Manipulating market sentiment

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Highlights

• We analyze the interaction between a large rational trader and naïve speculators.
• Naïfs trade after a streak of price deviations from the asset’s fundamental value.
• Naïfs’ trading rule does not follow a trend or respond to price trends.
• Nevertheless, the model gives rise to rich patterns of price fluctuations.
• The model synthesizes opposing views regarding the role of rational speculators.

Abstract

We analyze a simple model of an asset market, in which a large rational trader interacts with “noise speculators” who seek short-run speculative gains, and become active following a prolonged episode of mispricing relative to the asset’s fundamental value. The model gives rise to patterned like bubble dynamics, positive short-run correlation and vanishing long-run correlation of price deviations from the fundamental value. We argue that this example model sheds light on the question as to whether rational speculators abet or curb price fluctuations.

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1. Introduction

One of the main themes in the behavioral finance literature has been the effect that boundedly rational traders have on price fluctuations in financial markets. In seminal papers such as De Long et al. (1990a,b) and Hong and Stein (1999), conventionally rational traders coexist with “noise traders” (agents whose trading behavior follows some exogenous stochastic process), or with agents who follow trading rules, such as “fundamental trading” or “trend seeking”, based on an incomplete understanding of the market.

A maintained assumption in this literature has been that the market is competitive in that rational traders are price takers and have no market power. In many markets, however, some rational traders have genuine market power: a large hedge fund acting in a (relatively thin) derivative market, for example, or a large oil producing country in a market for oil-related securities.

This short paper is a modest attempt to explore the effects of boundedly rational trading on price fluctuations when some rational traders have market power. We analyze a simple example of speculative trade between a large rational trader and boundedly rational speculators who follow a trading rule that conditions on the observed price history. We use this example to show how rich patterns of asset price fluctuations can emerge from very simple boundedly rational trading rules, as a result of their interaction with a large rational trader. Specifically, although the
strictly decreasing and odd (i.e., function $v$ is defined as $v(x) = a \cdot \text{sign}(\theta^t - x)$ where $\theta^t$ is i.i.d. according to a density $f$ that is symmetric around zero, $w^t = 0$ when $B(h^t) < L$, and $w^t = a \cdot \text{sign}(\theta^t - x)$ when $B(h^t) \geq L$, where $L > 2$ is an integer and $a \in \{-1, 1\}$. Let $F$ be the cdf induced by $f$, and assume $F(a) < 1$.

The interpretation of the process governing $x^t$ is as follows. Noise speculators consist of one conventional noise trader and one naive speculator. The former agent’s net demand at period $t$ is $x^t$. The latter agent takes a buy or short position of one unit in each period; this position must be closed one period later. He takes a non-zero position only after a sufficiently long sequence of price discrepancies in the same direction and of sufficient magnitude. The naive speculator’s net position at $t$ is thus $w^t - w^{t-1}$. We say that a history $h^{t-1}$ is inactive if $w^{t-1} = w^{t-2} = 0$ and $B(h^{t-1}) < L$. At an inactive history, the naive speculator is "waiting" for a critical streak of price discrepancies to form and does not take a non-zero position. Since we allow $a$ to be either positive or negative, we can capture two types of “market sentiment”. When $a = -1$, it is apt to refer to the noise speculator as a “fundamental trader”, because he acts at period $t$ as if the market is about to correct the mispricing. On the other hand, when $a = 1$, we may refer to the noise speculator as a “momentum trader”. Our results can be extended to the case in which $a$ is stochastic. The large trader’s activity thus manipulates market sentiment in the sense that it helps activating the perception that the market is about to crash, or that it has gained momentum, etc.

If the large trader only faced arbitrages and conventional noise traders – i.e., if $w^t = 0$ for all $t$ – he would be unable to make any speculative gain, and his optimal policy would be to supply a zero quantity in every period. Thanks to the naive speculator, the large trader has an incentive to manipulate the market price, in order to induce the naive speculator to become active, and then lean against him when he does.

