# **Optimal Unemployment Insurance with Monitoring**

Ofer Setty \* Tel Aviv University

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## Abstract

I model job-search monitoring in the optimal unemployment insurance framework, in which jobsearch effort is the worker's private information. In the model, monitoring provides costly information upon which the government conditions the unemployment benefits. Using a simplified two-period model with log utility I show that the monitoring frequency increases and the sanction upon a bad signal decreases with the generosity of the welfare system. The quantitative exercise that follows shows that compared to optimal unemployment insurance, monitoring saves about 60%of the cost associated with moral hazard.

JEL Classification: E24; D82; J64; J65.

Keywords: Unemployment Insurance; Optimal Contract; Moral Hazard; Job-search Monitoring.

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## **1** Introduction

Most unemployment insurance (UI) programs in the United States include the monitoring of jobsearch effort (Grubb, 2000). A typical monitoring policy requires the unemployed worker to record her job-search activities by listing the employers she contacted in a given period. At the employment office, a caseworker occasionally evaluates whether the job-search requirements are met by verifying that the contacts are authentic. If the caseworker finds the report unsatisfactory, then she may impose sanctions, usually in the form of a reduction in benefits for a limited period.<sup>1</sup>

In this paper I incorporate monitoring into the principal-agent framework of optimal unemployment insurance as in Hopenhayn and Nicolini (1997). Monitoring allows the principal (planner) to acquire imperfect information that is related to the job-search effort of the worker. Using a twoperiod model with log utility I characterize the optimal contract. I then parameterize the infinitehorizon model to the US economy and use it to estimate the value of monitoring relative to the optimal contract without monitoring.

In optimal unemployment insurance, a risk-neutral planner insures a risk-averse worker against unemployment by setting transfers during unemployment and a wage tax or a subsidy during employment. During unemployment, the worker searches for a job by exerting effort, the level of which is private information. Since the planner cannot observe the job-search effort, the constant benefits that are implied by the first-best allocation would undermine the worker's incentives to search for a job. Therefore, to solve the incentive-insurance trade-off, benefits should continuously decrease during unemployment and the wage tax upon re-employment should continuously increase.

I include monitoring in this framework as follows. The planner monitors the unemployed worker with some history-dependent probability. When a worker is monitored, the planner pays a cost and receives a signal that is correlated with the worker's job-search effort. The planner uses that signal to improve the efficiency of the contract by conditioning future payments and the wage tax not only on the employment outcome, but also on the signal. These future payments create endogenous sanctions and rewards that, together with the random monitoring, create effective job-search incentives. The worker exerts a high job-search effort in order to increase the probability of a good signal and, consequently, of higher payments.

In order to analyze the model analytically I use a two-period model with log utility that cap-

<sup>&</sup>lt;sup>1</sup>Other countries, such as Australia, Canada, Switzerland and the United Kingdom, use job-search monitoring for unemployed workers as well (Grubb, 2000). Since the policy implementation defers across countries, I focus on job-search monitoring in the United States.

tures the same economic forces as the infinite-horizon model. The monitoring frequency and the dispersion of future utilities complement one another in creating the incentives for the worker to actively search for a job. The specific combination of those two components depends on promised utility, which represents the welfare system's generosity. As the generosity of the welfare system increases, the planner monitors the unemployed worker more frequently but imposes lower sanctions. The driving force of this result is that while the per-unit cost of monitoring is independent of the generosity of the welfare system, the cost of spreading out future utilities increases with it.

I then parameterize the infinite-horizon model to the US economy. I use the parameterized model to estimate the value of monitoring by comparing the results of this model to those of a model in which monitoring technology is unavailable. I find that at the balanced budget point the gain from adding monitoring equals 61% of the difference between the planner's value of the first-best and the value in the model without monitoring. These savings stem from the planner's ability to smooth the worker's consumption across states. Simulating individuals' consumption histories shows that monitoring reduces log-variance of consumption by 46%.

