

# EXPECTATIONS AND THE TIMING OF NEIGHBORHOOD CHANGE

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## ABSTRACT

We study the role of expectations when agents have a preference for segregation and households face moving frictions. In a fixed environment, there are multiple equilibria: agents' expectations determine whether an ethnic transition occurs. However, the outcome is unique if there is a deterministic trend that gradually makes the neighborhood more appealing to the outside group. It is unique also if the relative payoff from living in the neighborhood is subject to small shocks. In both cases, the insiders must leave at the *first possible moment*: when the outsiders would outbid them if an immediate ethnic transition were expected.

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A few houses were sold to [blacks] in 1955. “The selling of the third house convinced everyone that the neighborhood was destined to become mixed.” A year later 40 houses had been sold to [blacks].... In another two years the percentage had gone above 50%.<sup>2</sup>

## I. INTRODUCTION

Expectations seem to play an important role in neighborhood change. Yet research by economists has traditionally assumed myopic agents, who do not think about the future.<sup>3</sup> The focus of this paper is to study neighborhood change under rational expectations. We show that there can be multiple equilibria, but that exogenous trends or shocks can often eliminate this multiplicity, giving rise to unambiguous predictions of when a transition will take place.

We focus on a single neighborhood. There are two types of agents, “black” and “white.” (They might also be interpreted as rich and poor, Protestant and Catholic, Arab and Jewish, etc.) Agents prefer to live with others of the same type.<sup>4</sup> Over time, each resident receives random opportunities to sell her house and move out. This creates a role for expectations. An agent's decision to move or stay will depend on her assessment of how her neighborhood's ethnic composition will change while she waits for her next chance to move.

In the first formulation of the model, the environment is fixed; an agent's willingness to pay to live in a neighborhood depends only on its current and future expected ethnic composition. In this case, there can be multiple equilibria for a range of initial conditions. For example, consider an initially all-white neighborhood. In one equilibrium, the neighborhood remains segregated. In another, each white resident sells her home to a black at her first opportunity. One can interpret this as implying that expectations play a causal role: an ethnic transition either will or will not occur, depending on what the agents expect.

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<sup>2</sup> Schelling [14], p. 181, quoting from a 1960 study by A. J. Mayer of an all-white neighborhood of 700 single-family homes. The quotation marks are in the original text.

<sup>3</sup> Examples include Bènabou [2], Bond and Coulson [4], Miyao [13], and Schelling [14].

<sup>4</sup> More precisely, an increase in the proportion of one type in a neighborhood raises the static utility of a member of that type, *relative* to a member of the other type, from living in the neighborhood. We use this reduced form to simplify the model; in addition to racism, a preference for segregation might come from different tastes for public goods (Tiebout [16]), redistributive conflict (Epple and Romer [10]), educational complementarities, wealth differences, or credit constraints (Benabou [13]). A richer specification (in which we explicitly model how the preference for segregation comes from a source other than racism) would lead to the same qualitative conclusions at the cost of greater complexity.

The finding of multiple equilibria can be viewed as a deficiency of the model. One cannot predict whether a neighborhood will undergo an ethnic transition by examining only observable factors. Information about what agents expect is also needed, but this data cannot be directly observed. We address this by looking for factors that have the potential to eliminate the multiplicity. That is, the initial version of the model may have omitted features that, once included, would force agents to coordinate their expectations on a particular outcome.

One such factor is the presence of exogenous trends or shocks that differentially affect the payoffs of the two groups. We first study what happens in the presence of an exogenous trend that gradually raises the relative appeal of the neighborhood to the outside group. One example is the gradually increasing demand for black workers in Northern cities during and after World War I. In this period, demand for industrial workers in the North greatly exceeded the local labor supply. In response, Northern employers actively recruited blacks into jobs that had been traditionally held by whites. This progressive opening of the labor market to blacks led to a "Great Migration" in which many Southern rural blacks moved to Northern industrial cities (Collins [8]). The effects on many all-white neighborhoods was dramatic. Helper quotes real estate agents from Chicago's once mainly white south side in 1955-1956:

“The long and short of it is that when [blacks begin] to get close to a neighborhood, you can no more stop it than you can stop a million tons of snow from rolling down a mountain side. ... It's due to the expanding [black] population.” (Helper [12], p. 107)

We find that such exogenous changes in payoffs can sharply reduce the set of equilibria. We model this in two steps. First we consider a deterministic trend (such as a rising demand for black workers in the North) that progressively raises the city's appeal to blacks. We show that this forces an ethnic transition to occur at a unique time. The reason is backwards induction. Agents foresee that, after a sufficiently long time, the trend will drive the instantaneous payoff of a black from living in an all-white neighborhood above that of a white.<sup>5</sup> They know that when this happens, an ethnic transition *must* occur. Anticipating this, whites will

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<sup>5</sup> We model the dynamics of a single all-white neighborhood. The rising demand for black workers might affect other neighborhoods in the city as well. However, they have no bearing on what happens in the given neighborhood if (as we assume) there is a large pool of identical blacks who could enter the city. In this case, the entry of blacks into other neighborhoods does not prevent or impede the entry of blacks into the given neighborhood, so it can be studied in isolation.

sell out to blacks slightly before this point as well. This gives rise to an unravelling effect: as the date of the transition creeps backwards in time, whites move out earlier and earlier.

The unravelling continues as long as the neighborhood is more appealing to blacks than to whites under the expectation that a transition is about to occur. It stops when, under this expectation, blacks and whites have an equal willingness to pay for housing in the neighborhood. Thus, *a transition must occur at the first moment at which its expectation becomes a self-fulfilling prophecy*: when blacks are first willing to pay more if both groups expect an ethnic transition to begin immediately. In contrast, without the external trend, a transition may not occur even if its expectation is self-fulfilling, since whites may be willing to pay more if the neighborhood is expected to remain all-white. The external trend gives rise to an unravelling effect that eliminates this "no transition" equilibrium, since agents know a transition will eventually occur and reason backwards.

If agents were myopic, the transition would also occur eventually, but much later than under rational expectations. This is because myopic agents do not foresee the impending transition. With myopic agents, blacks move in only when the region of inevitability is actually reached: when their instantaneous utility rises above that of a white from living in the existing, all-white neighborhood. There is no unravelling.

There is anecdotal evidence against myopia. The first whites who exit seem to do so not because of some objective changes in the neighborhood *per se*, but rather because they foresee an impending influx of blacks. Helper reports the following statements of Chicago real estate agents:

“When the [blacks] are within 2 to 3 to 4 blocks, the whites begin to flee.” (Helper [12], p. 107)

“When the first threat that [blacks] are approaching comes, the major change occurs. It stops almost overnight --- I mean the chance to sell to white other than a speculator.” (Helper [12], p. 87)

The above analysis assumes that the external trend follows a known course. In fact, the forces driving blacks to move to Northern industrial cities were largely random. The North-South relative wage, manufacturing labor demand in the North, and the arrival rate of white European immigrants were all unpredictable to some extent (Collins [8]). With such stochastic elements, an ethnic transition would have been by no means inevitable in most

neighborhoods.