3. The result

Our objective is to provide a qualitative characterization of the price fluctuations that emerge from the large trader’s optimal net supply of the asset in each period. We first observe that the large trader faces a Markov decision problem. The naive speculator’s net demand at period $t$ following the history $h^{t-1}$ is a deterministic function of the state defined by $q(h^{t-1}) = (\text{sign}(\theta^t - y), B(h^{t-1})), (\text{sign}(\theta^t - x), B(h^{t-1})).$ Since the behavior of arbitrages and the conventional noise trader is entirely stationary, it follows that the large trader’s dynamic optimization problem is Markovian w.r.t. to the set of states $Q$ defined above. An inactive history $h^{t-1}$ corresponds to a state with $B(h^{t-1}) < 0$, $B(h^{t-1}) < L$. Let $V(q)$ be the value function given by a solution to this problem. Note that the arbitrages’ demand function $D$, the density $f$ and the naive speculators’ trading rule are all symmetric w.r.t. the sign of price discrepancies. Therefore, $V(q)$ is symmetric in the following sense. Let $q = ((i, B), (j, B))$ and $q = ((i-t, B), (j-t, B)).$ Then, $V(q) = V(q').$

The following notation will be useful. Consider an inactive history $h^{t-1}$ with $B(h^{t-1}) = B < L$, and $\theta^{t-1} > 0$. We denote the state that corresponds to this history by $p^B$. We use $F^B$ to denote the cdf of $x^t$ conditional on a history $h^{t-1}$ that corresponds to the state $q(h^{t-1}).$ The expected price deviation at period $t$ given $x^t$ and a history $h^{t-1}$ is thus

\[ E(\theta^t | h^{t-1}, x^t) = \int \Theta(x^t - w^t - w^{t-1}) \cdot dF^B(h^{t-1})(z^t). \]

Note that this expression is decreasing in $x^t$.

Proposition 1 (Bubble Dynamics). Let $x^t$ be a trading strategy that solves the large trader’s problem. Consider two inactive histories $h^{t-1}$.
and \( h^t \) for which \( B(h^t) < B(h^t') \leq L - 1. \) Then,
\[
E(\theta^{t+1} | h^t, x^t(h^t)) > E(\theta^{t+1} | h^t, x^t(h^t')) > 0 \quad \text{if} \ \theta^t, \theta^t > 0
\]
\[
E(\theta^{t+1} | h^t, x^t(h^t)) < E(\theta^{t+1} | h^t, x^t(h^t')) < 0 \quad \text{if} \ \theta^t, \theta^t < 0.
\]

**Proof.** We will only prove the first row of inequalities, as the remaining set is symmetric. Given an inactive history \( h \), define
\[
\pi(x, h) = x \int \Theta(x - z) dF_q(h)(z).
\]
Note that, if \( E(z | h) = 0 \), then \( \pi(0, h) = 0 \) and \( \pi(x, h) < 0 \) whenever \( x \neq 0 \).

Consider the state \( \rho^B \) corresponding to an inactive history. Recall that by definition, \( B < L \). Note that \( V(\rho^B) > 0 \), since the myopic maximization of \( \pi(x, \rho^B) \) leads with positive probability to a future history in which the expectation of \( z \) is non-zero. Since the fundamental value function is symmetric w.r.t. the sign of the history of price discrepancies, the Bellman equation for \( 0 \leq B \leq L - 1 \) is
\[
V(\rho^B) = \max_x \left\{ \pi(x, \rho^B) + \delta V(\rho^{B+1}) \left( 1 - F^B(x - D(\alpha)) \right) \right. \\
+ \delta V(\rho^B) (x^t(\rho^B) - D(\alpha)) \left. \right) \\
- F^B \left( x^t(\rho^B) - D(\alpha) \right) \right). \\
\]

Now,
\[
V(\rho^B) = \pi(x^t(\rho^B), \rho^B) + \delta V(\rho^B) ( x^t(\rho^B) - D(\alpha)) \left( 1 - F^B \left( x^t(\rho^B) - D(\alpha) \right) \right) \\
+ F^B \left( x^t(\rho^B) - D(\alpha) \right) \left( x^t(\rho^B) - D(\alpha) \right) \\
+ \delta V(\rho^B) \left( x^t(\rho^B) - D(\alpha) \right) \\
- F^B (x^t(\rho^B) - D(\alpha)) \right). \\
\]

Since \( \pi(\rho^B, x^t(\rho^B)) \leq 0, \) if \( V(\rho^B) \leq V(\rho^B), \) by simple substitution it is easy to verify that \( V(\rho^B) \leq 0, \) a contradiction. Thus, \( V(\rho^B) > V(\rho^B), \) now let \( B \) be the smallest \( B \leq L - 1 \) such that \( V(\rho^{B+1}) \leq V(\rho^B) \). Then
\[
V(\rho^B) \leq \pi(x^t(\rho^B), \rho^B) + \delta V(\rho^B) \\
which implies \( V(\rho^B) \leq 0, \) a contradiction. Thus, \( V(\rho^{B+1}) > V(\rho^B) \)
for any \( 0 \leq B \leq L - 1 \).

