Buridan Competition*

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Abstract

We analyze a model in which two agents compete in two-attribute alternatives over a decision maker who exhibits "trade-off avoidance", and thus follows a non-compensatory choice procedure that responds purely to ordinal rankings along the two dimensions. The decision maker chooses a dominant alternative when one exists. In the absence of domination, he chooses according to one of the attributes (with different probabilities that reflect their relative salience), or sticks to a default option (when one exists). The decision maker’s tendency to choose by default is a personal characteristic that represents his "indecisiveness". Our analysis of mixed-strategy Nash equilibria in the game between the agents highlights the effect of the decision maker’s trade-off avoidance on the value of offered alternatives and its distribution across attributes, the prevalence of domination, and the rate of switching away from default options. We discuss the potential implications of this analysis for recent discussions of "default architecture".

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1 Introduction

One of the biggest distinctions between economists’ and psychologists’ view of the decision process is the way they regard trade-offs. The standard economic approach assumes that the decision maker (DM henceforth) has well-defined preferences, and in the vast majority of applications these preferences are continuous and locally non-satiable. The economic DM is a "trade-off machine" who effortlessly weighs multiple considerations - Kreps (1988) fondly calls him Totrep ("Trade-Off Talking Rational Economic Person").

A viewpoint more typical of psychologists (e.g., Tversky (1972), Payne et al. (1993), Luce et al. (1999), Anderson (2003)) is that DMs generally try to avoid trade-offs, for a variety of reasons. First, trade-offs may be hard to calculate. Second, making a decision that relies on intangible, subjective "decision weights" is often emotionally difficult. Finally, when the DM is required to justify his choice to other people - a key motive in organizational behavior - he is expected to provide reasons for his choice, and decision weights are hard to justify. Trade-off avoidance implies that the DM will tend to use so-called "non-compensatory choice procedures" that rely purely on the ordinal rankings over alternatives along each dimension. In particular, if the DM has a default option that enables him to "decide not to decide" when facing the need to perform trade-offs, he may exercise this option.

In this paper, we explore the possible implications of this view of the decision process for competitive interaction in market or organizational contexts. Does trade-off avoidance lead to non-competitive equilibrium outcomes? If so, how large is the departure of from a rational-DM competitive benchmark? How ubiquitous in equilibrium are situations in which the DM is called to perform trade-offs (a task he avoids)? What is the role of the default specification for these questions, and how frequently does the DM switch away from the default option (when one exists)?

To address these questions, we study a simple model in which two agents (referred to as "competitors") compete in two-attribute alternatives for a
DM. The alternative offered by competitor $i$ is characterized by a pair $(q^1_i, q^2_i) \in \mathbb{R}^2_+$, where $q^k_i$ represents the alternative’s "quality" along dimension $k$. For instance, a car can be characterized by its safety, energy efficiency, etc. A dimension may also correspond to a state of Nature, such that the quality pair describes an investment project’s state-contingent performance.\(^1\) The DM’s choice set consists of the alternatives offered by the competitors, as well as an outside option represented by the quality pair $(0, 0)$. Agent $i$’s payoff is 0 when his alternative is not chosen by the DM, and $1 - c(q^1_i, q^2_i)$ when it is, where $c$ is a continuous and strictly increasing cost function. A conventionally rational DM would be endowed with a continuous, strictly increasing function $u(q^1, q^2)$, and would choose an alternative that maximizes $u$. In this case, our model would collapse into Bertrand-like competition: in Nash equilibrium, competitors would offer quality pairs $(q^1_i, q^2_i)$ that maximize $u$ subject to $c(q^1_i, q^2_i) = 1$.

In contrast, we assume that the DM follows a non-compensatory choice procedure, which is based solely on *ordinal quality rankings*. When one of the competitors’ alternatives dominates another (i.e., $q^k_i \geq q^k_j$ for both $k = 1, 2$, with at least one strict inequality), the DM chooses the dominant alternative; we refer to this situation as an "easy choice", because it does not require the DM to perform any trade-offs. When the competitors’ alternatives do not dominate one another - a situation we refer to as a "difficult choice" - the DM responds in two possible ways. He may resolve his tension by focusing entirely on a single, randomly selected attribute (again, avoiding trade-offs), where the probability with which each attribute is selected represents its relative salience; or he may procrastinate (that is, "decide not to decide") and adheres to a default option, if one exists. The probability with which the DM chooses by default in the presence of difficult choices is a personal characteristic that captures the degree of his "indecisiveness". What our DM

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\(^1\)We use the language of quality rather than price for convenience; one could easily think of at least one of the attributes as a price - see Section 2 for an elaborate discussion.
never does is resolving trade-offs according to some smooth utility function over \((q^1, q^2)\).  

Because the DM’s choice procedure exhibits default bias, it enables us to study the equilibrium implications of what Thaler and Sunstein (2008) call "default architecture", namely the design of default rules. This has been one of the most important policy ideas coming out of behavioral economics. However, to our knowledge this is the first paper that analyzes the equilibrium effects of default architecture in the context of an explicit behavioral model that generates default bias. Following the literature, we examine three default rules: (i) "opt in": the default is the outside option; (ii) "opt out": the default is one of the two competitors (each with equal probability, for simplicity); (iii) "no default": the DM is forced to make an active choice and cannot choose by default. Because the outside option is by assumption inferior to the competitors’ alternatives, the DM’s behavior under "opt in" is reminiscent of the proverbial Buridan’s Ass: unable to resolve the trade-off between two superior alternatives, he may delay choice and end up settling for the inferior outside option.

"Buridanic" situations of this nature have received considerable attention by researchers who studied empirically the intertwined phenomena of choice complexity and choice procrastination, in both experimental and "field" settings, notably retirement savings - see Iyengar et al. (2004), Madrian and Shea (2001) and Beshears et al. (2012), for a few important examples. Our contribution is to formulate a simple procedural model that exhibits trade-off avoidance and choice procrastination, and examine its implications for a Bertrand-like competitive interaction under various default rules. Our model is clearly an example, as it focuses on a specific environment with two competitors and two dimensions. In addition, it considers an extreme case in which the population from which the DM is drawn contains no Totreps.

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2The case in which both agents offer the same alternative poses an interpretational difficulty, but it turns out to matter little for the analysis.
Since equilibrium analysis even in this simple case turns out to be rich and non-trivial, we hope that this example will encourage other researchers to examine generalizations and extensions.

In its basic features, our model fits market environments in which firms compete for consumers who struggle to trade-off various product attributes. For example, think about weighing a car’s safety against its energy efficiency. Even if the consumer has access to precise data about each attribute, it may be hard for him to find the right scale for comparison. The difficulty is not only cognitive but also emotional, because the consumer ultimately needs to trade off the risk of injury or death against lower fuel costs. Similarly, when consumers choose between insurance policies that provide different levels of coverage in different contingencies, they have to perform complex actuarial calculations and imagine unpleasant future events.

In such market settings, the default option can be regarded as the consumer’s existing provider of a long-term service like insurance (when the consumer is new to the market, the default can be the outside option). What could be the meaning of default architecture in this environment? For example, "opt in" can be interpreted as a rule that outlaws automatic renewal of contracts: unless the consumer actively selects his old provider or switches to a different one, he ends up without a contract. In the same vein, "opt out" fits an environment in which all consumers have an existing provider, and the consumer’s contract is renewed automatically unless he actively switches.

Another interpretation of the model fits organizational settings. For instance, suppose the DM is a company official who considers several candidates for a construction job with several dimensions (total cost, speed of delivery, quality of materials, etc.). The official needs to justify his selection to his superiors. When the selection is not obvious and requires judgment, the official is more vulnerable to criticisms by his supervisors, both ex-ante and ex-post. As a result, it may be easier for him to justify his selection if he leaves out relevant dimensions, thus presenting the problem to his superiors.
as being simpler than it actually is, or if he goes for a default constructor (in case one exists). In this environment, default architecture can be an instrument that the organization employs to manipulate the official’s justifiability considerations.

We should emphasize that we do not try to approximate any of the above market and organizational real-life settings; the model is deliberately stylized and formulated abstractly. In particular, we stick to the extreme case of a single DM (or, equivalently, a population of DMs with identical underlying tastes). The reason is that as a first model of competition for DMs who exhibit trade-off avoidance, we want the benchmark of a conventionally rational DM to lead to an unambiguous Bertrand-like equilibrium outcome, such that the theoretical implications of trade-off avoidance for default architecture are as transparent as possible.

We analyze symmetric (mixed-strategy) Nash equilibria in the simultaneous-move game played between the two competitors. We first make the simple observation that as far as the game’s payoff structure is concerned, "no default" with symmetric attribute salience is equivalent to "opt out". Next, we show that in this case, if the cost function $c$ is weakly super-modular, the competitors’ alternatives never dominate one another in equilibrium: the realization of $q_2$ is a deterministic decreasing function of the realization of $q_1$. Thus, the competitors’ equilibrium response to "no default" or "opt out" rules removes easy choices from the DM’s landscape.

For our subsequent results, we strengthen the structure of $c$ and assume it is additively separable, such that quality along any dimension can be measured by its cost to the competitor, w.l.o.g. We provide a complete characterization of symmetric equilibrium under the "no default" rule for arbitrary attribute salience. As the two attributes become more equally salient, the expected total quality of the alternatives offered in equilibrium goes down, as does the quality variation across attributes. Thus, there is a sense in which greater quality is positively correlated with a greater amount of "obfusca-
tion" (defined as the gap between true average quality and the quality along the dimension the DM focuses on).

We then analyze the case of an "opt in" default rule. When the DM is maximally indecisive (i.e., he always sticks to the default in the presence of difficult choices), the competitors’ equilibrium behavior is "competitive", in the sense that they mix somehow over alternatives with $c(q^1, q^2) = 1$. Yet, since neither competitor’s alternative dominates another in equilibrium, the DM adheres to the inferior outside option. This is "Buridanic" behavior in the extreme. When the DM is "partially decisive" - i.e., he chooses by default with probability $\lambda \in (0, 1)$ when faced with a difficult choices - equilibrium analysis under "opt in" is significantly more complex and gives rise to multiple equilibria. We assume symmetric attribute salience for this case. In any symmetric equilibrium, domination occurs with positive probability, and as a result the DM switches away from the outside option with a probability that is greater than $1 - \lambda$.