We address this by letting the relative appeal of a neighborhood to the two groups depend on an exogenous parameter that changes randomly over time. The parameter may even fluctuate trendlessly. However, we still assume that the parameter has the potential to reach "regions of inevitability" where an ethnic transition must occur. For example, there is some small probability that the demand for black workers in the North would rise to such an extent that the instantaneous payoff of blacks would rise above that of whites even in an all-white neighborhood.

This assumption is fairly weak since the regions of inevitability can be arbitrarily far from the parameter's current value. Moreover, we focus on what happens in the limit as shocks to the parameter become small. This means that the regions of inevitability will be reached, if at all, only in the far distant future.

In the presence of these arbitrarily small shocks to the exogenous parameter, there is again a unique equilibrium. An ethnic transition can occur in either direction, depending on the shocks. In an initially all-white neighborhood, a transition will occur if the random changes make the neighborhood sufficiently better for blacks. Conversely, whites will enter an all-black neighborhood if the shocks are favorable enough to them. Surprisingly, the thresholds at which these transitions occur are the same as if a *deterministic* trend were causing them. That is, in an initially segregated neighborhood (black or white), an ethnic transition begins *once its expectation becomes self-fulfilling*: once the outside group would outbid the inside group on the belief that a transition is about to occur at the fastest possible rate. While the result is the same as with a deterministic trend, the intuition is more involved. It is explained in section V.

This paper contributes some new results to the extensive literature on residential segregation. Mixed neighborhoods are unstable in our model because agents have a taste for segregation: group A is willing to pay more than group B to live near members of group A. Bailey [1] was the first to suggest and study such preferences in a spatial model in which households care only about their immediate neighbors. Schelling [14] and Miyao [13] study "bounded neighborhood models" in which agents care about the composition of an entire neighborhood. Schelling assumes that an agent is willing to live in a neighborhood if and only

if no more than a given percentage of the residents are of the other group.<sup>6</sup> He shows that some mixing may be possible. In contrast, Miyao [13] assumes that an agent's utility depends continuously on her neighborhood's ethnic composition. He shows that mixing is unstable if housing and transportation costs are homogeneous within a neighborhood. Our model modifies that of Miyao [13] by considering rational expectations and by adding exogenous trends and shocks to payoffs.

Our model is also related to the phenomenon of "tipping." Bond and Coulson [4] and Schelling [14,15] show that the entry of a few members of one group may "tip" a neighborhood, causing that group to enter until the neighborhood is completely segregated. In Bond and Coulson [4], as in our model, the initial entry occurs because of an external trend (in their case, the aging of the housing stock). These models assume myopic behavior. We show that with rational expectations, the tipping point occurs much earlier because of an unravelling effect in which agents rationally anticipate the ethnic transition and reason backwards.

Our analysis relies on the approach of Burdzy, Frankel, and Pauzner [6] for finding unique equilibria in models with shocks. These techniques were subsequently applied to a development model in Frankel and Pauzner [11]. The present paper is the first to apply this approach to urban economics as well as the first application in which there is a traded asset (housing). It also extends the results in Burdzy, Frankel, and Pauzner [6] to the case of a deterministic trend.

## II. THE MODEL

In this section we present the model and derive conditions for blacks to outbid whites and vice-versa. We also give plausible conditions under which, consistent with intuition, prices fall monotonically before, during, and after blacks begin to move into an all-white neighborhood.

We study a small neighborhood with a continuum of houses, each of which contains one agent. Agents are risk-neutral and live forever. They can borrow and lend freely at the market interest rate, which equals the discount rate  $\theta$ .<sup>7</sup> There are two types of agents: black

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<sup>6</sup> Clark [7] provides some evidence on the empirical distribution of these thresholds.

<sup>7</sup> The assumption that the interest rate equals the discount rate is necessary because agents' utility is linear in consumption.

and white. The total measure of agents of each type is much larger than the measure of houses in the neighborhood, so only a small fraction of blacks or whites can live in the neighborhood at any given time. Time  $t$  is continuous. There are frictions: agents in the neighborhood cannot move at will. Rather, moving opportunities arrive at random times, according to (independent) Poisson processes, with common arrival rate  $\delta$ .<sup>8</sup> The frictions, while nonzero, can be arbitrarily small (that is,  $\delta$  can be arbitrarily large). An agent with a moving opportunity need not sell, but if she does, the sale and transfer of her house occur instantaneously. A house is sold to a member of the ethnic group with the highest willingness to pay. All homes are owner-occupied.<sup>9</sup>

We denote by  $b_t$  the proportion of blacks in the neighborhood, and let  $z_t$  be an exogenous parameter that affects the relative appeal of the neighborhood to the two groups. Both  $b$  and  $z$  are commonly observed. (When no confusion arises, we omit time subscripts.) A black agent who lives in the neighborhood and has no housing costs receives, relative to her best alternative, a utility flow of  $B(b,z)$  (measured in units of consumption).<sup>10</sup> The analogous utility flow of a white agent is  $W(b,z)$ .  $B(b,z)$  and  $W(b,z)$  can be interpreted as the rental rates that blacks and whites would be willing to pay if they could rent on a spot market. To capture agents' preference for segregation, we assume that the relative utility flow,  $D(b,z) = B(b,z) - W(b,z)$ , is strictly increasing in  $b$ . This says simply that blacks appreciate an increase in the proportion of blacks more than whites do. We also assume that  $D(b,z)$  is strictly increasing in  $z$ . For example,  $z$  might be the mean wage in jobs that are accessible to blacks living in the neighborhood, less the mean wage in jobs that are accessible to white residents.  $D(b,z)$  is continuously differentiable in  $b$  and  $z$ .

We now derive the condition under which blacks will move into the neighborhood. Let  $P_t^W$  and  $P_t^B$  be the prices that whites and blacks, respectively, are willing to pay for a house in the neighborhood at time  $t$ . The time- $t$  price is  $P_t = \max\{P_t^B, P_t^W\}$ . The price that a black

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<sup>8</sup> This captures the idea that searching for a new home takes an unpredictable amount of time.

<sup>9</sup> Since we assume risk neutrality, there are no wealth effects, so the results would be the same with absentee landlords.

<sup>10</sup> That is,  $B(b,z)$  is the utility flow of the agent from living in the neighborhood and bearing no housing costs, less her utility from living at her best alternative location and bearing the housing costs that prevail there.

resident is willing to pay at time  $v$  equals direct utility  $B(b_v, z_v)dv$  she will obtain during the infinitesimal period  $[v, v + dv]$ , plus the (discounted) expected value of the house at time  $v+dv$ . This value depends on whether she receives a moving opportunity, an event with probability  $\delta dv$ . If she does, this value is  $P_{v+dv}$ : since moving is costless, she is always willing to sell to the highest bidder (which may be herself). If not (probability  $1-\delta dv$ ), she is “obliged” to sell to herself, at price  $P_{v+dv}^B$ . Thus,

$$(1) \quad P_v^B = B(b_v, z_v)dv + e^{-\theta dv} (\delta dv \cdot E(P_{v+dv}) + (1 - \delta dv) \cdot E(P_{v+dv}^B)).$$