There is some empirical support that monitoring is both beneficial and required for reducing the duration of unemployment. The effect of job-search monitoring on unemployment duration is usually significant and positive.<sup>2</sup> Johnson and Klepinger (1994) use the Washington Alternative Work-Search Experiment, which includes random assignment of unemployed workers to treatment groups that differ in their job-search requirements. They find that waiving the weekly requirement to record three contacts increases the average unemployment spell by 3.3 weeks. Klepinger, Johnson, Joesch, and Benus (1997) evaluate the Maryland Unemployment Insurance Work Search Demonstration. They find that increasing the number of required contacts from two to four decreases the average unemployment spell by 5.9%. They also find that informing the unemployed workers that the contacts will be verified decreases the average unemployment spell by 7.5%.

The evidence on the effects of sanctions is limited yet positive. In two empirical studies conducted in the Netherlands, van den Berg, van der Klaauw, and van Ours (2004) and Abbring, van den Berg, and van Ours (2005) find that the unemployment exit rate doubles following a sanction. Lalive, van Ours, and Zweimüller (2005) use Swiss data on benefit sanctions and find that both warning about not complying with eligibility requirements and enforcement have a positive effect on the unemployment exit rate. In addition, increasing the monitoring intensity reduces the unemployment duration of non-sanctioned workers.

<sup>&</sup>lt;sup>2</sup>van den Berg and van der Klaauw (2006) consider a model where search efficiency is undermined because unemployed workers substitute formal for informal channels. For adverse effects of job search assistance see Van den Berg (1994) and Fougere, Pradel, and Roger (2009).

Finally, the need for this policy is evident in recent papers that use the American Time Use Survey to measure the time spent searching for jobs. For example, the study by Krueger and Mueller (2010) shows the following two observations. First, the average job search per weekday of the unemployed is very low (about 41 minutes). Second, unemployed workers who are eligible to UI benefits search on average only 14 minutes more per day than ineligible ones. Taken together, these empirical findings show that job-search monitoring can be an important policy instrument for workers on UI.

I now turn to review the theoretical literature on monitoring and sanctions. A typical assumption in principal-agent models that include costly state verification is that monitoring perfectly reveals the agent's hidden information (or action) to the principal. This simplifying assumption rests on Becker (1968) seminal paper "Crime and Punishment." In a standard environment, such signals allow the planner to get arbitrarily close to the first-best allocation, by using a combination of very low monitoring frequencies that cost very little and extremely severe punishments that will never be applied.

Allowing the signal to be imperfect, as I do here, has three salient implications. First, the monitoring probability becomes a choice variable. Second, the contract dictates endogenously limited sanctions and rewards. Third, sanctions are applied in equilibrium. These results are realistic for many applications of monitoring, including that of unemployment insurance benefits. Specifically, maximal sanctions are usually not practiced and monitoring is not applied at a minuscule probability. Moreover, it may be infeasible or too costly to perfectly verify the level of the worker's job-search effort.

Since the planner's ability to acquire imperfect information is common to many principal-agent settings, I review models of monitoring in various contexts, with either perfect or imperfect signals. Aiyagari and Alvarez (1995) extend the Atkeson and Lucas (1995) analytical framework by introducing costly monitoring technology. They assume a lower bound on the expected discounted utility that can be assigned to any agent at any date. As in Becker (1968), the monitoring technology is perfect. The solution to their problem, however, differs from Becker's because the presence of a lower bound prevents the principal from inflicting Becker's infinite punishment. Given their rich monitoring technology and the bound on utility they show that the monitoring probability, unlike in my model, is non-monotone.

Popov (2009) models verification of hidden information as reported by a worker. He keeps the problem nontrivial by assuming that the utility function is bounded from below and that the continuation utility is bounded. With this assumption, the contract delivers bounded sanctions and rewards, depending on the verification result. Popov finds that monitoring never occurs with certainty and that for a certain class of utility functions, the principal uses verification regardless of this cost.

Zhang and Ravikumar (2012) study optimal monitoring in a tax compliance context with hidden income in a model with CARA utility. In their model audits are more beneficial the later they are conducted because the likelihood of hidden income increases with time. Since the cost of auditing is constant the optimal application of monitoring consists of cycles: initially a low-income taxpayer is unaudited, but with time he faces a positive probability of auditing.

I now turn to review studies that model monitoring specifically in the context of unemployment insurance. The closest paper to the one I study is by Boone, Fredriksson, Holmlund, and van Ours (2007). They analyze the design of optimal unemployment insurance in a search equilibrium framework. They allow the signal to be imperfect but they restrict the set of policies from which the optimal policy is chosen. First, the planner does not condition benefits on the worker's history; second, the planner can apply only a fixed decrease in benefits for the remainder of the unemployment spell. Their model, however, has the advantage of general equilibrium, which my model lacks.