For this case of "opt in" with a partially decisive DM, we characterize a class of symmetric equilibria in which competitors mix over total quality according to some continuous density, and independently shift quality across attributes according to a discrete uniform distribution, such that DM faces an "easy choice" if and only if the realization of the "shifting" component is the same for both competitors. When the DM becomes more indecisive (i.e., when $\lambda$ goes up), the maximal probability of easy choices in these equilibria increases; however, it never exceeds $\frac{1}{3}$. As a result, the default rule that maximizes the DM's equilibrium welfare (measured by expected total consumed quality) is "opt out" (equivalently, "no default"). In the concluding section, we discuss the possible implications of these results for contemporary discussions of default architecture. In particular, we suggest that in settings like retirement saving, the success of "opt out" or "active choice" default architecture owes to a regulatory environment that effectively shuts down adverse equilibrium effects.
2 The Model

Two expected-payoff maximizing competitors play the following simultaneous-move game. Each competitor \( i = 1, 2 \) offers an alternative characterized by a pair \((q^i_1, q^i_2) \in \mathbb{R}^2_+\). For expositional simplicity, we refer to \( q^i_k \) as the "quality" of attribute \( k \) in alternative \( i \) - we discuss various interpretations below. The competitors face a single decision maker (DM) whose choice set consists of the competitors’ alternatives as well as an outside option represented by the quality pair \((0, 0)\). When competitor \( i \)'s alternative is selected by the DM, his payoff is \( 1 - c(q^i_1, q^i_2) \), where \( c \) is referred to as the competitors’ cost function. When the competitor is not chosen by the DM, he earns zero.

The DM chooses from among the three feasible alternatives according to the following procedure. When \( q^i_k \geq q^j_k \) for both \( k = 1, 2 \), with at least one strict inequality - i.e. when one competitor’s alternative dominates the rival alternative - the DM chooses the dominant alternative. We refer to such a situation as an "easy choice". When \( q^i_1 > q^j_1 \) and \( q^i_2 < q^j_2 \) - a case we refer to as a "difficult choice" - the DM resorts to a non-compensatory rule which is sensitive to the presence of default options. Suppose one of the three feasible alternatives is a designated default option. Then, with probability \( \lambda \), the DM acts "indecisively" and selects the default option; with probability \( \alpha^k(1 - \lambda) \), he acts "decisively" and selects the dominant available alternative along dimension \( k \). When choosing by default is impossible, the DM is forced to act "decisively", and he chooses the dominant alternative along dimension \( k \) with probability \( \alpha^k \). We will often denote \( \alpha^1 = \alpha \). The Parameter \( \lambda \) captures the degree of the DM’s "indecisiveness" or "propensity for default bias", while the parameter \( \alpha \) captures the relative salience of the first quality dimension.

To illustrate the implications of the DM's choice rule for the competitors’ payoffs, consider a strategy profile in which \( q^i_1 > q^j_1 \) and \( q^i_2 < q^j_2 \) (i.e., competitor \( i \) performs better along dimension \( k = i \)). Suppose the DM’s default is the outside option. Then, competitor \( i \) earns \( \alpha^i(1 - \lambda)((1 - c(q^i_1, q^i_2)) \).
Now suppose competitor \( i \) serves as the DM’s default. Then, competitor \( i \)'s payoff is \( (\lambda + \alpha^i(1 - \lambda))(1 - c(q_1^i, q_2^i)) \), while competitor \( j \neq i \) earns \( \alpha^j(1 - \lambda)(1 - c(q_1^j, q_2^j)) \). Finally, if the DM cannot choose by default, competitor \( i \) earns \( \alpha^i((1 - c(q_1^i, q_2^i)) \).

The DM’s choice behavior departs from rationality in several dimensions. First, it is sensitive to the default specification; this is a framing effect that conventionally rational DMs do not exhibit. Second, even when we hold the default rule fixed, we can observe violations of rationality. For instance, suppose the default is the outside option and let \( \lambda = 1 \). When the competitors’ alternatives are \((2, 2)\) and \((1, 1)\), the DM chooses the former. However, if we replace the latter alternative with \((1, 3)\), the DM chooses the default \((0, 0)\). This is a simple violation of the Weak Axiom of Revealed Preferences.

We have not specified how the DM chooses when both competitors offer the same quality pair. For expositional simplicity, we will assume that the DM reacts as if this is a "difficult choice". However, the case for such "Burdanic" behavior when the DM faces two literally identical alternative seems weaker than in the case in which he faces distinct alternatives that are hard to compare. A plausible alternative assumption is that when \( q_1 = q_2 > (0, 0) \), the DM chooses the competitors’ alternatives with probability \( \frac{1}{2} \) each. Fortunately, except for one special case (Proposition 3 in Section 3.3), \( q_1 = q_2 \) will typically be a zero-probability event in any symmetric Nash equilibrium (regardless of what we assume about how the DM chooses in this event).

The competitors’ costs and the DM’s welfare
Throughout the paper, we assume that \( c \) is continuous and strictly increasing, with \( c(0, 0) = 0 \). Also, there exist finite \( q^1 \) and \( q^2 \) such that \( c(0, q^2) = c(q^1, 0) = 1 \). When imposing additional structure on \( c \), one should bear in mind that since the DM’s choice procedure is based entirely on ordinal rankings, the cardinal meaning of the quality variables \( q^1 \) and \( q^2 \) is questionable, and therefore we refrain from making assumptions that are based on cardinal quality measurements. For some of the results, we will
require $c$ to be weakly supermodular - i.e., for every two quality pairs $q$ and $r$, $c(q \lor r) + c(q \land r) \geq c(q) + c(r)$, where $q \lor r = (\max\{q^1, r^1\}, \max\{q^2, r^2\})$, $q \land r = (\min\{q^1, r^1\}, \min\{q^2, r^2\})$. For other results, we will impose the stronger assumption of additive separability: $c(q^1, q^2) = c^1(q^1) + c^2(q^2)$. These properties of $c$ are stated purely in terms of the ordinal quality rankings along the two dimensions, hence they are invariant to monotone transformations of $q^k$. In particular, additive separability means that we can assume w.l.o.g that $c^k(q^k) = \frac{1}{2}q^k$, i.e. quality along each dimension is measured by (twice) its cost to the competitor, such that the cost of a competitor’s alternative is equal to its average quality.

We apply similar caution to the welfare analysis. Because the DM’s choice procedure does not involve utility maximization over quality pairs, his choices do not reveal welfare in the traditional revealed-preference sense. Therefore, generally refrain from evaluating the DM’s welfare in equilibrium, and we qualify our analysis when we do.

**Quality vs. prices**

Although we refer to the two attributes in terms of quality, it is of course possible to interpret each one of them as a price component. For instance, in a market setting (where competitors are firms and the DM is a consumer), dimension 1 can be a product price, such that $q^1 = 1 - p$, while dimension 2 is a "proper" quality dimension. In this case, it is natural to assume that $c$ is additively separable and linear in $q^1$; and the requirement that $q^1 \geq 0$ implies that the product’s price is bounded from above by 1. This upper bound may represent a budget constraint that the consumer faces, e.g., it is also possible to interpret both dimensions as prices. For instance, an alternative can be a service contract that specifies state-contingent payments. Under this interpretation, trade-off avoidance can be a consequence of deeply ambiguous beliefs, in line with Bewley’s (1986) notion of Knightian uncertainty as incomplete preferences. The requirement that $p^1, p^2 \leq 1$ may thus be interpreted as follows: the DM has a state-independent willingness
to pay of 1 for the service; he can choose not to use the service after the state is realized, if the payment exceeds the willingness to pay.\footnote{In market applications, the assumption that quality and price are chosen simultaneously is open to criticism, because the time scale for changing product quality is typically longer than for prices.}

**Heterogeneity among DMs**

Our model admits heterogeneity among the population from which the DM is drawn, in terms of their "decisiveness" - i.e., their willingness to take short cuts when facing complex decision problems. The parameter $\lambda$ measures the fraction of indecisive DMs in this population. An indecisive DM is reluctant to take any short cut and eschews active choice, unless the default architecture forces him to do so. A decisive DM is willing to simplify his problem by "editing out" one of the relevant attributes. However, all DMs in the population exhibit trade-off avoidance. It would be natural to augment the model by introducing a fraction of conventional Totreps into the population. This seemingly innocuous modification turns out to be very hard to analyze, and we do not know whether the model admits any mixed-strategy equilibrium in this case. The present model should be viewed as an extreme case, which is diametrically opposed to the conventional model that regards DMs as impeccable "trade-off machines".

In certain market applications of our model, it would make sense to assume that consumers differ in their default assignment. Think of a market for some long-term service, where automatic renewal of annual contracts is the norm. Thus, the current supplier is the default option for an existing consumer whose contract has expired. In contrast, for new consumers who enter the market, the default is the outside option. Our model can be adapted to this scenario, simply by reinterpreting the parameter $\lambda$ as a reflection of heterogenous default assignment rather than as a distribution of behavioral types.
3 Symmetric Equilibrium Analysis

We now turn to analysis of symmetric Nash equilibria in the game described in Section 2, under various default rules.

3.1 No Default

We begin with the case the DM cannot choose by default. Therefore, when the competitors’ alternatives fail to dominate one another, the DM chooses the best-performing competitor along dimension $k$ with probability $\alpha^k$. Without loss of generality, let $\alpha_1 = \alpha \geq \frac{1}{2}$. The case of $\alpha = 1$ is simple (it is formally a special case of Gabaix and Laibson (2006)). In Nash equilibrium, competitors will offer $q^2 = 0$, and $q^1$ will be determined by the equation $c(q^1, 0) = 1$. The reasoning is simple: since the DM never considers dimension 2, competitors have no incentive to compete in this dimension. In contrast, competitive pressures along dimension 1 drive its quality up in "Bertrand fashion", such that in equilibrium competitors must make zero profits.