Approximating  $e^{-\theta dv}$  by  $1 - \theta dv$  and ignoring terms of order  $dv^2$ , we obtain:

$$(2) \quad P_v^B = B(b_v, z_v)dv + \delta dv E(P_{v+dv}) + (1 - \theta dv - \delta dv)E(P_{v+dv}^B)$$

Multiplying both sides by  $e^{-(\theta+\delta)(v-t)}$  and integrating, we obtain

$$(3) \quad P_t^B = E \int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} [B(b_v, z_v) + \delta P_v] dv$$

By analogous reasoning,

$$(4) \quad P_t^W = E \int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} [W(b_v, z_v) + \delta P_v] dv$$

The difference between blacks’ and whites’ willingness to pay is therefore

$$(5) \quad E \left[ \int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} D(b_v, z_v) dv \right]$$

Since blacks and whites have the same rate of moving opportunities, their expected capital gains or losses (the  $P_v$  term) are the same, so they cancel out in (5). The difference in willingness to pay is just the present value of the difference in utility flows,  $D(b_v, z_v)$ . The discount rate  $\theta + \delta$  reflects both the pure rate of time preference and the finiteness of agents’ horizons due to their receiving moving opportunities at the rate  $\delta$ .

When (5) is positive, blacks outbid whites. The proportion of whites is  $1 - b$  and they receive moving opportunities at the rate  $\delta$ , so the proportion of blacks rises at the rate  $\dot{b} = \delta(1 - b)$ . When (5) is negative, whites outbid blacks; the proportion of blacks is  $b$  and they move out at the rate  $\delta$ , so the proportion of blacks falls at the rate  $\dot{b} = -\delta b$ .

We assume that the exogenous parameter  $z$  has *regions of inevitability*: if  $z_t$  is large enough,  $E \left[ \int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} D(0, z_v) dv \mid z_t \right] > 0$ , so that blacks will outbid whites even if the neighborhood is expected to remain all-white forever. Conversely, for  $z_t$  low enough,



$E[\int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} D(1, z_v) dv \mid z_t] < 0$ : whites will outbid blacks even if the neighborhood is expected to remain all-black forever.

We now derive an expression for the price of housing. In periods when blacks are willing to pay more than whites,  $P = P^B$ , so (2) becomes  $P_v = B(b_v, z_v)dv + (1 - \theta dv)E(P_{v+dv})$ .<sup>11</sup> Analogously, in periods when whites pay more, equation (2) becomes  $P_v = W(b_v, z_v)dv + (1 - \theta dv)E(P_{v+dv})$ . Combining these,  $P_v = M(b_v, z_v)dv + (1 - \theta dv)E(P_{v+dv})$ , where  $M(b_v, z_v)$  equals  $B(b_v, z_v)$  if blacks outbid whites at time  $v$  and  $W(b_v, z_v)$  if whites bid more than blacks. Multiplying both sides by  $e^{-\theta(v-t)}$  and integrating, we obtain  $P_t = E[\int_{v=t}^{\infty} e^{-\theta(v-t)} M(b_v, z_v) dv]$ . Thus, for prices to fall during a transition from all-white to all-black, it suffices for black to prefer to live with whites ( $B_b < 0$ ).<sup>12</sup> This is admissible under the model, which assumes only that  $W_b < B_b$ : an increase in the proportion of blacks raises the difference between black and white utility.

### III. A STATIC ENVIRONMENT

We first analyze the case in which  $z$  is constant over time. Suppose that it is dominant for whites to outbid blacks if  $z < \underline{z}$  and for blacks to outbid whites if  $z > \bar{z}$ .<sup>13</sup> For  $z$  between  $\underline{z}$  and  $\bar{z}$ , both all-black and all-white are steady state equilibria. That is, if only one type initially inhabits the neighborhood, it is an equilibrium for the neighborhood's composition to remain unchanged. However, whether a given steady state can be reached depends on the initial proportion of blacks in the neighborhood.

Figure 1 shows the set of equilibria for each  $z$  and for each proportion  $b$  of blacks. The proportion of blacks is measured on the vertical axis; the parameter  $z$  appears on the horizontal axis. In the rightmost region, blacks always outbid whites: the neighborhood converges to

<sup>11</sup> This is because if  $P_v^B = P_v$  and  $dv$  is infinitesimal, then  $E(P_{v+dv}) = E(P_{v+dv}^B)$ .

<sup>12</sup> If a transition to all-black is underway at time  $t$ , then blacks outbid whites at all times  $v \geq t$ , whence  $P_t = E[\int_{v=t}^{\infty} e^{-\theta(v-t)} B(b_v, z_v) dv]$ . If  $z$  is fixed, prices must therefore decline if  $B_b < 0$ .

<sup>13</sup>  $\underline{z}$  is defined by  $D(1, \underline{z}) = 0$  and  $\bar{z}$  by  $D(0, \bar{z}) = 0$ .

all-black. In the leftmost region, whites always bid more, so all-white is the only long run outcome. In the region between  $\underline{z}$  and  $\bar{z}$ , there are multiple equilibria: what happens depends on agents' expectations. Whites may win every home; then the proportion of whites will grow, raising the neighborhood's relative appeal to whites and making it indeed optimal for whites to bid more. Or blacks may bid more, lowering the neighborhood's relative appeal to whites and thus confirming their decision to bid less than blacks. These results are summarized in Theorem 1.

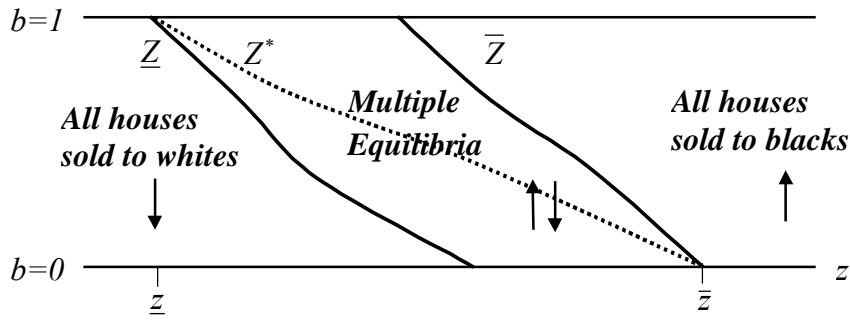


Figure 1: Unchanging Environment (Theorem 1)

**THEOREM 1.** There are decreasing functions  $\underline{z}(b) < \bar{z}(b)$  such that if  $z > \bar{z}(b)$ , there is a unique equilibrium, in which blacks always outbid whites. If  $z < \underline{z}(b)$ , there is a unique equilibrium, in which whites always outbid blacks. For  $z$  between  $\underline{z}(b)$  and  $\bar{z}(b)$  there are multiple equilibria; both all-black and all-white are long run outcomes.