Fredriksson and Holmlund (2006b) use a search model to compare between three different means of improving the efficiency of UI: the duration of benefit payments, monitoring in conjunction with sanctions, and workfare. Their analysis suggests that a system with monitoring and sanctions restores search incentives most effectively, since it brings additional incentives to search actively so as to avoid the sanction.

Pavoni and Violante (2007) consider monitoring as part of an optimal Welfare-to-Work program. In their model, the planner can observe the worker's job-search effort perfectly by paying some cost. As a result, the planner monitors this effort with certainty and sanctions or rewards are never needed.

The literature has also adressed other aspects of the interaction between the unemployed worker and the employment agency. Wunsch (2013) uses a framework similar to the one in this paper to study another aspects of the interaction between the case worker and the unemployed worker, when the planner can assist the worker by improving her job-search ability and interviewing skills. Pavoni, Setty, and Violante (2014) study policies that are based on allowing the unemployed worker to defer her job search to an agency at a cost.

Fuller, Ravikumar, and Zhang (2012) study concealed earnings, i.e., when an unemployed worker becomes employed and still continues to collect benefits. They show that in the optimal contract the planner monitors the worker at fixed intervals. Similar to my findings, unemployment

benefits are relatively flat between verifications but decrease sharply after a verification.

Fredriksson and Holmlund (2006a) survey studies on the design of UI in the context of time profile, monitoring with sanctions, and workfare. In addition, using a unified theoretical model, they show how the three instruments are different ways of imposing a penalty on less active job search.

The remainder of the paper is organized as follows. In Section 2, I describe the model. In Section 3, I provide analytic results using a simplified two-period model. In Section 3, I parameterize the model to the US economy. In Section 4, I provide numerical results based on the infinite-horizon model. I parameterize the model and use it to derive the optimal instruments of the monitoring policy and to assess the value of monitoring relative to optimal unemployment insurance. Section 5 concludes.

## 2 The model

In this section I describe the model in detail.

## 2.1 The economy

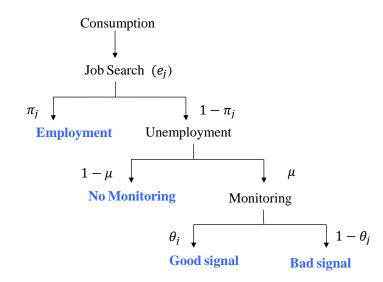
**Preferences:** Workers have a period utility u(c) - a, where c is consumption, a is disutility from a job-search effort or work, and u is strictly increasing and strictly concave. Workers discount the future at the discount factor  $\beta$ .

**Employment and unemployment:** The worker is either employed or unemployed. During employment, which is assumed to be an absorbing state, the worker exerts a constant effort level  $e_w$  and receives a fixed periodic wage w.<sup>3</sup>

During unemployment, the worker searches for a job with an effort level  $a \in \{e_l, e_h\}$  that is either low or high. This effort is the worker's private information. The job-finding probability, denoted by  $\pi_j$ , increases with the job-search effort level  $j \in \{l, h\}$ . The low job-search effort is interpreted as not actively looking for a job; I therefore normalize  $e_l$  to 0 and set  $\pi_l = 0$ . For brevity of notation, I henceforth denote  $e_h$  as e, and  $\pi_h$  as  $\pi$ .

<sup>&</sup>lt;sup>3</sup>The assumption that employment is an absorbing state is widely used in the literature (e.g., Hopenhayn and Nicolini 1997, Pavoni 2009, and Pavoni and Violante 2007). This assumption allows us to analyze one unemployment spell at a time, and does not affect the qualitative characteristics of the optimal policy. Hopenhayn and Nicolini (2009) characterize the optimal unemployment insurance contract in environments in which workers experience multiple unemployment spells.

Figure 1: The timing of the model



Notes: The timing of the model is shown here with the four possible outcomes: employment, unmonitored unemployment, monitored unemployment with a good signal, and monitored unemployment with a bad signal.