The case of $\alpha \in [\frac{1}{2}, 1)$ is more interesting.

**Proposition 1 (No easy choices)** Consider the "no default" rule and let $\alpha \in [\frac{1}{2}, 1)$. If $c$ is weakly supermodular, then for any symmetric Nash equilibrium, there exist $\bar{q}^1, \bar{q}^2 > 0$ such that the support of the equilibrium strategy is a continuous and strictly decreasing curve that connects the points $(0, \bar{q}^2)$ and $(\bar{q}^1, 0)$.

Thus, when the DM is forced to make active choices and cannot choose by default, any symmetric Nash equilibrium has the feature that no competitor’s alternative ever dominates another. That is, the DM always faces difficult choices. The key argument in the proof is that if there were two quality pairs $q$ and $r$ in the support of the equilibrium strategy that dominate one
another, then deviating to \((q_1, r_2)\) or \((r_1, q_2)\) would be profitable, thanks to supermodularity of \(c\).

What is the significance of this result? Note that one interpretation of our model is that making difficult choices involves a mental cost, which the DM could successfully avoid if he were allowed to choose by default. When forced to make an active choice, the mental cost is incurred whenever he faces a difficult choice. Our result means that in that case, spontaneous competitive forces "conspire" to maximize this mental cost. Arguably, if there were a prior stage (before the competitors present their offers) in which the DM could avoid the interaction altogether, anticipation of this mental cost could impel him to do so.

For our next results, we strengthen the structure of \(c\) and assume it is additively separable. Recall that w.l.o.g, \(c(q^1, q^2) = \frac{1}{2}(q^1 + q^2)\).\(^4\)

**Proposition 2** Assume the "no default" rule.

(i) Let \(\alpha = \frac{1}{2}\). The game has a unique symmetric Nash equilibrium, in which with probability one competitors play \(q^1 + q^2 = 1\) where \(q^1 \sim U[0, 1]\), such that total cost is \(\frac{1}{2}\).

(ii) Let \(\alpha \in (\frac{1}{2}, 1)\). The game has a unique symmetric Nash equilibrium, in which competitors mix over total cost \(c = \frac{1}{2}(q^1 + q^2)\) according to the cdf

\[
G(c) = \frac{1 - \alpha}{2\alpha - 1} \left[ \frac{\alpha}{1 - c} - 1 \right]
\]

defined over the interval \([1 - \alpha, \alpha]\). The quality along each dimension is a deterministic function of \(c\):

\[
q^1 = \frac{2\alpha}{2\alpha - 1} [c - (1 - \alpha)] \\
q^2 = \frac{2(1 - \alpha)}{2\alpha - 1} [\alpha - c]
\]

\(^4\)The characterization can be extended to any supermodular \(c\). However, since the equilibrium strategy does not have a closed form in this case, we decided to omit it.
When $\alpha \in (\frac{1}{2}, 1)$, competitors mix over average quality $c$ in equilibrium. The greater the asymmetry in the attributes’ salience, the greater the range of values that $c$ gets in equilibrium. Note that the $\alpha \to \frac{1}{2}$ and $\alpha \to 1$ limits of this equilibrium characterization coincide with our analysis for these extreme cases. The expectation of $c$ is

$$E_G(c) = 1 - \frac{\alpha(1 - \alpha)}{2\alpha - 1} \ln \left( \frac{\alpha}{1 - \alpha} \right)$$

which is strictly increasing in $\alpha$ in the range $(\frac{1}{2}, 1)$. In equilibrium $q^1$ takes values in $[0, 2\alpha]$, while $q^2$ takes values in $[0, 2(1 - \alpha)]$. The two components are linked deterministically by the linear equation

$$q^2 = 2(1 - \alpha) - \frac{1 - \alpha}{\alpha}q^1$$

The following corollary describes the competitors’ equilibrium payoffs.

**Corollary 1** The firms’ payoff in symmetric equilibrium is $\alpha(1 - \alpha)$.

To derive this result, consider the quality pair $(q^1, q^2) = (2\alpha, 0)$, which is an extreme point in the support of the equilibrium strategy. When a competitor plays this vector, it wins the DM if and only if he focuses on attribute 1. Therefore, the competitor’s equilibrium payoff is $\alpha \cdot [1 - \frac{1}{2}(0 + 2\alpha)] = \alpha(1 - \alpha)$. Thus, equilibrium payoffs go down and average quality goes up when attribute salience becomes more asymmetric. The intuition is that when $\alpha$ approaches 1, the DM’s choice behavior becomes more predictable, and this strengthens competitive pressures among competitors. At the same time, more asymmetric attribute salience also implies a greater cross-attribute variation in quality, in the sense that the range of values that $|q^2 - q^1|$ becomes wider as $\alpha$ gets closer to 1.
Comment: Rational-choice interpretation

When the "no default" rule is held fixed, the DM’s choice behavior has a simple rational-choice interpretation: a fraction $\alpha^k$ of the population from which the DM is drawn is genuinely interested only in attribute $k$ - i.e. they have a well-defined utility function that is increasing in $q^k$ and constant in $q^{-k}$. From this perspective, it is not surprising that when discrimination is impossible, greater heterogeneity in DMs’ preferences (captured by shifting $\alpha$ toward $\frac{1}{2}$) results in a less competitive equilibrium outcome. For this interpretation to be valid, it is important to assume that the bundling of the two quality attributes is intrinsic, such that competitive pressures will not lead to their "unbundling". And of course, it is implausible when the two attributes are interpreted as price and overall quality. Most importantly, the rational-choice interpretation does not fit our model on the whole, as explained in the previous section, because it breaks down under the "opt in" rule.

Spurious attributes

Throughout this paper, we view the two attributes as intrinsic features of the alternatives. However, suppose that the framing of alternatives as if they have two attributes is spurious, in the sense that from the competitors’ point of view, quality is fully characterized by the scalar $x = q^1 + q^2$. In particular, their cost is purely a function of $x$. For instance, $q^k$ could be interpreted as provision of a certain quantity in a state of Nature indexed by $k$, where the only thing that is relevant for the cost is the total quantity across the two states. Under this assumption, weak supermodularity of $c$ implies that it is a weakly convex function of $x$, i.e. $c''(x) \geq 0$ for every $x \geq 0$. Proposition 2 can be easily extended to this case (as the proof makes transparent). When $\alpha > \frac{1}{2}$, the characterization is the same, except that some parameters are defined implicitly; we omit it for brevity. When $\alpha = \frac{1}{2}$, however, the extension is simple and conveys a novel insight (the proof of Proposition 2 covers this case).
Claim 1 Let $\alpha = \frac{1}{2}$ and assume that $c$ is a weakly convex function of $x = q^1 + q^2$. Under the "no default" rule, the game has a unique symmetric Nash equilibrium, in which competitors play $q^1 \sim U[0, x^*]$, and $q^2 = x^* - q^1$ with probability one, where

$$x^* = \frac{1 - c(x^*)}{c'(x^*)}$$

Thus, the total quality that competitors offer in equilibrium is exactly what a single agent maximizing $x(1 - c(x))$ would choose. If we interpret $x$ as a quantity, this expression means quantity multiplied by the profit per unit sold. In this sense, competing for a trade-off avoiding DM (under the "no default" rule with $\alpha = \frac{1}{2}$) causes competitors to behave like monopolists.

Another way of interpreting $x^*$ is that each competitor $i$ chooses $x_i$ as if competitor $j$ randomly draws $x_j$ from $U[0, 1]$ and the DM rationally chooses the competitor that offers the highest $x$. This is a "dual" interpretation: rather than positing that competitor $j$ plays $x^*$ deterministically and the DM chooses randomly (because of the random breakdown of $x^*$ into the two quality components), here we assume that competitor $j$ chooses $x_j$ randomly and the DM chooses deterministically.

### 3.2 Opt Out

Let us now assume that the DM's default option is one of the competitors' alternatives. To maintain the game's symmetry, suppose that each competitor serves as the DM's default with probability $\frac{1}{2}$, and that the competitors make their strategic decisions before the default is realized. We refer to this default specification as "opt out". As far as the competitors’ payoff function is concerned, "opt out" is equivalent to the "no default" rule with an appropriately adjusted $\alpha$. To see why, note first that defaults are irrelevant when one competitor’s alternative dominates another. Now suppose a strategy profile in which no competitor’s alternative dominates another, and that competitor $i$ outperforms his opponent in some dimension $k$. Then, the ex-
ante probability that the DM ends up choosing competitor $i$ under "opt out" is $\lambda \cdot \frac{1}{2} + (1 - \lambda) \cdot \alpha^k$.

It follows that the DM’s final choice probabilities mimic the "no default" case in which the salience of dimension 1 is $\alpha' = \frac{1}{2} \lambda + \alpha(1 - \lambda)$. By our equilibrium analysis of the "no default" case, the DM will never face easy choices in equilibrium when the default rule is "opt out". In symmetric equilibrium, each competitor is equally likely ex-ante to outperform his opponent on any given dimension. As a result, the DM will switch away from his default option in symmetric equilibrium with probability $\frac{1}{2}(1 - \lambda)$.

### 3.3 Opt In

Let us now assume that the default rule is "opt in" - i.e., the DM’s outside option serves as his default. Throughout this section, we assume that $c$ is additively separable - i.e. $c(q^1, q^2) = \frac{1}{2}(q^1 + q^2)$. Let us first consider the extreme case of a maximally indecisive DM.

**Proposition 3** Let $\lambda = 1$. The set of symmetric Nash equilibrium strategies consists of all probability measures over quality pairs $(q^1, q^2)$ satisfying $q^1 + q^2 = 2$. The DM chooses the outside option in any equilibrium.

The intuition is that a maximally indecisive DM switches away from the outside option only when facing an easy choice. Thus, competitors have nothing to gain from making comparison between alternatives hard. This in turn implies strong competitive pressures, which raise quality along both dimensions and push the competitors’ payoff toward zero. However, since in equilibrium neither competitor’s alternative dominates the other, the DM ends up sticking to his default alternative. The equilibrium outcome is somewhat paradoxical: the offered alternatives have the highest possible total quality (higher quality would result in negative payoffs for the competitors), yet the
DM does not pick them. This extreme "Buridanic" state of affairs is of course a consequence of the assumption that the DM is maximally indecisive.