**Proof of Theorem 1:**

Let us take the initial proportion  $b_0$  of black residents as given. When it is an equilibrium for each white resident to be replaced by a black at her first moving opportunity? It suffices to check that a white resident who gets a moving opportunity at time zero would sell to a black if she expects all other white residents to do the same. This is because the growth of  $b$  raises  $D(b,z)$ , raising the gap between black and white bids. Under these expectations,  $b$  grows at the rate  $\dot{b} = \delta(1-b)$ : every white resident leaves at her first chance, there are  $1-b$  white residents, and chances to leave arrive at the rate  $\delta$ . Therefore, the difference between black and white bids equals:

$$\underline{U}(b_0, z) = \int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(b_t^\uparrow, z) dt$$

where  $b_t^\uparrow = 1 - (1-b_0)e^{-\delta t}$ . It is an equilibrium for blacks to win at  $(b_0, z)$  iff  $\underline{U} \geq 0$ . Since  $\underline{U}$  is increasing in both arguments, there is decreasing function  $\underline{z}(b)$  such that it is an equilibrium

for blacks to buy houses whenever  $z \geq \underline{Z}(b)$ . ( $\underline{Z}$  satisfies  $\underline{U}(b, \underline{Z}(b)) = 0$ .) A similar argument shows that it is an equilibrium for whites to buy houses whenever

$$\bar{U}(b_0, z) = \int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(b_t^\downarrow, z) dt \leq 0$$

where  $b_t^\downarrow = b_0 e^{-\delta t}$ . Define  $\bar{Z}$  by  $\bar{U}(b, \bar{Z}(b)) = 0$ . Since  $b_t^\downarrow$  is always below  $b_0$  and  $b_t^\uparrow$  is always above, whenever  $\bar{U}$  (which is proportional to a weighted average of  $D(b_t^\downarrow, z)$  for all  $t > 0$ ) equals zero,  $\underline{U}$  (which is proportional to a weighted average of  $D(b_t^\uparrow, z)$ ) must be positive. This implies that  $\underline{Z}(b) < \bar{Z}(b)$ . Q.E.D.

For an initially all-white neighborhood, there is a large region of multiple equilibria. For all  $z$  between  $\underline{Z}(0)$  and  $\bar{Z}(0)$ , the neighborhood can remain all-white forever; alternatively, each white who gets a moving opportunity can be replaced by a black. Both prophecies are self-fulfilling. This contrasts with the case of myopic behavior, on which the prior literature on neighborhood dynamics has focused. The dotted curve  $Z^*$  in Figure I is the myopic indifference line, given by  $D(b, z) = 0$ .<sup>14</sup> On this curve, the instantaneous utility flows of the two races are equal. With myopic agents, blacks simply outbid whites to the right of  $Z^*$  and whites bid more to the left of  $Z^*$ . In an initially all-white neighborhood, for example, the assumption of myopic behavior discards equilibria for  $z$  between  $\underline{Z}(0)$  and  $Z^*(0)$  that involve self-fulfilling expectations of a transition to all-black. As we will see, with an external trend or small shocks there *must* be a transition to all-black in this parameter range under rational expectations.

The forms of  $\underline{Z}$  and  $\bar{Z}$  depend on the arrival rate of moving opportunities  $\delta$ . When  $\delta$  is small, moving opportunities arrive rarely and hence changes in the neighborhood's composition will be slow. In this case, even slight impatience of the agents ( $\theta > 0$ ) will make them care mostly about an initial period in which the neighborhood's composition has not changed. This implies that agents behave like myopic agents: as  $\delta \rightarrow 0$ ,  $\underline{Z}$  and  $\bar{Z}$  converge to  $Z^*$ . In contrast, when  $\delta$  is large,  $\underline{Z}$  and  $\bar{Z}$  become further apart; in the limit, the upper endpoint of  $\bar{Z}$  and the lower endpoint of  $\underline{Z}$  are at the same value of  $z$ .

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<sup>14</sup> This curve is downwards sloping since  $D$  is increasing in both arguments.

#### IV. A DETERMINISTIC TREND

We now examine what happens if  $z$  follows a deterministic trend that progressively makes the neighborhood more appealing to blacks relative to whites:  $z_t = z_0 + kt$  for some positive constant  $k$ . (The case of a negative trend is analogous and presented in the end of this section.) Going from no trend ( $k = 0$ ) to even a slightly positive trend leads the region of multiple equilibria to shrink discontinuously. This is shown in Figure 2, which depicts the limit as the trend  $k$  approaches zero. The boundaries of the region with multiple equilibria converge to  $\underline{Z}'$  and  $\bar{Z}'$ . While  $\underline{Z}'$  coincides with the left boundary  $\underline{Z}$  from the case of no trend (Figure 1),  $\bar{Z}'$  does not equal the corresponding right boundary  $\bar{Z}$ . Hence, even an arbitrarily small positive trend leads to a unique equilibrium in part of the region of multiplicity (the area between  $\bar{Z}'$  and  $\bar{Z}$ ).

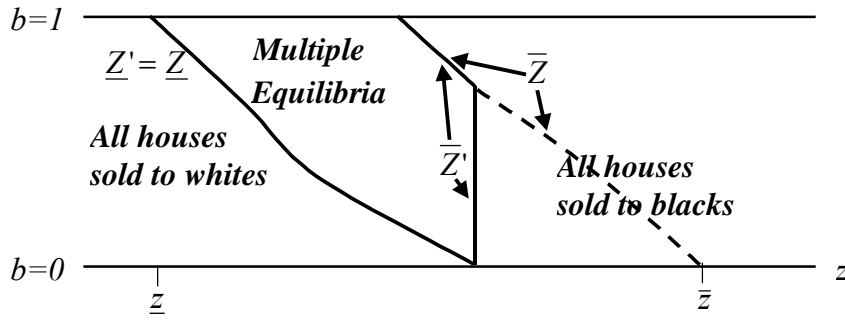


Figure 2: A Small Positive Trend in  $z$  (Lemma 1)

**LEMMA 1.** Assume that the payoff parameter  $z$  has a positive trend  $k > 0$ . There are decreasing functions  $\underline{Z}'(b) \leq \bar{Z}'(b)$  such that whenever  $z > \bar{Z}'(b)$ , blacks outbid whites; when  $z < \underline{Z}'(b)$ , whites outbid blacks. When  $z$  is between  $\underline{Z}'(b)$  and  $\bar{Z}'(b)$  there are multiple equilibria: either whites or blacks may make the higher bids. The lower endpoints of the two boundaries coincide:  $\underline{Z}'(0) = \bar{Z}'(0)$ . In the limit as the trend  $k$  approaches zero,  $\underline{Z}'(b) \rightarrow \underline{Z}(b)$  and  $\bar{Z}'(b) \rightarrow \min\{\bar{Z}(b), \underline{Z}(0)\}$  for any  $b$  (see Figure 2).<sup>15</sup>

**Proof:** see appendix.

<sup>15</sup>  $\underline{Z}(b)$  and  $\bar{Z}(b)$  are defined in Theorem 1.

The timing of the ethnic transition is described in Theorem 2. The theorem says that a transition occurs at the earliest possible moment:

**THEOREM 2.** Assume that the neighborhood is initially all-white and that the trend  $k$  is positive. The equilibrium path is unique: blacks start outbidding whites at the moment at which the expectation of an ethnic transition first becomes self-fulfilling. This is the first time  $t$  at which  $\int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} D(1 - e^{-\delta(v-t)}, z_0 + kt) dv > 0$ .

