductible at the firm's level and investment is debt-financed, the corporate tax rate (θ) does not affect the marginal tax rate applicable to capital income.

Similarly, the average tax is defined as follows: The extra profit generated by the new investment made by an ε−firm is $\pi(\varepsilon) - rI(\varepsilon) - (1+r)C$ before taxes and $(1-t-t^*)(1-\theta)[\pi(\varepsilon) - rI(\varepsilon) - (1+r)C]$ after taxes. Therefore, the average tax rate is:

$$t_A = 1 - (1-\theta)(1-t-t^*) = 1 - (1-\theta)(1-t_M) = \theta + (1-\theta)t_M. \tag{10}$$

Note that the corporate tax rate does not affect the marginal tax rate, but it does affect the average tax rate. Unlike the equity-finance case, the average tax rate is independent of ε in the debt-finance case. In both cases the average tax rate is higher than marginal tax rate.

5 Conclusion

This stylized model illustrates the distinction between the marginal tax rate and the average tax rate. The former may affect the level of investment in the equity-finance case, whereas the latter is a key factor determining the location of investment Neither rate affects the level or location of investment in the debt-finance case.

6 Appendix: A Dynamic Extension

In this appendix we provide an extension of the static, two-period model to an infinite horizon, dynamic framework.

In each period t, ε_t is an idiosyncratic productivity shock. Furthermore, we assume that ε_t is independent of ε_{t-1}, ε_{t-2}, $\varepsilon_{t-3}, \ldots$. At each period, investment decisions are made after the realization that ε_t is known. Each firm carries with it the capital stock that was established in the preceding period. Specifically, at period $t = 1$, for instance, the ε_1-firm is already endowed with a stock of capital $(1-\delta)K_0$. This K_0 varies from one firm to another, depending on the realization of ε_0 each firm experienced (at period $t = 0$). In this case, the stock of capital of an ε_t firm (that is, a firm that experienced an ε_t shock in period t) chooses in period t depends also on its initial stock of capital $(1-\delta)K_{t-1}$. That is, $K_t = f[(1-\delta)K_{t-1}, \varepsilon_t]$.

Let us now turn to the derivation of the value function of the ε_t-firm. Suppose an ε_t-firm is endowed with a stock of capital $(1-\delta)K_{t-1}$, carried over from the previous period. Its value $V[(1-\delta)K_{t-1}, \varepsilon_t]$ is equal to:

$$
\begin{aligned}
& W^{+}[(1-\delta)K_{t-1}, \varepsilon_t] \\
= \; & \underset{K}{Max} \left\{ \begin{array}{c} (1-t-t^{*})(1-\theta)[F(K)(1+\varepsilon_t) - \delta K - C](1+r^{*})^{-1} \\ +(1+r^{*})^{-1}C + (1+r^{*})^{-1}EV[(1-\delta)K, \varepsilon_{t+1}] = [K-(1-\delta)K_{t-1}+C] \end{array} \right\}
\end{aligned}
$$

if it chooses K above $(1-\delta)K_{t-1}$ (and thus, incurs the setup cost C). However, it has also the option of not investing at all and producing with the existing stock of capital which is $(1-\delta)K_{t-1}$. If it exercises this option, its value is:

$$
W^{-}[(1-\delta)K_{t-1}, \varepsilon_t)] = \frac{1}{1+r^{*}} \left\{ \begin{array}{c} [F[(1-\delta)K_{t-1}](1+\varepsilon_t) - \delta(1-\delta)K_{t-1}]\,(1-\theta)(1-t-t^{*}) \\ +EV[(1-\delta)^2 K_{t-1}, \varepsilon_{t+1}]. \end{array} \right\}
$$

Hence, the value of an ε_t-firm that carried over a stock of capital of $(1-\delta)K_{t-1}$ from the previous period is:

$$V[(1-\delta)K_{t-1},\varepsilon_t] = Max\left\{W^+[(1-\delta)K_{t-1},\varepsilon_t], W^-[(1-\delta)K_{t-1},\varepsilon_t]\right\}.$$

The cutoff (productivity) tax ε_t^0 between investing and incurring the setup cost C or not investing at all and avoiding this setup cost is a function of $(1-\delta)K_{t-1}$, denoted by $\varepsilon_t^0 = h[(1-\delta)K_{t-1}]$. The latter function is related to the investment function $K_t = f[(1-\delta)K_{t-1},\varepsilon_t]$ by:

$$(1-\delta)K_{t-1} = f\left\{(1-\delta)K_{t-1}, h[(1-\delta)K_{t-1}]\right\}.$$

That is, a firm which experienced a productivity shock of $\varepsilon^0[(1-\delta)K_{t-1}]$ in period t will be indifferent between investing and not investing, so that $K_t = (1-\delta)K_{t-1}$. This ε_t^0 depends on the average tax rate.

In period $t-1$ there is a cumulative distribution function of K_{t-1}. Denote it by $\Psi_{t-1}(\cdot)$. A firm that went into period t with an initial stock of capital of $(1-\delta)K_{t-1}$, has a probability of $G(\varepsilon_t^0) = G\{h[(1-\delta)K_{t-1}]\}$ of not investing at all in period t. Thus, the proportion of firms that will not invest in period t, as perceived in period $t-1$, is:

$$E_{\Psi_{t-1}}[G(\varepsilon_t^0)] = E_{\Psi_{t-1}}\left(G\{h[(1-\delta)K_{t-1}]\}\right).$$

Similarly, there exists a probability of $1 - G(\varepsilon_t^0) = 1 - G\{h[(1-\delta)K_{t-1}]\}$ that a firm that started period t with a capital stock of $(1-\delta)K_{t-1}$ will choose to make positive investment, that is, $K_t > (1-\delta)K_{t-1}$. Thus, given K_{t-1}, the cumulative distribution function of K_t is:

$$\varphi(K_t;K_{t-1}) = \begin{cases} 0 & \text{if } K_t < (1-\delta)K_{t-1} \\ G\{h[(1-\delta)K_{t-1}]\} & \text{if } K_t = (1-\delta)K_{t-1} \\ G\{e[K_t,(1-\delta)K_{t-1}]\}(1-G\{h[(1-\delta)K_{t-1}]\})^{-1} & \text{if } K_t > (1-\delta)K_{t-1}, \end{cases}$$

where $e[K_t,(1-\delta)K_{t-1}]$ is the productivity level that will generate a capital stock of K_t in period t, given K_{t-1}. That is, e is defined implicitly by:

$$K_t = f\left\{(1-\delta)K_{t-1}, e[K_t,(1-\delta)K_{t-1}]\right\}.$$

The expected value of the stock of capital in period t (that is, the average over all firms) is:

$$E_{\Psi_{t-1}}\left[E_{\varphi}(K_t;\ K_{t-1})\right],$$

where $E_{\varphi}(K_t;\ K_{t-1})$ is the expected value of the stock of capital (K_t) in period t, given K_{t-1}. This economy-wide average depends on both the average and marginal tax rates.

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