Proposition 3 highlights the interpretational difficulty pointed out in Section 2: if the symmetric equilibrium is in pure strategies, both competitors offer the same alternative with probability one, and in this case the assumption that the DM acts like Buridan’s Ass seems less plausible. The critique does not apply when the symmetric equilibrium strategy is a smooth density over the line $q^1 + q^2 = 2$. As we shall see, when we perturb the extreme maximal-indecisiveness assumption, by taking the limit $\lambda \rightarrow 1$, we are able to sidestep this difficulty entirely, because $q_1 = q_2$ is a zero-probability event in equilibrium.

The case of "opt in" with $\lambda \in (0, 1)$ turns out to be considerably more difficult to analyze, and there are multiple symmetric Nash equilibria. Let us begin with the following observation.

**Proposition 4** When $\lambda \in (0, 1)$, competitors’ alternatives dominate one another with positive probability in any symmetric Nash equilibrium.

Thus, when the DM is partially decisive, easy choices occur with positive probability in any symmetric Nash equilibrium. Therefore, the probability that the DM switches away from the outside option is above $1 - \lambda$, which is the probability the DM acts decisively. This observation also means that the structure of equilibrium strategies is more elaborate than under the previous specifications, since $q^2$ and $q^1$ can no longer be linked deterministically.

Figure 1 represents the support of a symmetric equilibrium strategy for $\alpha = \lambda = \frac{1}{2}$. It consists of three 45-degree line segments. Each of these segments is realized with probability $\lambda$. The distribution over average quality is given by some $cdf$ over $[0.4, 0.8]$, which is independent of the segment. Note that two quality pairs in the support constitute an easy choice if and only if
they belong to the same segment. Hence, the probability of easy choices is \( \frac{1}{3} \).

For the remainder of this section, we assume \( \alpha = \frac{1}{2} \). We characterize a family of symmetric equilibria, of which the strategy illustrated by Figure 1 is an instance. For this purpose, we introduce some new notation. First, we represent a pure strategy \((\theta_1, \theta_2)\) by the pair \((\pi, \varepsilon)\), where \(\pi = 1 - \frac{1}{2}(\theta_1 + \theta_2)\) is the profit that the quality pair generates for the competitor conditional on being chosen, and \(\varepsilon = \frac{1}{2}(\theta_1 - \theta_2)\) is the quality variation the strategy exhibits.

Second, for any positive integer \(m\), denote

\[
\Lambda = 1 - \frac{1}{2} \cdot \mu_1 - \frac{1}{2} \cdot \mu_2 \cdot \mu_3
\]

The interpretation of \(\sigma\) is simple: it is the probability the DM would be a "captured customer" thanks to his default bias, if the probability of easy choices were \(\frac{1}{n}\).

Let \(n \geq 3\) be an integer and let \(d > 0\). Define \(s^*(n, d)\) to be a mixed strategy that consists of independent randomizations over \(p\) and \(e\), where:
(i) $p$ is distributed according to the \textit{cdf}

$$G(p) = (1 + \sigma n) \left( 1 - \frac{d \sigma n}{p} \right)$$

defined over the interval $[d \sigma n, d \sigma n + d]$.

(ii) $e$ is uniformly distributed over the discrete set

$$\left\{ d \left( k - \frac{1}{2}(n - 1) \right) \right\}_{k=0,1,\ldots,n-1}$$

The strategy illustrated in Figure 1 is thus $s^*(3, 0.4)$.

**Claim 2** Let $\alpha = \frac{1}{2}$ and $\lambda \in (0, 1)$. If $n \in \left[ 1 + \frac{1}{\lambda}, 1 + \frac{3}{\lambda} \right]$ and $d = \frac{2}{\lambda + n(2 - \lambda)}$, then $s^*(n, d)$ is a symmetric Nash equilibrium strategy.

Let us elaborate on the properties of this class of equilibrium strategies.

**Structure of the support and domination probability.** The support of the equilibrium strategy is divided into $n$ line segments, which are vertical in the $(p, e)$ representation (they have a 45° slope in $(q^1, q^2)$ space, as in Figure 1). Each segment corresponds to a different value of $e$. The distance between adjacent segments is $d$, which is also the range of values that $p$ can get. Therefore, domination occurs only within each segment - i.e., two realizations $(p_1, e_1), (p_2, e_2)$ constitute an easy choice if and only if $e_1 = e_2$. Thus, the "easy comparison" binary relation defined over the support is an equivalence relation. A larger $n$ corresponds to an equilibrium with weaker comparability. The probability the DM faces an easy choice is thus $\frac{1}{n}$.

**Switching rates.** The above characterization of the structure of domination implies that the DM will switch away from the outside option with probability
1 − λ + λ · \frac{1}{n}. The restriction on the values that \( n \) can get implies that the switching rate is bounded from above by

\[(1 - \lambda) + \lambda \cdot \frac{\lambda}{1 + \lambda} = \frac{1}{1 + \lambda}\]

Because \( n \) only gets integer values, these upper bounds on switching are not tight. We will see below that the maximal switching rate overall is \( \frac{1}{3} \) (obtained when \( \lambda \) is large). We conjecture that this upper bound is valid for all symmetric equilibria.

**Quality and payoff.** The marginal distributions over \( q^1 \) and \( q^2 \) are identical, with support \([0, dn]\). The upper bound of this interval is strictly above 1 - in this sense, competitors incur a loss along any of the dimensions with positive probability. The expected equilibrium average quality is

\[1 - dn(1 + \sigma n) \ln \left( \frac{1 + \sigma n}{\sigma n} \right)\]

Each competitor’s expected payoff is

\[d \sigma (\sigma n + 1)\]

It can be verified that the expected average quality is strictly greater than \( \frac{1}{2} \), and the payoff is lower than \( \frac{1}{4} \). That is, expected average quality is lower and the competitors’ payoff is higher than when the DM is maximally decisive. In this sense, a less decisive population of DMs makes the equilibrium market outcome more competitive under "opt in".

**Limit equilibria and the DM’s welfare.** As \( \lambda \) tends to 0 (approaching a maximally decisive population of DMs), the permissible values of \( n \) diverge, and the collection of line segments becomes infinitely dense, approximating the line \( q^1 + q^2 = 1 \), i.e. \( p = \frac{1}{2} \) and \( e \sim U \left[ -\frac{1}{2}, \frac{1}{2} \right] \). Equilibrium switching rates thus converge to zero. On the other hand, there are two limit equilibrium distributions when \( \lambda \to 1 \) (i.e., when the DM is almost perfectly indecisive).
In both of them, \( q^1 + q^2 = 2 \) with probability one, i.e. \( p = 0 \); in one of them, \( n = 3 \), such that \( e \) is uniformly distributed over \( \{-\frac{1}{2}, 0, \frac{1}{2}\} \); while in the other, \( n = 4 \), such that \( e \) is uniformly distributed over \( \{-\frac{3}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{3}{5}\} \).

Thus, when the case of a maximally indecisive DM is slightly perturbed, the switching rate is at most \( \frac{1}{3} \). Suppose we define the DM’s welfare as the average quality he ends up getting (recall that given the DM’s behavioral model, this definition lacks "revealed-preference foundations"). Then, the DM’s welfare in the unique symmetric equilibrium under "opt out" (equivalently, "no default") is \( \frac{1}{2} \) (for any \( \lambda \)). Under "opt in", the limit equilibria for \( \lambda \to 1 \) induce expected welfare of at most \( \frac{1}{3} \), whereas the limit equilibrium for \( \lambda \to 0 \) induces an expected welfare of \( \frac{1}{2} \).

For an arbitrary \( \lambda \), the DM’s expected welfare is given by the expected switching rate net of the industry expected profit:

\[
1 - \lambda + \frac{\lambda}{n} - 2d\sigma(\sigma n + 1) = \frac{(1 - \lambda) n^2 + \lambda n + 1}{(2 - \lambda) n^2 + \lambda n}
\]

Maximal welfare (attained at the minimal permissible \( n \) for any given \( \lambda \)) is in the interval \((\frac{1}{3}, \frac{1}{2})\) and decreasing in \( \lambda \) almost everywhere. Thus, given the set of equilibria we have focused on under "opt in", this default regime is inferior to "opt out" (equivalently, "no default") in terms of the DM’s welfare: the increase in the participation rate thanks to the switch from "opt in" to "opt out" outweighs the decrease in the quality of alternatives that competitors offer in equilibrium.

What is the broad intuition behind the relatively low upper bound \((\frac{1}{3})\) on the equilibrium probability of easy choices (in the class of equilibria we have isolated)? When \( \lambda \) is low (i.e., the DM is relatively decisive), competitors have an incentive to increase the probability of hard choices. They can do so by introducing large quality variation across attributes. On the other hand, when \( \lambda \) is high, competitors have an incentive to induce a high degree of comparability in order to attract the indecisive DM away from the outside
option. However, comparability in this model is established by domination; and since the quality variation $e$ can take values in a wide range, a competitor has to offer high quality along both dimensions in order to attain a substantial probability of domination. Thus, attaining domination is quite costly for the competitor (in terms of his payoff conditional on being chosen by the DM), and this restraints the incentive to increase comparability.

The class of equilibria under consideration turns out to be fully characterized by two properties that can be distilled from the above description. A mixed strategy $\sigma$ satisfies independence if it induces statistically independent distributions over $\pi$ and $\epsilon$. We say that $\sigma$ satisfies constant comparability if $\Pr\{(\pi_1, \epsilon_1) \text{ dominates } (\pi_2, \epsilon_2) \mid (\pi_1, \pi_2)\}$ is the same for almost all $(\pi_1, \pi_2)$, where $(\pi_1, \epsilon_1)$ and $(\pi_2, \epsilon_2)$ are two independent draws from $\sigma$.