Monitoring technology: The monitoring probability  $\mu \in [0, 1]$  is one of the planner's choice variables. When the worker is monitored, the planner receives a signal on the worker's job-search effort that is either good (g) or bad (b). The probability of a *good* signal given job-search effort  $j \in \{l, h\}$  is  $\theta_j$ . The signal is informative only if  $\theta_h \neq \theta_l$ . Hence, I assume, without loss of generality, that  $\theta_h > \theta_l$ . This means that following a high rather than a low job-search effort, a monitored worker is more likely to receive the good signal.<sup>4</sup>

Allowing  $\theta_h$  to be *smaller than 1* indicates that the planner receives imperfect information regarding the worker's effort. This *false negative* option is a realistic feature of the unemployment insurance system, representing a verification that unjustifiably fails. Allowing  $\theta_l$  to be *greater than*  $\theta$  is another source of imperfection, representing a *false positive* result. This imperfection occurs, for example, as a result of an administrative failure or caseworker over generosity. The cost of monitoring is linear in the monitoring frequency and equal to  $\kappa \mu$  per period.<sup>5</sup>

Information structure: Both the worker and the planner observe the employment state, the moni-

<sup>&</sup>lt;sup>4</sup>Note that this technology does not restrict the value of  $\theta_h$  to be higher than 0.5. Indeed, there may be some strict monitoring tests generating a useful signal for which  $\theta_h$  can be very small.

<sup>&</sup>lt;sup>5</sup>Given that the administrative institutions for unemployed workers already exist, I assume that monitoring involves no additional fixed cost.

toring signal, and the on-the-job effort level.<sup>6</sup> The worker's job-search effort level is private information. This leads to the moral hazard problem.

**Timing:** Figure 1 shows the model's time frame and the four possible outcomes. At the beginning of each period, the planner delivers consumption c to the worker. Then, the worker looks for a job with an effort level  $e_j$  and finds a job with probability  $\pi_j$ . If the worker becomes employed, the planner does not apply monitoring.<sup>7</sup> If, however, the worker remains unemployed, the planner applies monitoring with probability  $\mu$ . When monitoring takes place, the planner pays the cost  $\kappa$  and receives the signal  $s \in \{g, b\}$ .

Given the realization of the employment state, monitoring, and the signal, the four possible outcomes are employment (e), unmonitored unemployment (n), monitored unemployment with a good signal (g), and monitored unemployment with a bad signal (b).

### 2.2 The planner's problem

The optimal contract between the planner and the worker requires the benefits and the wage tax to be conditioned on the worker's entire history. Abreu, Pearce, and Stacchetti (1990) find that all the relevant information for the recursive contract is contained in a one-dimensional object. In the monitoring contract, as in the unemployment insurance contract, this one-dimensional state is the expected discounted utility U promised to the worker at the beginning of each period. This value is updated at the end of each period, according to the outcomes. Hence, this state (U) is governed by all the relevant information in the worker's history. Although this state is not a primitive of the model, using it makes the problem tractable. Once the model is solved, the state can be used to recover the allocation for each type of worker. I maintain the standard assumption that the principal is able to fully commit to the contract.

In what follows, I present the planner problems for an employed worker and for an unemployed one.

<sup>&</sup>lt;sup>6</sup>This assumption is standard in the optimal unemployment insurance literature. Wang and Williamson (2002) consider the case where the worker's effort level affects the probability of transitions both from unemployment to employment and vice versa.

<sup>&</sup>lt;sup>7</sup>When a worker becomes employed, the effort level is perfectly revealed to the planner because  $\pi_l = 0$ ; hence monitoring such a worker is never optimal.

### 2.2.1 The planner's problem for an employed worker

Let W(U) be the planner's value from an employed worker who has promised utility U. The planner's problem for an employed worker is:

$$W(U) = \max_{c,U^e} -c + w + \beta W(U^e)$$
(1)  
s.t. :  
$$U = u(c) - e_w + \beta U^e,$$

where  $U^e$  is the future promised utility contingent on employment. If c > w, the planner delivers the difference to the worker as a wage subsidy; if c < w, the planner extracts the difference as a wage tax. The *promise-keeping* constraint in the problem imposes that the planner delivers in expected terms the utility promised to the worker.

With no moral hazard during employment, the solution to the employment problem is full insurance and constant benefits. This implies a constant wage tax or subsidy.