The intuition for Theorem 2 is as follows. When  $z$  crosses  $\bar{z}$ , blacks must outbid whites under any belief, even if the neighborhood is expected to remain all-white forever. Therefore, just before  $z$  reaches  $\bar{z}$ , all agents must *know* that an ethnic transition is about to start. Since  $z$  is close to  $\bar{z}$ , blacks would bid almost what whites would bid even if the neighborhood were expected to remain all-white forever; hence, since agents actually expect blacks to begin to enter, blacks must bid strictly *more* than whites. But now, expecting the transition to begin slightly *before*  $z$  reaches  $\bar{z}$ , blacks will outbid whites even earlier than this! This backward induction argument can be iterated again and again – as long as  $z$  is high enough that, under the expectation of an immediate ethnic transition, blacks would outbid whites. On the other hand, if  $z$  is low enough that whites would outbid blacks even under the expectation of an immediate ethnic transition, then whites must outbid blacks under any beliefs. Thus, blacks must begin to outbid whites precisely when  $z$  crosses the level above which the expectation of an ethnic transition is self-fulfilling.

This result relies on the presence of some frictions, though they can be arbitrarily small. To see why, suppose a white agent has a chance to move out while the neighborhood is still all-white. Assume that an ethnic transition is expected to begin at the myopic starting point: when the static payoff of a black living in an all-white neighborhood rises above that of a white in the same neighborhood (i.e., at  $\bar{z}$  in Figure 2). If there were no moving frictions, the agent could simply wait until this myopic starting point and move out when others do so. This would thus be an equilibrium. But with frictions, if the agent expects a transition to begin at the myopic starting point, she will actually want to move out if she gets the chance to do so somewhat before this point is reached. If she fails to move out earlier, she will almost certainly be stuck in the neighborhood while its ethnic composition changes. But then moving out even

earlier is optimal. Thus, white agents have an incentive to "leapfrog" each other, substantially lowering the transition threshold even with arbitrarily small moving frictions.

For the sake of completeness, we now present the results for the case of negative trend ( $k < 0$ ). Since the reasoning is analogous (the two cases are symmetric), we omit the proofs.

**LEMMA 1'.** Assume that the payoff parameter  $z$  has a negative trend  $k < 0$ . There are decreasing functions  $\underline{Z}'(b) \leq \bar{Z}'(b)$  such that whenever  $z > \bar{Z}'(b)$ , blacks outbid whites; when  $z < \underline{Z}'(b)$ , whites outbid blacks. When  $z$  is between  $\underline{Z}'(b)$  and  $\bar{Z}'(b)$  there are multiple equilibria: either whites or blacks may make the higher bids. The upper endpoints of the two boundaries coincide:  $\underline{Z}'(1) = \bar{Z}'(1)$ . In the limit as the trend  $k$  approaches zero,  $\bar{Z}'(b) \rightarrow \bar{Z}(b)$  and  $\underline{Z}'(b) \rightarrow \max\{\underline{Z}(b), \bar{Z}(1)\}$  for any  $b$  (see Figure 2').

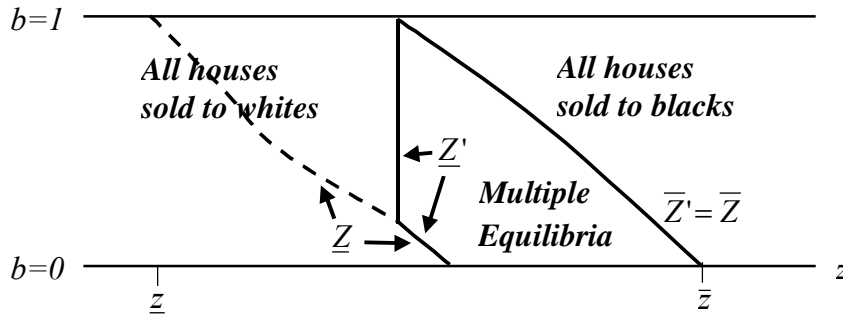


Figure 2': A Small Negative Trend in  $z$  (Lemma 1')

**THEOREM 2'.** Assume that the neighborhood is initially all-black and that the trend  $k < 0$ .

The equilibrium path is unique: whites start outbidding blacks at the moment at which the expectation of an ethnic transition first becomes self-fulfilling. This is the first time

$$t \text{ at which } \int_{v=t}^{\infty} e^{-(\theta+\delta)(v-t)} D(e^{-\delta(v-t)}, z_0 + kt) dv \leq 0.$$

## V. EXOGENOUS SHOCKS

We have seen that, by guarantying an eventual ethnic transition, a deterministic trend causes transitions to occur very early. However, the assumption of a *deterministic* trend might be too strong. External events that affect the utility from living in a neighborhood are to some extent random.

This section addresses this concern by replacing the trend with a (possibly trendless) stochastic process. We assume that  $z$  is a Brownian motion with variance  $\sigma^2 > 0$  and trend  $\lambda$ . (Hence, the change in  $z$  over time  $\varepsilon$  is normal with variance  $\sigma^2\varepsilon$  and mean  $\lambda\varepsilon$ .<sup>16</sup>) The trend  $\lambda$  may be positive, negative, or zero.

We focus on the case in which  $z$  changes very slowly.<sup>17</sup> Theorem 3 states that in the limit as  $\sigma^2$  and  $\lambda$  approach zero, there is a unique curve  $Z$  that splits the set of possible starting points into two regions (Figure 3). Whenever the state is to the left of  $Z$  (i.e., if the payoff parameter  $z$  or the proportion  $b$  of blacks is small enough), any black in the neighborhood sells her house to a white at her first opportunity. To the right of  $Z$ , every white sells to a black when she gets the chance. The dotted curves in Figure 3 are the region boundaries from Figure I. Between these dotted curves, there were multiple equilibria in the case of constant  $z$ . With small shocks to  $z$ , in contrast, the outcome is uniquely determined throughout the state space.

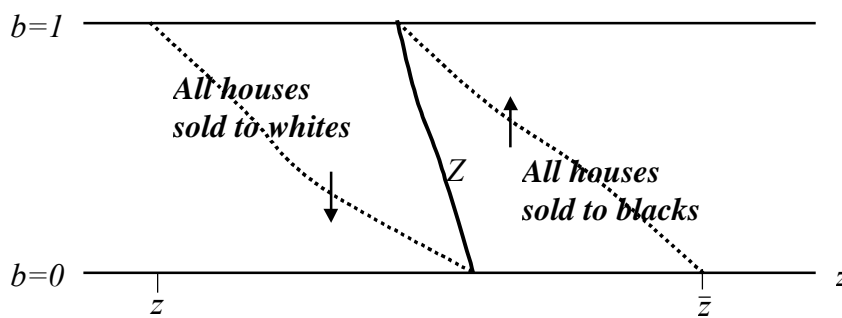


Figure 3: Small Shocks (Theorem 3)

<sup>16</sup> All of our results hold if the fixed trend  $\lambda$  is replaced by a variable trend  $\lambda \cdot \mu(t, z)$  where  $\mu$  is a continuously differentiable function that is bounded in  $t$  for any given  $z$ . Then our results hold in the limit as  $\lambda \rightarrow 0$ .