**Proposition 5** Let $\alpha = \frac{1}{2}$ and $\lambda \in (0, 1)$. If a symmetric Nash equilibrium strategy satisfies independence and constant comparability, it must take the form $s^*)(n, d)$, where $n \in \left[1 + \frac{1}{\lambda}, 1 + \frac{3}{\lambda}\right]$ and $d = \frac{2}{\lambda + n(2 - \lambda)}$.

While the two properties lack an a priori justification, they are of interest because they provide a link to models of price competition under limited comparability. In Varian (1980), the fraction of consumers who make price comparisons is assumed to be an exogenous constant. Therefore, equilibria in our model that exhibit constant comparison probability may be viewed as a "foundation" for this constant. In Piccione and Spiegler (2012), independence and constant comparability are logically linked by an underlying property of the comparability structure.

### 4 Related Literature

This paper is the first to analyze competition for "trade-off averse" DMs who follow an entirely non-compensatory choice procedure. In an independent,
somewhat related paper, Papi (2014b) analyzes a model in which consumers are limited in the number of attributes they can trade off. When market products contain more attributes, consumers focus on a subset which is a function of firms' marketing strategies. Thus, one key difference between the two works is that in our model, consumer choice is entirely based on ordinal rankings, whereas in Papi's model, consumers maximize a continuous utility function over a restricted set of attributes. Papi (2014a) axiomatizes a choice procedure that mixes compensatory and non-compensatory elements, and applies it to a Stackelberg model. In his model, the DM uses a non-compensatory procedure only to shrink the choice set into a small "consideration set", to which he applies a well-defined utility function. Bachi (2014) studies uni-dimensional price competition when consumers are unable to perceive small price differences.

Models of individual choice with an explicit non-compensatory component have been studied in the literature. Rubinstein (1988) analyzed a choice procedure related to ours, where the DM regards one alternative in $\mathbb{R}^2$ as dominating another if it is "approximately the same" along one dimension and significantly better along the other. Mariotti and Manzini (2007) axiomatized a "sequentially rationalizable" choice procedure that employs a succession of binary relations to eliminate alternatives from the choice set.

More generally, choice models in which the DM tends to stick to a status-quo / default alternative when facing "too little dominance" have been studied by Masatlioglu and Ok (2005, 2014) and Teper and Riella (2014), among others - extending a tradition of multi-utility representations of incomplete preferences due to Bewley (1986) and Ok (2002). A key feature in Masatlioglu and Ok's axiomatizations is that they examine choice problems with and without a default; this mirrors the distinction between the "opt in" and "no default" regimes in our model. Dean (2008) conducted an experimental test of axioms that characterize various families of models of decision avoidance. Finally, the randomness in our DM’s response to "hard choices"
(which results from the "decisiveness" typology) links it to recent models of stochastic choice, where randomness derives from new primitives (e.g., consideration sets in Manzini and Mariotti (2014)), unlike conventional random-utility models.

The market interpretation of our model links it to the growing literature on "behavioral industrial organization" (see Spiegler (2011) for a textbook treatment). Within this literature, two papers are most closely related. Gabaix and Laibson (2006) analyze a model in which two firms compete in price pairs, where a fraction of consumers are unaware of dimension 2, and thus choose purely on the basis of price rankings along dimension 1, while the remaining consumers are conventionally rational and choose the firm with the lowest true price. Consumers have an outside option, the value of which is correlated with their type (the interpretation is that more sophisticated consumers are more likely to find a good outside option). Indeed, the "no default" version of our model can be interpreted in terms of unawareness: the DM focuses on one dimension because he is unaware of the other. An aspect of Gabaix and Laibson (2006) which is not dealt with in this paper is the endogeneity of attribute salience due to firms' disclosure decisions.

Spiegler (2006) analyzes a model in which $n$ firms choose price $cdfs$ over $(-\infty, 1]$. A firm's profit conditional on being chosen is the expected price according to its own $cdf$. The consumer chooses by taking a sample point from each of the $cdfs$ and selecting the cheapest firm in his sample. As Spiegler (2006) notes, this can be viewed as a reduced form of a model in which firms choose infinite-dimensional price vectors and the consumer chooses according to the price ranking in a randomly selected dimension. This interpretation forms a clear link with the present model, and suggests an interesting generalization of our model to the case of $n$ firms and $K$ dimensions, in which consumers choose according to some probabilistic aggregation of the ordinal rankings along each dimension. From this perspective, Spiegler (2006) assumes a specific aggregation rule - random dictatorship - and takes the limit
$K \to \infty$. From a technical point of view, this general problem has affinities with the majority auction games studied by Szentes and Rosenthal (2003).

Piccione and Spiegler (2012) present an alternative approach to modeling market competition when consumers have limited ability to make comparisons. A market alternative consists of a "real price" and a "price format", and consumers are able to make a price comparison (and thus choose the cheapest firm) if and only if the two firms’ price formats are comparable, according to some exogenous comparability structure. Carlin (2009) and Chioveanu and Zhou (2013) study special cases of this limited comparability formalism and extend them to the many-firms case. All these papers can be viewed as extensions of Varian (1980), who studied price competition when an exogenous fraction of the consumer population does not make comparisons. The new models essentially endogenize this parameter as a consequence of the firms equilibrium obfuscation tactics. Piccione and Spiegler (2012) assume that consumers have no outside option. Following the example of the present paper, Spiegler (2015) extends the Piccione-Spiegler model to incorporate an outside option, and performs a rudimentary comparison between the "opt in" and "opt out" default rules.

Finally, the role of salience in choices between multi-attribute objects has been recently studied by Koszegi and Szeidl (2013) and Bordalo et al. (2013a). These papers model salience as a systematic distortion of decision weights, while the present paper captures the salience of an attribute by the probability it is considered by a trade-off avoiding DM. Bordalo et al. (2013b) and Spiegler (2013) analyze market models in which consumers’ decision weights are endogenously determined by firms’ pricing and marketing equilibrium strategies.
5 Concluding Remarks

This paper explored the theoretical implications of trade-off avoidance for competitive behavior, in the simplest possible environment, in which two competitors compete in two-dimensional objects for a single DM, such that if the DM were conventionally rational the interaction would collapse to standard Bertrand competition. We assumed instead that the DM follows a (probabilistic) non-compensatory choice procedure that purely responds to the ordinal rankings over feasible alternatives. Crucially, we allowed the DM's procedure to exhibit a "default bias", in the sense of selecting a default alternative with positive probability when faced with a "difficult choice" where none of the alternatives offered by the competitors dominates another. We saw that the specification of the default option has important implications for the structure of equilibrium behavior. Depending on the magnitude of the DM's default bias, a switch from an "opt in" to an "opt out" or "no default" rule leads to a decrease in the expected quality of the alternatives that are offered in equilibrium, coupled with an increase in the probability of "difficult choices".

Although our inquiry in this paper was abstract and not tied to any particular application, we believe that the lessons from our analysis can have interesting applications for more concrete market situations, where our competitors are interpreted as firms and our DM is interpreted as a consumer.

Horizontal and vertical differentiation

Economists' conventional account of horizontal product differentiation in markets is that it is a response to heterogeneity of tastes among consumers. An alternative point of view, proposed by Piccione and Spiegler (2012), is that some of the observed differentiation can be a consequence of firms' response to consumers' limited ability to compare market alternatives. In more traditional models of price competition with "comparability frictions" (mainly Varian (1980) and its variants), firms compete in scalars (prices)
and the probability that consumers make a comparison is exogenous. Such models cannot capture horizontal differentiation - by construction, the only differentiation they admit is vertical, along the scalar dimension.

The present paper (and, using a different approach to modeling limited comparability, Piccione and Spiegler (2012)) provides tools for capturing horizontal differentiation as a response to limited comparability. In our model, symmetric mixed-strategy equilibrium can be said to exhibit "unambiguous vertical differentiation" when two different realizations of the equilibrium strategy dominate one another. Likewise, an equilibrium can be said to exhibit "unambiguous horizontal differentiation" when two realizations of the equilibrium strategy induce the same total quality. We saw that under "opt out" or "no default", unambiguous vertical differentiation never occurs in equilibrium. Moreover, when \( \alpha = \frac{1}{2} \), total quality is constant in equilibrium, such that the equilibrium strategy exhibits maximally unambiguous horizontal differentiation. This differentiation has nothing to do with heterogenous consumer preferences, and everything to do with the firms' attempts to take advantage of consumers' default bias in response to "difficult choices". In contrast, under "opt in", the equilibrium strategies we focused on display patterns of both unambiguous vertical differentiation and unambiguous horizontal differentiation.

We believe that our modeling approach can generate interesting patterns of vertical and horizontal differentiation in models that try to provide a more faithful approximation of real-life market situations, and thus illuminate the question of how much of observed product differentiation in markets is due to limited comparability as opposed to underlying taste heterogeneity - especially when default or status-quo options are an observed parameter of consumers' choice sets.

**Implications for "Default Architecture"**

The observation that people tend to stick to default options in the presence of difficult choices implies that "default architecture" can have dramatic im-


lications for eventual choice patterns. This has led researchers (e.g. Thaler and Sunstein (2008), Beshears et al. (2012)) to advocate switching from opt-in default regimes to "no default" (i.e. active choice) or "opt-out" rules, in settings such as retirement savings by employees. It should be emphasized that in these settings, spontaneous and direct competition among firms over employees is not the norm; instead, contracts are mediated by employers. In other words, these markets are regulated de facto, except that the regulator is not the government but the employer.

The modeling exercise in this paper helps clarifying the respective roles of this de-facto regulation and default architecture in such settings, and speculating about the equilibrium effects of redesigning defaults if employers ceased to mediate the interaction between funds and savers, and spontaneous and direct competition among funds over savers were the norm. What would be the consequences of default architecture in this case? Our analysis suggests that a shift from "opt in" to "opt out" or "no default" would indeed raise the overall level of participation in retirement saving programs, but at the same time it would raise the management fees that funds charge. The magnitude of this effect will be large if the population of savers is relatively "indecisive". The intuition is that the opt-out rule gives funds greater effective market power; because they benefit from default bias, they have a stronger incentive to induce it by creating difficult choices for savers.