### 2.2.2 The planner's problem for an unemployed worker

For an unemployed worker, the planner has six decision variables: consumption c, monitoring probability  $\mu$ , and four continuation values, one for each possible outcome: employment  $U^e$ , unmonitored unemployment  $U^n$ , monitored unemployment with a good signal  $U^g$ , and monitored unemployment with a bad signal  $U^b$ .

In addition to these six decisions, the planner recommends a job-search effort level.<sup>8</sup> The high job-search effort recommendation needs to be supported by appropriate incentives. This is achieved with the *incentive-compatibility* constraint, which guarantees that the expected utility for a worker who exerts high job-search effort is at least as high as that for a worker who exerts low job-search effort.<sup>9</sup>

<sup>&</sup>lt;sup>8</sup>If the planner recommends the low effort level, there is no need to set incentives. The solution is constant benefits and a constant wage tax. This solution can be achieved because while  $\pi > 0$ , the probability of finding a job associated with zero effort is zero. The planner therefore knows that a worker who received a job-offer must have expended high effort when searching for that job. The planner can use this observation to apply punishments severe enough to discourage workers from not following the low job-search effort recommendation.

<sup>&</sup>lt;sup>9</sup>For sufficiently high promised utility, creating incentives by spreading future promised utilities is too costly; hence, the planner recommends low job-search effort and implements full insurance (Pavoni and Violante (2007) refer to this state as Social Assistance). To fully characterize the optimal monitoring policy, I describe the monitoring policy while assuming that it is always desirable to create incentives to expend high job-search effort. According to the calibration, social assistance is optimal only for those values of promised utility associated with consumption levels that are more than 10 times the government's balanced-budget point.

Let V(U) be the planner's value from an unemployed worker who has promised utility U. The planner's problem for an unemployed worker is:

$$V(U) = \max_{c,U^{e},U^{n},U^{g},U^{b},\mu} -c + \beta \{\pi W(U^{e}) + (1-\pi)(1-\mu)V(U^{n}) + \mu \left[\theta_{h}V(U^{g}) + (1-\theta_{h})V(U^{b})\right] - \kappa\mu\}\}$$
  
s.t. :  

$$U = u(c) - e + \beta \pi U^{e} + \beta (1-\pi) \left[(1-\mu)U^{n} + \mu \left(\theta_{h}U^{g} + (1-\theta_{h})U^{b}\right)\right]$$
  

$$U \ge u(c) + \beta \left[(1-\mu)U^{n} + \mu \left(\theta_{l}U^{g} + (1-\theta_{l})U^{b}\right)\right],$$
(2)

where the objective function includes the cost of consumption payments to the worker and the discounted weighted values of the four possible outcomes. The constraints are *promise keeping* and *incentive compatibility*, discussed above.<sup>10</sup>

## **3** Analytic results

In order to analyze the model analytically I use in this section a two-period model that captures the same economic forces as the infinite-horizon model. Specifically, the possible outcomes, their probabilities, and the choice variables of the planner are all identical to the infinite-horizon problem. The following adjustments take place in the two-period model: W(U) becomes  $w - c^e$ ;  $U^i$  becomes  $u(c^i)$  for  $i \in \{e, n, g, b\}$ ; and  $V(U^i)$  becomes  $(-c^i)$  for  $i \in \{n, g, b\}$ . I impose logarithmic utility from consumption and discuss more general preferences in Section 4.4.<sup>11</sup> Finally, I assume that  $\beta = 1$  for simplicity.

<sup>&</sup>lt;sup>10</sup>Formally both constraints should state that LHS $\geq$ RHS. I show in Claim 1 in Appendix A that both constraints are tight. The same line of proof can be used to show equality in the constraints in (3) below.

<sup>&</sup>lt;sup>11</sup>In an unpublished manuscript, available at http://www.tau.ac.il/ ofers/MHEM.pdf (Setty (2015)) I show that in a model where the principal chooses the monitoring *precision* instead of the monitoring *frequency*, monitoring precision increases with the agent's promised utility *iff* the coefficient of constant relative risk aversion is strictly greater than 0.5.