<sup>17</sup> Even without taking limits, shocks lead to a unique equilibrium; this can be shown using the approach in Frankel and Pauzner [11]. We focus on the limiting case of small shocks since this permits us to derive a closed-form expression for when a transition occurs in an initially segregated neighborhood.

**THEOREM 3.** There is a decreasing function  $Z(b)$  (defined in the appendix), satisfying  $Z(0) = \underline{Z}(0)$  and  $Z(1) = \overline{Z}(1)$ , such that in the limit as the shocks shrink ( $\lambda \rightarrow 0$  and  $\sigma^2 \rightarrow 0$ ):

1. Whenever  $z > Z(b)$ , whites remain in their houses and any black in the neighborhood sells her house to a white at her first opportunity;
2. Whenever  $z < Z(b)$ , blacks remain in their houses and any white in the neighborhood sells her house to a black at her first opportunity.

More precisely, for all  $\varepsilon > 0$  there are  $\underline{\lambda} > 0$  and  $\underline{\sigma}^2 > 0$  such that if  $|\lambda| < \underline{\lambda}$  and  $\sigma^2 \in (0, \underline{\sigma}^2)$  then blacks outbid whites whenever  $z > Z(b) + \varepsilon$  while whites outbid blacks whenever  $z < Z(b) - \varepsilon$ .

**Proof:** see appendix.

One implication of this is that, in the limit as the shocks shrink, ethnic transitions occur whenever their expectation is self-fulfilling, even if the trend is not deterministic. That is, in an all-white neighborhood, blacks must outbid whites if and only if the expectation of a transition to all-black would be self-fulfilling in a static environment. Likewise, in an all-black neighborhood, whites must outbid blacks if and only if the expectation of a transition to all-white would be self-fulfilling in a static environment.<sup>18</sup>

This may seem counterintuitive: although the shocks can in principle make either blacks or whites like the neighborhood more, a transition in an initially segregated neighborhood must occur at the *same* point as with a deterministic trend that makes the neighborhood progressively more appealing to the outside group. We now explain this result.

Recall our assumption that if  $z$  is sufficiently high, blacks outbid whites regardless of their expectations. This ‘inevitability region’ may be far from the current value of  $z$ , making it very improbable that  $z$  will reach it before an agent gets another chance to move. However, the mere existence of this region starts an unravelling effect that spreads to regions where, without shocks, there would be multiple equilibria. We denote the left boundary of this region, by ‘ $Z_0$ ’

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<sup>18</sup> This follows from Theorem 3, since  $Z(0)$  (respectively,  $Z(1)$ ), the point where a transition in the stochastic case, equals  $\underline{Z}(0)$  (respectively,  $\overline{Z}(1)$ ), the point where the expectation of a transition becomes self-fulfilling with fixed  $z$ .



(see Figure 4). The curve is downward sloping since either a higher  $z$  or a higher  $b$  makes the neighborhood better for blacks (under a given belief about the future).

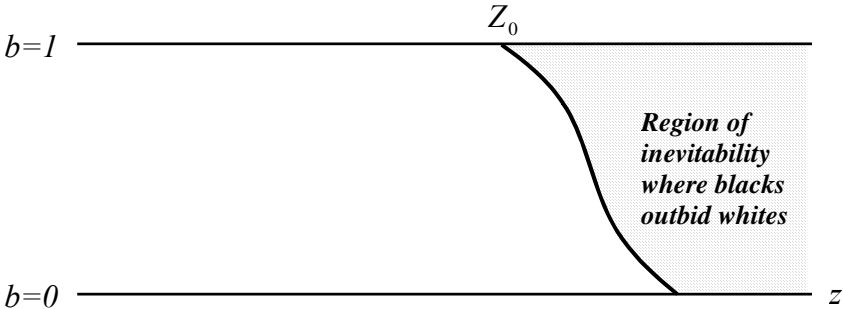


Figure 4

Knowing that blacks always outbid whites to the right of  $Z_0$ , blacks will actually bid more than whites at states that are slightly to the left of the curve as well. Why? On  $Z_0$  the bids are equal *on the worst belief for blacks*: that all houses auctioned thereafter will be won by whites. But agents now know that blacks will actually win when the state is to the right of  $Z_0$ . Since  $z$  changes stochastically, agents who bid when the state is slightly to the left of  $Z_0$  expect it to spend some time to the right of  $Z_0$  before their next moving opportunity. At such times, some houses must be bought by blacks. Since this raises agents' assessment of the future proportion of blacks in the neighborhood, blacks and whites no longer make the same bids at states on  $Z_0$ ; blacks bid strictly more. Therefore, there is a new curve  $Z_1$ , that lies strictly to the left of  $Z_0$ , such that blacks must outbid whites when to the right of  $Z_1$ .

This reasoning can be repeated, giving curves  $Z_2, Z_3$ , and so on *ad infinitum*. Let  $Z_\infty$  be the limit of this sequence (Figure 5). We know that blacks must outbid whites when to the right of  $Z_\infty$ .

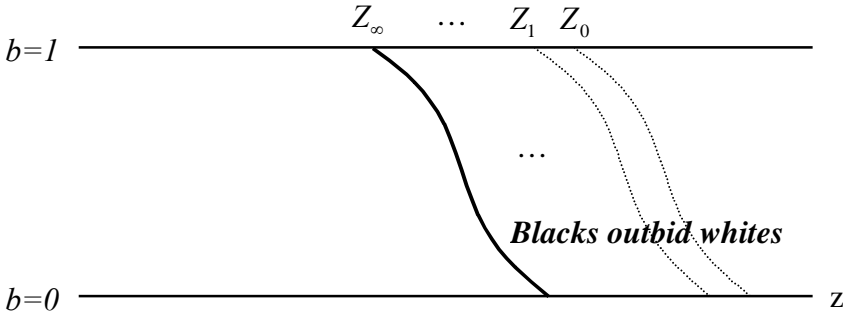


Figure 5

A crucial feature of  $Z_\infty$  is that the iterations *stop* there. This means that on the worst-case belief for blacks that is consistent with blacks bidding more to the right of  $Z_\infty$ , whites must bid more to the left of  $Z_\infty$ . Otherwise, we could push  $Z_\infty$  further to the left. Thus, blacks and whites are willing to pay the *same amount* if they expect all future home auctions to be won by blacks at states to the right of  $Z_\infty$  and by whites at states to the left of  $Z_\infty$ .

The dynamics implied by this belief are shown in Figure 6. When to the right of  $Z_\infty$ ,  $b$  rises at the rate  $\dot{b} = \delta(1-b)$ : every white resident leaves at her first chance, there are  $1-b$  white residents, and chances to move arrive at the rate  $\delta$ . When to the left of the curve,  $b$  falls at the rate  $\dot{b} = -\delta b$  by analogous reasoning.