A broad lesson from our exercise is that when we wish to analyze regulatory interventions that address consumer decision errors, it is important to have an explicit procedural model of consumer choice, which provides a concrete "story" behind the consumers’ errors, and enables us to speculate about the market equilibrium’s response to the intervention. For further exploration of this theme, see Spiegler (2014).
References


Appendix: Proofs

Let us introduce some notation that will serve us in what follows. A symmetric mixed equilibrium strategy is a probability measure \( \mu \) over the set \( \{(q^1, q^2) \mid c(q^1, q^2) \leq 1\} \). Let \( F^k \) denote the marginal cdf over \( q^k \) induced by \( \mu \). That is,

\[
\begin{align*}
F^1(q^1) &= \int_{q^2} \int_{r^1 \leq q^1} d\mu(r^1, q^2) \\
F^2(q^2) &= \int_{q^1} \int_{r^2 \leq q^2} d\mu(q^1, r^2)
\end{align*}
\]

Let \( F^{k-} \) be the left limit of \( F^k \), i.e. \( F^{k-}(x) = \lim_{y \to x^-} F^k(y) \). Let \( \bar{q}^k \) denote the supremum of the support of \( F^k \).

When \( c \) is additively separable, namely \( c(q^1, q^2) = \frac{1}{2}(q^1 + q^2) \), we will sometimes denote \( p^k = 1 - q^k \) and interpret it as the profit, or "price", associated with dimension \( k \), and use \( p = \frac{1}{2}(p^1 + p^2) \) to denote the competitor’s profit conditional on being chosen. In the same vein, we sometimes denote \( p(q) = 1 - c(q) \).

Proposition 1

Let us first establish that \( \mu \) is continuous, such that \( F^{k-} \equiv F^k \). Assume, in contradiction, that w.l.o.g \( F^1 \) contains an atom on some \( q^1 \). Consider the lowest \( q^2 \) such that the support of \( \mu \) contains \((q^1, q^2)\). If \( c(q^1, q^2) = 1 \) then competitors make zero profits in equilibrium. However, this is impossible when \( \alpha \in (\frac{1}{2}, 1) \), because competitors can secure a strictly positive profits, by playing a mixed strategy with full support on \( \{(q^1, q^2) \mid c(q^1, q^2) \leq 1\} \). Now consider the case in which \( c(q^1, q^2) < 1 \). Here a conventional "undercutting" argument applies: if a competitor deviates to \((q^1 + \varepsilon, q^2)\), where \( \varepsilon > 0 \) is
arbitrarily small, the increase in the competitor’s probability of being chosen overweights his loss in profit conditional on being chosen.

Next, we show that the supports of \( F^1 \) and \( F^2 \) contain no gaps - that is, for each \( k = 1, 2 \), the support of \( F^k \) is \([0, \bar{q}^k]\), where \( \bar{q}^k > 0 \). Assume the contrary. W.l.o.g, let \([a, b]\) be a maximal interval such that \( F^1(q^1) = c \) for any \( q^1 \in [a, b] \). Let \((b, q^2)\) be some element in the support of \( \mu \). If a competitor deviates to \((q^1, q^2)\) where \( q^1 \in (a, b) \), the competitor’s probability of being chosen does not change, but his profit conditional on being chosen increases, hence the deviation is profitable.

Having established these two properties of \( F^1 \) and \( F^2 \), we reach the key argument in the proof. Suppose that the support of \( \mu \) contains two pairs \( q = (q^1, q^2), r = (r^1, r^2) \), such that \( r > q \). By the previous argument, \( F^k(r^k) - F^k(q^k) = \varepsilon^k > 0 \). Both \( q \) and \( r \) are best-replies to \( \mu \). In order for deviations to either \((q^1, r^2)\) or \((q^2, r^1)\) to be unprofitable, we must have

\[
(1 - c(q^1, r^2))(\alpha^1 F^1(q^1) + \alpha^2 F^2(r^2)) \leq (1 - c(q^1, q^2))(\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2))
\]

\[
(1 - c(r^1, q^2))(\alpha^1 F^1(r^1) + \alpha^2 F^2(q^2)) \leq (1 - c(r^1, r^2))(\alpha^1 F^1(r^1) + \alpha^2 F^2(r^2))
\]

where the R.H.S (L.H.S) of the first inequality is the payoff from \( q \ ((q^1, r^2)) \), and the R.H.S (L.H.S) of the second inequality is the payoff from \( r \ ((r^1, q^2)) \). Adding up the two inequalities and rearranging, we obtain

\[
[\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)] [c(q^1, r^2) + c(q^2, r^1) - c(q^1, q^2) - c(r^1, r^2)] \\
\geq \alpha^1 \varepsilon^1 [c(r^1, r^2) - c(r^1, q^2)] + \alpha^2 \varepsilon^2 [c(r^1, r^2) - c(q^1, r^2)]
\]

While the R.H.S of this inequality is strictly positive, weak supermodularity of \( c \) implies that the L.H.S is non-positive (note that \( q = (q^1, r^2) \land (r^1, q^2) \), 
\( r = (q^1, r^2) \lor (r^1, q^2) \)), a contradiction.

Since we have established that the support of \( F^k \), \( k = 1, 2 \), is \([0, \bar{q}^k]\), where \( \bar{q}^k > 0 \), and that the support does not contain points that dominate one another, it follows that the support is a continuous curve that connects
(0, \bar{q}^2) \) and \((\bar{q}^1, 0)\).

**Proposition 2**

By Proposition 1, we can describe the support of \(\mu\) by a continuous and strictly decreasing function \(\gamma : [0, \bar{q}^1] \to [0, \bar{q}^2]\), where for each \(q^1\) in the support of \(F^1\), \(g(q^1)\) is the unique \(q^2\) for which \((q^1, q^2)\) is in the support of \(\mu\). Therefore,

\[
F^2(g(q^1)) = 1 - F^1(q^1)
\]

for every \(q^1 \in [0, \bar{q}^1]\). Since \(F^1\) and \(F^2\) are strictly increasing, they are differentiable almost everywhere, such that the slope of \(g\) is

\[
g' = -\frac{dF(q^1) / dq^1}{dF^2(q^2) / dq^2}
\]

for almost every \((q^1, q^2)\) along the graph of \(g\).

Let us now write down an individual competitor’s payoff function when the opponent plays \(\mu\):

\[
\pi(q^1, q^2) = [1 - c(q^1, q^2)] [\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)]
\]

In equilibrium, first-order conditions must hold. Thus, for both \(k = 1, 2\), the equation

\[
[\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)] \cdot \frac{\partial c(q^1, q^2)}{\partial q^k} = \alpha^k \cdot \frac{dF^k(q^k)}{dq^k} \cdot [1 - c(q^1, q^2)]
\]

must hold almost everywhere along the graph of \(g\). Let us now invoke the property that \(c\) is purely a function of \(q^1 + q^2\), and with a sight abuse of notation, write \(c(q^1 + q^2)\). Then, the L.H.S of the equations for \(k = 1\) and \(k = 2\) are identical, and so we obtain

\[
\frac{dF^1(q^1)}{dF^2(q^2)} = \frac{\alpha^2}{\alpha^1}
\]
By (2), we conclude that

$$g'(q^1) = -\frac{\alpha^2}{\alpha^1}$$

almost everywhere along the graph of $g$. Therefore, we can write $g$ as follows:

$$g(q^1) = \bar{q}^2 - \frac{\alpha^2}{\alpha^1}q^1$$

(4)

Let us now distinguish between two cases.

(i) $\alpha^1 = \frac{1}{2}$. Then, $g' = -1$. This means that $\bar{q}^1 = \bar{q}^2 = q^1 + q^2$ and $c(q^1 + q^2)$ is constant for every $(q^1, q^2)$ in the support of $\mu$. Plug (1) into (3) and obtain the simplified equation

$$\frac{\partial c(q^1 + q^2)}{\partial q^k} = \frac{dF^k(q^k)}{dq^k} \cdot [1 - c(q^1 + q^2)]$$

Since $c(q^1 + q^2)$ is constant, $\frac{\partial c(q^1 + q^2)}{\partial q^k}$ is constant as well. Thus, $\frac{dF^k(q^k)}{dq^k}$ is constant as well, which implies that $q^k$ is distributed uniformly over $[0, \bar{q}^k]$ and

$$\frac{dF^k(q^k)}{dq^k} = \frac{1}{\bar{q}^k}$$

for almost every $q^k \in [0, \bar{q}^k]$ and (3) can be written as:

$$c'(x) \cdot x = 1 - c(x)$$

where $x = \bar{q}^k$. Finally, plug $c(q^1 + q^2) = \frac{1}{2}(q^1 + q^2)$, and obtain $x = 1$, which pins down the characterization.

(ii) $\alpha^1 \in (\frac{1}{2}, 1)$. The two extreme points in the support, $(0, \bar{q}^2)$ and $(\bar{q}^1, 0)$, must both generate the equilibrium payoff:

$$\alpha^1 \cdot (1 - c(\bar{q}^1 + 0)) = \alpha^2 \cdot (1 - c(0 + \bar{q}^2)) = \pi$$

(5)
The two points are also linked by (4), if we plug \( g(\bar{q}) = 0 \). Combining these two equations, we obtain a solution for \( \bar{q}, q^2 \) and for the equilibrium payoff \( \pi \). Moreover, according to (4), every realization of total cost \( c(q^1 + q^2) \) in this interval is associated with a unique \((q^1, q^2)\), as given in the statement of the proposition. Let us derive \( F^1 \). Since every \( (q^1, q^2) \) in the support of \( \mu \) must be a best-reply, we must have that for every \( q^1 \in [0, \bar{q}] \):

\[
[1 - c(q^1 + g(q^1))] \cdot \left[ \alpha F^1(q^1) + \alpha^2 F^2(g(q^1)) \right] = \pi
\]

(6)

Since every \( q^1 \) is associated with a unique \( c = c(q^1 + g(q^1)) \) which increases with \( q^1 \), \( F^1(q^1) = G(c(q^1)) \), where \( G \) is the induced cdf over \( c \). Plugging (1) and (4) into (6), we obtain an explicit expression for \( F^1 \) over \([0, \bar{q}]\), and hence also for \( G \):

\[
G(c) = \frac{1}{2\alpha - 1} \left[ \frac{\pi}{1 - c} - (1 - \alpha) \right]
\]

Let us now plug \( c(q^1 + q^2) = \frac{1}{2}(q^1 + q^2) \). By (4) and (5), \( \bar{q}^1 = 2\alpha \) and \( \bar{q}^2 = 2(1 - \alpha) \), such that the equilibrium payoff is \( \pi = \alpha(1 - \alpha) \). This pins down \( G \) and \( g \), hence the values that \( c \) can get, as well as the values of \((q^1, q^2)\) as a function of \( c \).