The problem of the planner for an unemployed worker is then:

$$V(U) = \max_{\substack{c,c^{e},c^{n},c^{g},c^{b},\mu \\ s.t. :}} -c - \pi (c^{e} - w) - (1 - \pi) \{ (1 - \mu) c^{n} + \mu [\theta_{h}c^{g} + (1 - \theta_{h}) c^{b}] + \kappa \mu \}$$
  

$$s.t. :$$

$$U = u(c) - e + \pi u(c^{e}) + (1 - \pi) [(1 - \mu) u(c^{n}) + \mu (\theta_{h}u(c^{g}) + (1 - \theta_{h}) u(c^{b}))]$$

$$U \ge u(c) + [(1 - \mu) u(c^{n}) + \mu (\theta_{l}u(c^{g}) + (1 - \theta_{l}) u(c^{b}))].$$
(3)

I start the characterization of the optimal contract with two results regarding the relative values of consumption levels. The first result refers to the ranking of the future consumption levels.

**Lemma 1** In the optimal solution,  $c^e > c^g > c^n > c^b$ .

All proofs are relegated to appendix B.

According to Lemma 1, the monitoring signal creates, relative to unmonitored unemployment, an endogenous prize  $c^g - c^n > 0$  when the good signal is realized, and an endogenous sanction  $c^n - c^b > 0$  when the bad signal is realized.

The next result refers to the relationship between the three consumption levels that are associated with unemployment:  $\{c^n, c^g, c^b\}$ .

**Lemma 2** In the optimal solution consumption upon the state *n* is equal to an average of consumption levels upon the signal, weighted by the probability of a good signal given a high effort.

$$c^n = \theta_h c^g + (1 - \theta_h) c^b \tag{4}$$

Thus, in the optimal contract, the prize and the sanction balance each other. Also observe that as the precision of the monitoring signal increases, the ratio of sanctions  $c^n - c^b$  to prizes  $c^g - c^n$ ,  $\frac{c^n - c^b}{c^g - c^n} = \frac{\theta}{1 - \theta}$ , increases. For high precision signals, monitoring allocates modest prizes with a high probability and severe sanctions with a low probability. In the extreme case, when  $\theta_h = 1$ , the sanction explodes relative to any other spread, as for example in Becker (1968). In the other extreme case of a non informative signal-that is, when  $\theta_h = \theta_l$  both  $c^g$  and  $c^b$  are equal to  $c^n$ .

Note that as long as  $\theta_h > \theta_l$ ,  $\theta_h = 1$  provides a perfect signal regardless of the value of  $\theta_l$ . This is so because upon receiving a bad signal, the planner knows with certainty that the worker deviated from the recommended level of effort. This event, regardless of its probability, can be leveraged as much as needed to provide the incentives for the worker to exert the high level of effort.

I now move to the effect of the worker's state on the monitoring frequency and the spread of future consumption.

### **Proposition 1** The optimal monitoring frequency increases with U.

Monitoring frequency is one of the two instruments of monitoring policy. The second instrument is the choice of the planner to spread out future utilities (henceforth, spreads). Define the spreads as the difference between promised utility of outcomes  $\{i, j\} \in \{e, n, g, b\}$  is  $u(c^i) - u(c^j)$ such that  $c^i \ge c^j$ . The next Lemma establishes that the dynamics of one spread can be used to characterize all the spreads.

### **Lemma 3** When either $\mu$ , U, or both, change, all the spreads move in the same direction.

The next Proposition complements the first by showing how the spreads respond to changes in promised utility.

### **Proposition 2** The spreads decrease with promised utility.

Taken together, the two propositions above suggest that as the promised utility of the worker increases, the planner monitors the unemployed more frequently but imposes more moderate sanctions. Note that this result holds for any parametrization of the model as long as utility from consumption is logarithmic.

The intuition for this result is the cost of monitoring relative to the cost of spreading out utilities. The cost of monitoring does not depend on the generosity of the welfare system. However, as I show formally in Appendix C, the cost of spreading out utilities for log-utility preferences increases with promised utility. Hence, as the generosity of the welfare system increases the cost of spreading out utilities relative to the cost of monitoring increases and the planner uses monitoring more intensively and imposes lower sanctions.

## 4 Numerical results

The previous section showed the interplay between the two instruments of monitoring – the monitoring frequency and the utility spread – using a simple two-period model. In this section I parameterize the infinite-horizon model and use it to quantitatively characterize the optimal contract and to assess the value of monitoring by comparing the results of the model to those of optimal UI where the monitoring technology is unavailable.