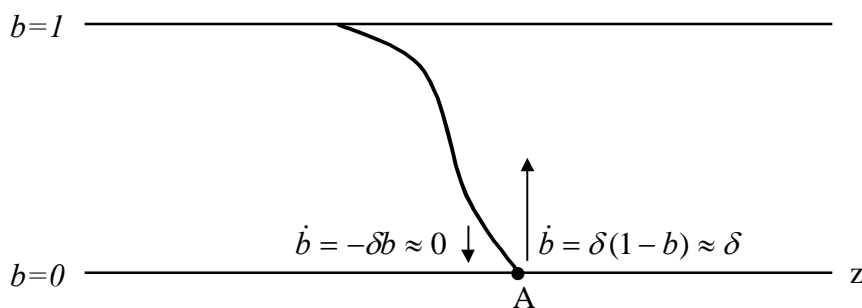


Figure 6

Assume now that a house is vacated at point A, when the neighborhood is all-white and blacks and whites bid the same. Under the dynamics of Figure 6, almost all future auctions are won by blacks if  $\lambda$  and  $\sigma^2$  are small. Why? When the shocks take  $z$  to the right of  $Z_\infty$ , the state moves up at a rate of about  $\delta$ : almost all houses are occupied by whites, so in every vacated house a black replaces a white. However, when  $z$  moves to the left of  $Z_\infty$ , the rate at which the state moves down is close to 0: the probability that a vacated house has a black owner is very small since the neighborhood is almost all white. This asymmetry causes the state, on average, to move upwards.<sup>19</sup> Since  $Z_\infty$  is downward sloping,<sup>20</sup> as the state moves up it moves away from the curve. For small  $\lambda$  and  $\sigma^2$ , the negative shocks to  $z$  are small and thus

<sup>19</sup> This is because the change in  $z$  over a short time interval  $\varepsilon$  has a large random component: its standard deviation is of order  $\sqrt{\varepsilon}$ . (Its variance must be of order  $\varepsilon$  for the variance of changes in  $z$  over a fixed, longer interval to be nontrivial; this is just a consequence of  $z$  having independent increments.) Since the trend is a linear function of time, its effect is of order only  $\varepsilon$  and thus is swamped by the random part. Hence, the shocks govern the short run behavior of the system.

<sup>20</sup> This can be shown by an inductive argument.

unlikely to move the state back to  $Z_\infty$  after it has been moving up for even a brief period. Hence, with high probability, the state begins very quickly to move upwards and does not return to the curve for a very long time. This means that very quickly, blacks begin winning all auctions and they continue to do so indefinitely. Since blacks and whites bid the same at point A,  $Z_\infty(0)$  must be computed by equating the willingness to pay of blacks and whites under the belief that a transition is about to start.

## V. CONCLUSION

We study the role of expectations in neighborhood change in the presence of external fluctuations that differentially affect insiders and outsiders. The traditional approach, with myopic agents, implies that a demographic transition will occur when the outsiders' willingness to pay to live in the neighborhood under its *current* composition surpasses that of the inside group. In contrast, when agents have rational expectations about the future, transitions occur much earlier: when the outsiders' willingness to pay first exceeds the insiders' *under the belief that a transition will start immediately*. We prove this assuming that the external changes follow a deterministic trend that is destined eventually to cause a transition. We also show that the same result extends, somewhat more surprisingly, to the case of small random external changes, which can be equally likely to move in either direction.

The unique equilibrium in a stochastic environment is characterized by a threshold for the external parameter. Above this threshold, blacks outbid whites; below it, whites outbid blacks. Importantly, the threshold is a decreasing function of the proportion of blacks currently in the neighborhood (see Figure 3). This means that there is *hysteresis*. For example, in an initially all-white neighborhood, blacks will begin to enter if the external parameter moves above a given threshold. Once blacks begin to move in, their entry makes the neighborhood more appealing to them (relative to whites). This lowers the threshold for the external parameter. Consequently, the longer the black entrance continues, the farther the parameter must subsequently drop to draw whites back into the neighborhood. Similarly, if the parameter moves below its threshold, the white influx will raise the threshold, which makes it less likely that blacks will enter in the near future.

## APPENDIX

### Proof of Lemma 1:

We first compute  $\underline{Z}'$ : the left boundary of the region where blacks can outbid whites. A necessary and sufficient condition for this to be consistent with equilibrium is that blacks will outbid whites if all agents expect them to continue to outbid whites forever: that

$$\underline{U}'(b_0, z) = \int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(b_t^\uparrow, z + kt) dt \geq 0$$

This is a necessary condition since (given that  $D(b, z)$  is increasing in  $b$ ) the path  $(b_t^\uparrow)_{t \geq 0}$  maximizes the black-white difference in willingness to pay. It is sufficient since as time passes,  $b$  and  $z$  both rise, which *raises* the black-white difference in willingness to pay.  $\underline{Z}'$  is defined by  $\underline{U}'(b, \underline{Z}'(b)) = 0$ ; since  $\underline{U}(b, \underline{Z}(b)) = 0$  and  $\lim_{k \downarrow 0} \underline{U}'(b, z) = \underline{U}(b, z)$ , it follows that  $\lim_{k \downarrow 0} \underline{Z}'(b) = \underline{Z}(b)$ .  $\underline{Z}'$  is decreasing since  $D$  is increasing in both arguments.

Now consider the right boundary  $\bar{Z}'$  of the region where whites can outbid blacks. This boundary is finite because if  $z$  is sufficiently large, whites must lose to blacks. Clearly,  $\bar{Z}'(b) \geq \underline{Z}'(b)$  for all  $b$ . Let  $\bar{Z}''$  be the right boundary of the region where whites will outbid blacks if they expect every future home to be bought by a white. That is,  $z_0 \leq \bar{Z}''(b_0)$  if and only if

$$\int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(b_t^\downarrow, z_0 + kt) dt \leq 0$$

where  $b_t^\downarrow = b_0 e^{-\delta t}$ . (The path  $b_t^\downarrow$  cannot actually occur in equilibrium since the trend in  $z$  must lead blacks eventually to outbid whites.) Clearly, if  $z_0 > \bar{Z}''(b_0)$ , whites can never buy (or retain) any homes in the neighborhood: blacks must *initially* outbid whites; and as  $b$  and  $z$  rise, the black-white bid differential rises, guaranteeing that blacks will *continue* to outbid whites. So  $\bar{Z}' \leq \bar{Z}''$ . Clearly,  $\bar{Z}''$  converges to  $\bar{Z}$  as  $k \rightarrow 0$ .

We now show that  $\bar{Z}' \leq \underline{Z}'(0)$ . Consider any initial state  $(b_0, z_0)$  such that  $z_0 > \underline{Z}'(0)$ . Let  $(b_t, z_t)_{t \geq 0}$  be any equilibrium path starting from  $(b_0, z_0)$ , and let  $\nu$  be the supremum of times at which whites outbid blacks. As  $t \uparrow \nu$ , whites who bid at time  $t$  outbid blacks even though they know that virtually all future houses will be purchased by blacks. But since  $\underline{Z}'$  is

decreasing,  $z_t > \underline{Z}'(b_t)$  for all  $t$ . This is a contradiction unless blacks *always* outbid whites (in which case  $\nu = 0$ ). This shows that  $\bar{Z}' \leq \underline{Z}'(0)$ .