The last step is checking that there are no profitable deviations. It suffices to consider deviations to pure strategies \((q^1, q^2) \in [0, \bar{q}] \times [0, \bar{q}]\). It is easy to verify that given the explicit expressions for \( F^1 \) and \( F^2 \), the payoff function

\[
\pi(q^1, q^2) = [1 - c(q^1 + q^2)] \left[ \alpha F^1(q^1) + \alpha^2 F^2(q^2) \right]
\]

is decreasing (increasing) in both arguments when \((q^1, q^2)\) is above (below) the graph of \( g \), hence the maximal payoff is obtained at the points along \( g \).

**Proposition 3**
Assume there exists a symmetric equilibrium strategy that assigns positive
probability to $q^1 + q^2 < 2$. Let $x^* < 2$ denote the lowest value of $q^1 + q^2$ in the support of the equilibrium strategy $\mu$. Consider some $(q^1, q^2)$ in the support of $\mu$ for which $q^1 + q^2 = x^*$. By definition, $(q^1, q^2)$ does not dominate any quality pair in the support of $\mu$ and therefore generates zero profits. However, if competitor 1, say, deviates by mixing with full support over \( \{(r^1, r^2) \mid x^* < r^1 + r^2 < 2\} \), it will dominate competitor 2’s quality pair with positive probability, and thus make strictly positive profits, a contradiction.

In the other direction, assume that competitor 1 plays some strategy $\mu$ in which $q^1 + q^2 = 2$ with probability 1. Competitor 2 can dominate $\mu$ only by playing $q^1 + q^2 > 2$, which results in a strictly negative payoff. Any strategy that dominates $\mu$ with zero probability (including $\mu$ itself) yields a zero profit and therefore is a best response against $\mu$.

**Proposition 4**

First, recall that when the DM acts "decisively" (an event that occurs with probability $1 - \lambda$), he chooses purely according to $q^k$ with probability $\alpha^k$, where $\alpha^1 + \alpha^2 = 1$ and $\alpha^1 \geq \frac{1}{2}$. Second, the proof that in symmetric Nash equilibrium, $F^1$ and $F^2$ are atomless and their supports contain no "holes" (see the proof of Proposition 1) can be extended to the $\lambda > 0$ case. We omit the proof for brevity - the arguments essentially are the same.

Now suppose that contrary to the claim, there is a symmetric Nash equilibrium in which domination occurs with zero probability. Then, we can describe the support of the equilibrium strategy $\mu$ by a continuous and strictly decreasing function $g : [0, \bar{q}^1] \to [0, \bar{q}^2]$, where for each $q^1$ in the support of $F^1$, $g(q^1)$ is the unique $q^2$ for which $(q^1, q^2)$ is in the support of $\mu$. Thus, $F^1(q^1) = 1 - F^2(q^2)$ for $(q^1, q^2)$ along the support.

Consider a pure strategy $(q^1, q^2)$ in the interior of the support of $\mu$. The payoff from this strategy is

$$\left[1 - \frac{1}{2}(q^1 + q^2)\right] \cdot (1 - \lambda) \cdot \left[\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)\right] \quad (7)$$
hence this is the competitors’ equilibrium payoff. Let \( q, r \) be two points in the support of \( F \). Assume w.l.o.g \( q^1 < r^1, q^2 > r^2 \). Consider deviations to \( q \land r = (q^1, r^2) \) and \( q \lor r = (r^1, q^2) \). If a competitor deviates to \( q \land r \), his payoff will be

\[
1 - \frac{1}{2}(q^1 + r^2) \cdot (1 - \lambda) \cdot [\alpha^1 F^1(q^1) + \alpha^2 F^2(r^2)]
\]

On the other hand, if the competitor deviates to \( q \lor r \), his payoff will be

\[
1 - \frac{1}{2}(r^1 + q^2) \cdot [(1 - \lambda) (\alpha^1 F^1(r^1) + \alpha^2 F^2(q^2)) + \lambda (F^1(r^1) - F^1(q^1))]
\]

In order for \( \mu \) to be an equilibrium, both expressions must be weakly below the payoff at \( q \), which is the same as the payoff at \( r \).

Denote \( A = F^1(q^1), B = F^1(r^1) - F^1(q^1) \) and \( C = 1 - F^1(r^1) \). Then, the payoffs at the four points \( q, r, q \land r \) and \( q \lor r \) can be written as follows:

\[
\begin{align*}
\pi(q) &= p(q) (1 - \lambda) (\alpha^1 A + \alpha^2 B + \alpha^2 C) \\
\pi(r) &= p(r) (1 - \lambda) (\alpha^1 A + \alpha^1 B + \alpha^2 C) \\
\pi(q \land r) &= p(q \land r) (1 - \lambda) (\alpha^1 A + \alpha^2 C) \\
\pi(q \lor r) &= p(q \lor r) (1 - \lambda) \left( \alpha^1 A + \alpha^1 B + \alpha^2 B + \frac{\lambda}{1-\lambda} B + \alpha^2 C \right)
\end{align*}
\]

It follows that

\[
\begin{align*}
\pi(p) + \pi(q) - \pi(q \land r) - \pi(q \lor r) \\
&= B (1 - \lambda) \left( \alpha^1 (p(r) - p(q \lor r)) + \alpha^2 (p(q) - p(q \lor r)) - \frac{\lambda}{1-\lambda} \right) (8)
\end{align*}
\]
Note that $B > 0$. If $q$ and $r$ are sufficiently close, we have

\[
p(r) - p(q \lor r) < \frac{\lambda}{1 - \lambda 2\alpha^1} \quad \frac{1}{1 - \lambda 2\alpha^2}
\]

such that expression (8) is strictly negative, which means that the deviation to $q \lor r$ or $q \land r$ is profitable, a contradiction.

**Claim 2**

Let $n \in [1 + \frac{1}{3}, 1 + \frac{3}{3}]$ and $d = \frac{2}{\lambda + n(2 - \lambda)}$. The strategy $s^*(n, d)$ induces a marginal distribution over $q^k$, with support $[0, \bar{q}^k]$ where $\bar{q}^k = nd$, $k = 1, 2$. It is easy to verify that each point on the support of $s^*(n, d)$ yields the same payoff. Clearly, when we look for profitable deviations from $s^*(n, d)$, we need only look for pure strategies $(q^1, q^2) \in [0, \bar{q}^1] \times [0, \bar{q}^2]$. From now on, we adhere to the $(p, e)$ representation of strategies. We index the $n$ values that $e$ by $k = 0, 1, ..., n - 1$. Let $l = d \sigma n$ and $h = d (\sigma n + 1)$ denote the lowest and highest values in the support of the marginal distribution over $p$. Define $L^k = \{(p, e) \mid p \in [l, h] \text{ and } e = e^k\}$. That is, $L^k$ is one of the $n$ line segments that constitute the support of $s^*(n, d)$, which is associated with $e^k$. There are three cases to consider.

**Case 1**: Deviation to $(p, e)$ where $p \geq h$.

For any $p \geq h$, it suffices to look for the most profitable deviation $(p, e)$. The fact that $e$ is uniformly distributed over evenly spaced values independently of $p$, and that $d = h - l$ and $d = e^k - e^{k-1}$, the total length of the $L^k$ segments that $(p, e)$ is dominated by is independent of $e$. Moreover, the number of segments that partially dominate $(p, e)$ is at most 2. Because of the concavity of $G$, it is more profitable to be partially dominated by one segment (the dominating prices on that segment being $[l, l + x + y]$ for some $x$ and $y$) than being partially dominated by two segments (where the dominating prices are $[l, l + x]$ and $[l, l + y]$). This implies that for a given $p$ the most
profitable \( e \) maximizes the number of line segments \( L^k \) that entirely dominate \((p, e)\). Therefore, in the sequel we restrict attention w.l.o.g to \( e = 1 - p \), i.e., to \((0, q)\), where \( q < 1 - h \), in the \((q^1, q^2)\) representation.

Consider a deviation to \( p = h + (m + x) \frac{d}{2} \), \( m = 0, 1, ..., n - 2 \), \( x \in [0, 1] \).

The payoff is

\[
\left( h + (m + x) \frac{d}{2} \right) \left( \frac{1 - \lambda}{2} + \frac{1 + \lambda}{2} \frac{1}{n} (1 + m + G (l + dx)) \right)
\]

Note that for \( x = 0 \), the payoff at \( m = 0 \) (which corresponds to no deviation) is higher than at \( m = 1 \) if and only if \( n \leq 1 + \frac{3}{\lambda} \). Second, if this is the case, then the payoff continues to decrease for any \( m > 1 \) (\( n \leq 1 + \frac{3}{\lambda} \) is a sufficient condition for the derivative of the payoff with respect to \( m \) is negative for \( m > 1 \) and \( x = 0 \)).

Moreover, the derivative of the payoff function w.r.t \( x \) (for a given \( m \)) is increasing. Thus, for each \( m \), the maximal payoff is achieved at \( x \in \{0, 1\} \). This, together with the previous result, imply that deviations to \( p \geq h \) are unprofitable if and only if \( n \leq 1 + \frac{3}{\lambda} \).

**Case 2**: Deviation to \((p, e)\) where \( p \leq l \).

By the same argument as in Case 1, the most profitable deviation for a given \( p \leq l \) is to \( e \) that maximizes the number of entire segments \( L^k \) which are dominated by \((p, e)\). Therefore, in the sequel we restrict attention w.l.o.g to \( e = (l + e^0) - p \), i.e., to \((\bar{q}^1, q)\), where \( q > 1 - l \), in the \((q^1, q^2)\) representation.