Table 1: Parameters

Parameter	Symbol	Value	Source	
Discount factor	β	0.9959	5% Annual interest rate	
Wage	w	\$3,900	National compensation survey (201-	
Unemployment exit rate	$\pi$	0.42	Shimer (2012)	
Disutility from effort	$e, e_w$	0.67	Pavoni and Violante (2007)	
Good signal probability given <b>high</b> effort	$ heta_h$	0.98	See text	
Good signal probability given <b>bad</b> effort	$\theta_l$	0.08	See text	
Monitoring cost	$\kappa$	\$37	See text	

### 4.1 Parameterization

Table 1 lists the model's parameters. The unit of time is set to one month. Preferences are log utility in consumption.<sup>12</sup> The monthly discount factor  $\beta$  is set to 0.9959 to match an annual interest rate of 5%. Monthly earnings, w, are set to \$3,900, which is, approximately the mean monthly earnings of all workers (DOL, 2014). The job-finding probability  $\pi$  is set to 0.42, based on the CPS-derived data constructed by Shimer (2012) for 2000–2006. The disutility of work effort,  $e_w$  is equal to 0.67, as in Pavoni and Violante (2007). For simplicity I assume that disutility of job-search effort, e is equal to that of work effort.<sup>13</sup>

The monitoring technology is characterized by three parameters: the probabilities of a good signal given high and low job-search efforts ( $\theta_h$  and  $\theta_l$ , respectively), and the monitoring cost per unit of monitoring  $\kappa$ .<sup>14</sup>

The parametrization of  $(\theta_h, \theta_l)$  is challenging for two reasons. First,  $\theta_h$  and  $\theta_l$  are related to a system imperfection that is unobservable by either the case worker or the economist. Second, while in the model all workers search for a job with high effort, the available data on monitoring are based on an unknown fraction of high-effort workers. I therefore denote by  $\xi$  the fraction of workers who search for a job with a high effort in the current system.<sup>15</sup>

<sup>&</sup>lt;sup>12</sup>I study more general preferences in section 4.4.

<sup>&</sup>lt;sup>13</sup>Krueger and Mueller (2010) show that the time spent in search in unemployment is quite low, thus calling for  $e \ll e_w$ . Taking this into account (e.g., setting  $e = \frac{e_w}{3}$ ) has almost no effect on the results. The reason is that unemployment spells are short (less than 3 months on average) and therefore the value for the planner is heavily dependent on the effort cost during employment.

<sup>&</sup>lt;sup>14</sup>The analysis here is based on the actual monitoring technology applied in the US. An alternative approach is the optimal choice of a monitoring technology as in Boone, Fredriksson, Holmlund, and van Ours (2007).

<sup>&</sup>lt;sup>15</sup>This parameter will not be used in the model. It is only required for the parametrization of the signal parameters

For the signal's calibration I use the systematic and detailed analysis of the adequacy of unemployment benefits, provided by the US Department of Labor (Woodbury, 2002; DOL, 2006b; Vroman and Woodbury, 2001). These audits reveal the fraction of overpayment and underpayment (denial errors) paid to workers specifically for non-separation errors, thus excluding reasons such as ineligibility due to insufficient previous earnings and quitting.

I proceed by providing explicit equations that connect  $\theta_h$ ,  $\theta_l$  and  $\xi$  to observed data. First, the fraction of *overpayment*, denoted by  $z_1$ , is equal to those who did not exert high effort yet received payments relative to all those who received payments:

$$\frac{(1-\xi)\,\theta_l}{(1-\xi)\,\theta_l+\xi\theta_h}.\tag{5}$$

Similarly, the fraction of *underpayment*, denoted by  $z_2$ , is equal to:

$$\frac{\xi \left(1-\theta_{h}\right)}{\left(1-\xi\right)\theta_{l}+\xi\theta_{h}}.$$
(6)

Finally, the fraction of monitored workers who were sanctioned, denoted by  $z_3$ , is equal to

$$z_3 = \xi * (1 - \theta_h) + (1 - \xi) * (1 - \theta_l).$$
(7)