This implies that  $\bar{Z}'(0) = \underline{Z}'(0)$ , as claimed. Hence, for an initially all-white neighborhood, the state is always either to the left of  $\underline{Z}'$  or to the right of  $\bar{Z}'$ . Whites win all homes until the payoff parameter reaches the threshold  $z^* = \bar{Z}'(0) = \underline{Z}'(0)$ , which must satisfy

$$\int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(1 - e^{-\delta t}, z^* + kt) dt = 0; \text{ with the change of variables } b = 1 - e^{-\delta t} \text{ this becomes}$$

$$\int_{b=0}^1 (1-b)^{\theta/\delta} D(b, z^*) db = 0.$$

We have shown that  $\bar{Z}'(b) \leq \min\{\bar{Z}''(b), \underline{Z}'(0)\}$  for all  $b$ ; hence,  $\lim_{k \rightarrow 0} \bar{Z}'(b) \leq \min\{\bar{Z}(b), \underline{Z}(0)\}$ . To show that  $\lim_{k \rightarrow 0} \bar{Z}'(b) = \min\{\bar{Z}(b), \underline{Z}(0)\}$ , we must show that for any initial state  $(b_0, z_0)$  such that  $z_0 < \min\{\bar{Z}(b_0), \underline{Z}(0)\}$ , there is a  $k^* > 0$  such that if  $k < k^*$ , there is an equilibrium path in which whites initially outbid blacks. If  $k$  is sufficiently small, then if whites win all houses for a long enough time this will eventually reduce  $b$  to such an extent that the state will move to the left of  $\underline{Z}'$  without previously moving to the left of (or even reaching)  $\bar{Z}''$ . Let  $\nu$  be the earliest time at which  $z_0 + k\nu \leq \underline{Z}'(b_0 e^{-\delta\nu})$ . At some time  $\nu' > \nu$ , the trend in  $z$  must cause the state to reach (and subsequently cross)  $\underline{Z}'$ . Consider the path in which whites win all houses until time  $\nu'$ , after which blacks always win. Clearly, this path is consistent with equilibrium after  $\nu'$ . Between times  $\nu$  and  $\nu'$ , whites will outbid blacks regardless of agents' expectations, so the path is consistent here as well. By taking  $k$  to be small, time  $\nu'$  can be made arbitrarily large relative to time  $\nu$ . Hence, agents who bid before time  $\nu$  believe that for a very long time all houses will be won by whites. Since before time  $\nu$ , the state is always to the right of  $\bar{Z}''$  (the right boundary of the region where whites will outbid blacks if they expect whites to buy *all* future houses), for  $k$  sufficiently small, whites will outbid blacks before time  $\nu$  as well. This shows that  $\lim_{k \rightarrow 0} \bar{Z}'(b) = \min\{\bar{Z}(b), \underline{Z}(0)\}$ . Q.E.D.

**Definition of  $Z(b)$  from Theorem 3:**

For any  $b$ ,  $Z(b)$  is the value of  $z$  at which the "expected static payoff" of blacks and whites from living in the neighborhood is equal. This "expected static payoff" is computed as

if agents do not know the ethnic composition of the neighborhood for sure. Rather, they put a probability  $p(b'|b)$  on the proportion of blacks being  $b'$ , where this probability also depends on the current proportion of blacks,  $b$ . To be precise,  $Z(b)$  is the solution to:

$$(1) \quad \int_{b'=0}^1 p(b'|b) D(b', Z(b)) db' = 0$$

where the probability  $p(b'|b)$  equals  $c[b'/b]^{\theta/\delta}$  if  $b' \leq b$  and  $c[(1-b')/(1-b)]^{\theta/\delta}$  if  $b' \geq b$ . ( $c$  is a constant that guarantees that the probabilities integrate to 1.) Note that the probabilities are single peaked at  $b' = b$ , so that the current proportion of blacks,  $b$ , has the highest probability.

### **Proof of Theorem 3:**

We perform iterative elimination of strictly dominated strategies using *translations* of  $Z$ . Define  $Z_0(b) = Z(b) + k_0$ , where  $k_0$  is large enough that blacks must outbid whites whenever  $z_t > Z(b_t) + k_0$ . Blacks must outbid whites at any state that lies to the right of  $Z_0$ . Inductively let  $Z_n$  be the leftmost translation of  $Z$  such that blacks must outbid whites to the right of  $Z_n$  if agents expect blacks to outbid whites to the right of  $Z_{n-1}$ .<sup>21</sup> The sequence must be weakly decreasing since knowing that blacks outbid whites in more scenarios makes blacks bid more and whites bid less. Let  $Z_\infty$  be the limit of this sequence; blacks must outbid whites when to the right of  $Z_\infty$ . Moreover, since the iterations stop at  $Z_\infty$ , there must be a point  $A$  on  $Z_\infty$  at which blacks and whites are willing to pay the *same amount* if they expect all future home auctions to be won by blacks at states to the right of  $Z_\infty$  and by whites at states to the left. If there were no such point - i.e., if blacks outbid whites at all points on  $Z_\infty$  - then we could continue to iterate, moving the curve further to the left.

Under this belief, when to the right of  $Z_\infty$ ,  $\dot{b} = \delta(1-b)$ ; when to the left of the curve,  $\dot{b} = -\delta b$ . These dynamics are unstable, since the movement in  $b$  always pulls the state *away* from  $Z_\infty$ . (With a bit of algebra, one can verify that  $Z$ , defined in (1), is strictly downward sloping if  $\theta > 0$ ; thus,  $Z_\infty$  is also strictly downwards sloping.) When the shocks to  $z$  are small, the movement in  $b$  is fast relative to the movement in  $z$ , so the system very quickly *bifurcates*, either upwards (leading all subsequent houses to be won by blacks for a very long time) or

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<sup>21</sup>  $Z_n(b) = Z(b) + k_n$ , where  $k_n$  is large enough that blacks must outbid whites whenever  $z_t > Z(b_t) + k_n$  if all agents expect blacks to outbid whites whenever  $z_t > Z(b_t) + k_{n-1}$ .

downwards (in which case whites win every house for a long while).

By Burdzy, Frankel, and Pauzner [5] (Theorem 2 and Corollary 1), as the variance and trend of  $z$  shrink to zero, the amount of time that passes before a bifurcation occurs goes to zero. Moreover, the chance of bifurcating up converges to  $1 - b$ , while the chance of bifurcating down goes to  $b$ . Hence, the difference between black and white bids is approximately

$$(1 - b) \cdot \int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(b_t^\uparrow, Z_\infty(b)) dt + b \cdot \int_{t=0}^{\infty} e^{-(\theta+\delta)t} D(b_t^\downarrow, Z_\infty(b)) dt$$

where  $b_t^\uparrow = 1 - (1 - b)e^{-\delta t}$  and  $b_t^\downarrow = be^{-\delta t}$ . This must equal zero at point A since, by assumption, blacks and whites bid the same here. By performing the changes of variables  $b = b_t^\uparrow$  and  $b = b_t^\downarrow$ , one can verify that  $Z_\infty(b) \approx Z(b)$ . Since the two curves have the same shape, in the limit blacks win at all states lying to the right of  $Z$ . An analogous argument (using iterations of  $Z$  from the left rather than the right) shows that whites must win to the left of  $Z$ . In fact,  $Z$  is the set of states  $(b, z)$  at which blacks and whites are willing to pay the same for a home if they believe that an immediate bifurcation will occur and with probability  $1 - b$  it will be upwards.

Q.E.D.

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