Consider a deviation to \( p = l - (m + x) \frac{d}{2} \), \( m = 1, ..., n - 2 \), \( x \in [0, 1] \). The payoff is:

\[
\left( l - (m + x) \frac{d}{2} \right) \left( \frac{1 - \lambda}{2} + \frac{1 + \lambda}{2} \frac{1}{n} (1 + m + (1 - G (h - dx))) \right)
\]

Note that the payoff at \( m = 0 \) (corresponding to no deviation) is higher than at \( m = 1 \) if and only if \( n \leq 1 + \frac{\lambda}{(1-\lambda)\lambda} \). Second, if this is the case, then the payoff continues to decrease for any \( m > 1 \). Note that \( n \leq 1 + \frac{3}{\lambda} \) implies
\[ n \leq 1 + \frac{3- \lambda}{(1- \lambda) \lambda}. \]

The derivative of this function w.r.t \( x \) implies the following: (i) it is increasing in \( x \) for \( m \leq \sigma n - 1 \); (ii) it is negative for \( m > \sigma n - 1 \). Thus, it is enough to check for deviation to \( x = 0 \) and \( m \leq \sigma n - 1 \), and by the previous result, these deviation are unprofitable for \( n \leq 1 + \frac{3}{\lambda} \).

**Case 3:** Deviation to \((p, e)\) where \( l \leq p \leq h \).

Fix \( p \in [l, h] \). Because any \((p, e)\) where \( p \) is in this interval is comparable to points in at most 2 segments, and because all segments have the same probability distribution, it is enough to check for deviations from \((p, e^0)\) to \((p, e^0 + x)\), where \( x \in (0, \frac{d}{T}) \). Thus, \((p, e^0 + x)\) is comparable only to points on \( L^0 \) and \( L^1 \). Consider these three cases:

(i) \( p + x \leq h \) and \( p - x \geq l \). In this case \((p, e^0 + x)\) is not dominating, nor being dominated by, any point in \( L^1 \). As \( x \) increases, \((p, e^0 + x)\) is dominated by less points on \( L^0 \) but also dominates less. The competitor’s net gain of market share is

\[
\frac{1}{n} \left( \frac{1 - \lambda}{2} [G(p) - G(p - x)] - \frac{1 + \lambda}{n} \frac{1}{2} [G(p + x) - G(p)] \right)
\]

Substituting \( G \), we obtain the following condition for the deviation’s profitability:

\[
\frac{p + x}{p - x} > \frac{1 + \lambda}{1 - \lambda}
\]

It is easy to verify that the L.H.S is maximized at \( p = l + \frac{d}{T} \) and \( x = \frac{d}{2} \), and the inequality is satisfies iff \( n < 1 + \frac{1}{\lambda} \).

(ii) \( p + x > h \). In this case \((p, e^0 + x)\) is dominated by some prices in \( L^0 \) and in \( L^1 \), but not dominating any point. Because the total length of the segments of \( L^0 \) and \( L^1 \) that dominate \((p, e^0 + x)\) is constant for any such \( x \), the concavity of \( G \) implies that it is more profitable to be dominated by \( L^0 \).
alone than by both. That is, this deviation is strictly less profitable than the
devention to \((p, e^0 + h - p)\) which is covered in case \((i)\).

\((iii)\) \(p - x < l\). In this case \((p, e^0 + x)\) is dominating some prices in \(L^0\) and
in \(L^1\), but not being dominated by any point. Because the total length of
segments of \(L^0\) and \(L^1\) that \((p, e^0 + x)\) dominates is constant for any such
\(x\), the concavity of \(G\) implies that it is more profitable to dominated \(L^0\)
alone. That is, this deviation is strictly less profitable than the deviation to
\((p, e^0 + p - l)\), which is covered in case \((i)\) as well.

**Proposition 5**

Consider a symmetric Nash equilibrium strategy \(\mu\) that satisfies indepen-
dence and constant comparability. The feature that the induced marginal
distribution over \(q^k\) has no atoms and no holes carries over to the present
setting. From now on, we adhere to the \((p, e)\) representation of pure strate-
gies. The proof proceeds stepwise.

**Step 1:** The marginal distribution over \(p\) is atomless.

**Proof:** Assume the contrary - i.e., that some price \(p\) is realized with posi-
tive probability. Then, with positive probability \(p_1 = p_2 = p\). In this case,
\((p_1, e_1)\) and \((p_2, e_2)\) necessarily do not dominate one another. Thus, condi-
tional on \(p_1 = p_2 = p\), the probability of domination is zero. By constant
comparability, the probability of domination must be zero in equilibrium, in
contradiction to Proposition 4.

The following two steps state properties that hold for *almost all* pairs
of realizations of a symmetric equilibrium strategy. For expositional conven-
ience, we state and prove the claims with slight imprecision, as if they hold
for *all* realizations.

**Step 2:** If \((p', e')\) is dominated by \((p, e)\), then \((p'', e')\) is dominated by \((p, e)\)
for every \(p'' \in (p, p')\).
**Proof**: Let \( p' > p'' > p \) be three prices in the support of the marginal distribution over \( p \). By definition, if \((p'', e')\) is dominated by \((p, e)\), then \((p', e')\) is dominated by \((p, e)\) as well. Now, calculate the probability of domination conditional on \((p_1, p_2) = (p', p)\), by integrating over all possible values of \( e_1, e_2 \), and do the same for \((p_1, p_2) = (p'', p)\). By independence, \( e_1 \) and \( e_2 \) are i.i.d. Therefore, if (contrary to the claim) there is positive probability that \((p', e')\) is dominated by \((p, e)\) yet \((p'', e')\) is not dominated by \((p, e)\), we will get a violation of constant comparability, because the domination probability conditional on \((p', p)\) will be strictly higher than the domination probability conditional on \((p'', p)\).

**Step 3**: For every \((p, e)\) and \((p', e')\) in the support of \( \mu \) with \( e \neq e' \), \( |e' - e| \geq |p' - p| \).

**Proof**: Assume the contrary, i.e., \( |e - e'| < h - l \) for \( e, e' \) in the support of the marginal obfuscation distribution (where \( h \) and \( l \) are as defined in the proof of Claim 2). By Step 1, we can find a price \( p \in (l, h) \) in the support of the marginal distribution over \( p \), such that \( p - l < |e - e'| \). This means that \((h, e)\) will be dominated by \((l, e')\) and yet \((p, e)\) will not be dominated by \((l, e')\), contradicting Step 2.

**Step 4**: The marginal equilibrium distribution over \( e \) is uniform with support \( \{e_0, ..., e^{n-1}\} \), where \( e^{k+1} - e^k = h - l \) for every \( k = 0, ..., n-2 \), and \( h + e^{n-1} = h - e^0 = 1 \).

**Proof**: Step 3 immediately implies that the gap between two adjacent realizations \( e < e' \) cannot be less than \( h - l \). Assume the gap is strictly greater than \( h - l \). Then, a competitor can profitably deviate from \((h, e)\) to \((h, e + \delta)\), where \( \delta > 0 \) is arbitrarily small. The reason is that since the distribution over \( p \) is atomless, it assigns positive probability to prices arbitrarily close to \( h \). Thus, by switching to \((h, e + \delta)\), the competitor reduces the probability of being dominated by strategies \((p, e)\) for \( p < h \), without affecting the probability of being dominated by strategies \((p, e''), e'' \neq e\). Since the marginal
distributions over $q^k$ have no holes, $h + e^{n-1} = h + e^0 = 1$. Finally, the reason
that the distribution is uniform is as follows. In equilibrium, competitors
are indifferent among all $(h, e^k)$. By construction, the payoff from $(h, e^k)$ is
$h \cdot \frac{1 - \lambda}{2} \cdot (1 - \Pr(e^k))$, because $(h, e^k)$ is dominated by $(p, e)$ if and only if $p < h$
and $e = e^k$.

**Step 5:** $h = 1 - \frac{n-1}{2}(h - l)$, $l = 1 - \frac{n+1}{2}(h - l)$.

**Proof:** Recall that $h + e^{n-1} = h - e^0 = 1$. Therefore, $e^0 = -e^{n-1}$. Since
values of $e$ are evenly spaced by intervals of length $h - l$, it follows that the
distribution of $e$ is symmetric around zero, such that $e^{n-1} = \frac{n-1}{2}(h - l)$, and
the result follows.

To complete the proof, we add the equation that the profits at $l$ and $h$
coincide:

$$h \cdot \frac{1 - \lambda}{2} \cdot (1 - \frac{1}{n}) = l \cdot \left[\frac{1 - \lambda}{2} + (1 - \frac{1 - \lambda}{2}) \cdot \frac{1}{n}\right]$$

This equation, coupled with Step 5, gives us the solutions to $h$ and $l$, as well
as the equilibrium profit, as a well-defined function of $n$. We can retrieve the
marginal distribution $G$ over $p$ from the following equation:

$$\frac{\sigma(2\sigma n + 2)}{2\sigma n + n + 1} = p \left[\frac{1 - \lambda}{2} \cdot \left(1 - \frac{1}{n} + \frac{1}{n} \cdot (1 - G(p))\right) + (1 - \frac{1 - \lambda}{2}) \cdot \frac{1}{n} \cdot (1 - G(p))\right]$$

Since this equation holds for every $p$ in the support of $G$, the support of
$G$ cannot have holes inside $[l, h]$, for otherwise there would be an atom,
contradicting Step 1.

Thus, any symmetric equilibrium strategy takes the form $s^*(n, d)$ where
d $= h - l$. As for the bounds on $n$, it is easy to verify that if $n > 1 + \frac{3}{\lambda}$ then a
deviation to $(p, e)$ where $p = h + \frac{d}{2}$ and $e = 1 - p$ is profitable and if $n < 1 + \frac{1}{\lambda}$
then a deviation to $(p, e)$ where $p = l + \frac{d}{2}$ and $e = e^0 + \frac{d}{2}$ is profitable.