Since \( \pi(\cdot, \cdot) \) and \( F^B(\cdot) \) are symmetric around zero and \( V(\rho^B) > V(\rho^B), \) we have that \( x^t(\rho^B) \leq 0. \) Otherwise, choosing \(-x^t(\rho^B) \) would yield a higher payoff. By definition, for \( 0 < B < L - 1 \),
\[
V(\rho^B) \geq \pi(x^t(\rho^B), \rho^B) + \delta V(\rho^{B+1}) \\
\times \left( 1 - F^B \left( x^t(\rho^{B+1}) - D(\alpha) \right) \right) \\
+ \delta V(\rho^B)(x^t(\rho^{B+1}) - D(\alpha)) \\
+ \delta V(\rho^B)(x^t(\rho^{B+1}) - D(\alpha)) \\
- F^B(\rho^{B+1}) \left( x^t(\rho^{B+1}) - D(\alpha) \right) \right)
\]
and
\[
V(\rho^{B+1}) \geq \pi(x^t(\rho^B), \rho^{B+1}) + \delta V(\rho^{B+2}) \\
\times \left( 1 - F^B \left( x^t(\rho^{B+2}) - D(\alpha) \right) \right) \\
+ \delta V(\rho^B)(x^t(\rho^{B+2}) - D(\alpha)) \\
\]
4. Concluding remarks

Our objective in this short paper was merely to illustrate that the combination of rational traders with market power and price-taking, boundedly rational speculators can generate price fluctuation patterns of interest. However, we believe it also sheds some light on an ancient debate regarding the role of rational speculators in financial markets. Some argue that speculators sow instability and create excess volatility, whereas others argue that speculators have a stabilizing role, as they bring prices back to fundamentals by spotting arbitrage opportunities. Our model synthesizes both views. Even if the large rational trader did not exist, episodes of persistent mispricing would occur spontaneously from time to time, and this would lead to large price fluctuations due to the activity of the naïve speculator. The rational trader’s strategic, forward-looking behavior in our model can make these episodes more likely to happen, thereby raising the frequency of large price movements. Nevertheless, the amplitude of the large price movements due to the naïve speculator’s activity is reduced, because the rational trader leans against him. Although the large rational trader precipitates episodes of large volatility, he curbs their amplitude when they occur. Thus, if we view price volatility as a “problem”, then rational speculators are both part of the problem and part of its solution.

Related literature

There is a huge literature in behavioral finance that addresses price fluctuations due to the presence of noise/naive traders. Our paper adds a dimension to this literature by introducing rational, forward-looking traders with market power. As a result, our modeling style is different from the competitive equilibrium methodology that characterizes this literature.

Our paper is also related to a smaller literature that asks whether a large rational speculator can make profits in a market for a financial asset or a storable commodity by manipulating prices (see Hart, 1977, Hart and Kreps, 1986, Jarrow, 1992). This literature has not addressed effects on price dynamics, and has treated the behavior of traders that the large speculator faces as a black box without deriving it from behavioral rules. The only exceptions we are aware of are Mei et al. (2004) and Rubinstein and Spiegler (2008). The former paper analyzes a finite-horizon model with one large manipulator, a population of rational arbitrageurs and a set of traders prone to the disposition effect (a tendency to avoid selling losing assets), and derive some asset price anomalies. The latter paper analyzes the interaction between a large rational trader and boundedly rational speculators whose trading rule responds to the ergodic price distribution. As a result, the Rubinstein–Spiegler model cannot be reduced to a Markov decision problem. Finally, there is a large literature on information-based manipulation of market prices, where a large informed trader exploits informational asymmetries and the presence of noise traders to make speculative profits at the expense of rational, uninformed traders (Kyle, 1985, Allen and Gale, 1992 are key references in this literature).

References