Those three equations can be rewritten as an explicit unique solution of  $\{z_1, z_2, z_3\}$ :

$$\begin{aligned}
\theta_h &= \frac{1-z_1}{z_2+1-z_1} \\
\theta_l &= \frac{z_1 * (1-z_3)}{z_3 + (1-z_3) (z_1-z_2)} \\
\xi &= (1-z_3) [z_2+1-z_1].
\end{aligned}$$
(8)

Based on various sources, the values for  $\{z_1, z_2, z_3\}$  are  $\{1.4\%, 1.9\%, 16\%\}$ , respectively.<sup>16</sup> The implied values for the monitoring technology are  $\theta_h = 0.98$ ,  $\theta_l = 0.08$  and  $\xi = 0.84$ .<sup>17</sup>

that are based on the actual UI policy in the US. Evidently, and contrary to the model, some workers in the US do not search with high effort.

<sup>&</sup>lt;sup>16</sup>The basis for  $z_1$  and  $z_2$  is Table 1 in Woodbury (2002), which gives the percentage of overpayment as 7.2% and of underpayment as 3.4%. The fraction of overpayment due to non-separation errors is 19.8% (DOL (2006)) and for wrongful denials it is 57% (Vroman and Woodbury (2001)).  $z_3$  is equal to the monthly probability of sanctions ( $\phi$ ) of 3.3% (Grubb, 2000), over the monthly monitoring frequency,  $\mu^{ACT}$ , of 0.20 (see Appendix D).

<sup>&</sup>lt;sup>17</sup>The derivation of those values assumes that the audit reveals all the false positive and false negative cases. In practice, workers may manipulate the system by deceiving both the caseworker and the audit. In this case, the value of overpayment  $(z_1)$  is understated and so is the level of  $\theta_l$ . In the other direction – because of bureaucracy, difficulty of

The monitoring cost  $\kappa$  is based on data from the Minnesota Family Investment Program (MFIP 2000), in which each caseworker was responsible for 100 clients and, among other tasks, was assigned to apply sanctions, assist with housing, and document client activities. Based on monthly gross earnings of \$3,700 per caseworker and the described case load, the value of  $\kappa$  is \$37 per month per each unemployed worker monitored.<sup>18</sup> This value is an upper bound because the caseworkers also engaged in activities other than monitoring. Interestingly, although Boone, Fredriksson, Holmlund, and van Ours (2007) use completely different data sources, their equivalent value of  $\kappa =$ \$27 is not far from to the value I find.

## 4.2 Optimal monitoring policy

In this section I discuss the characteristics of the optimal monitoring policy.<sup>19</sup> The optimal contract is described recursively by the following six functions of the state  $U : \{c, U^e, U^n, U^g, U^b, \mu\}$ . I begin with the mapping of current promised utility to the next period's promised utility. In the optimal contract, the four future values, corresponding to the four possible outcomes, endogenously create implicit rewards and sanctions.

Figure 2 shows the mapping of promised utility by outcome across periods. The horizontal axis is promised utility at the beginning of the period. The vertical axis is the next period's promised utility. The four future promised utilities are ordered as follows:  $U^e, U^g, U^n, U^b$ . This follows directly from the likelihood ratios, with  $l^e > l^g > l^n > l^b$ , where:  $l^e = 1, l^g = \frac{(1-\pi)\theta_h - \theta_l}{(1-\pi)\theta_h}, l^n = -\frac{\pi}{1-\pi}, l^b = \frac{(1-\pi)(1-\theta_h)-(1-\theta_l)}{(1-\pi)(1-\theta_h)}$ . Thus, the monitoring signal implies endogenous prize and sanction relative to unmonitored unemployment.<sup>20</sup> If a good signal is realized then the worker receives a sanction of  $U^n - U^b > 0$ .

Upon employment – an outcome that can happen in the model only if the worker exerts a high job–search effort - promised utility increases. Upon monitoring with a good signal, the worker receives a reward that is lower but indistinguishable in the figure from that of employment. Upon unmonitored unemployment, the promised utility changes only slightly. Finally, upon monitoring

proof and other reasons – some workers may give up benefits to which they are rightfully entitled to. I complement the analysis with a sensitivity analysis over various values of the technology parameters.

<sup>&</sup>lt;sup>18</sup>Mean annual earnings for *Community and Social Services Occupations* in the US, based on data from 2010-2013 was \$44,710 (Department of Labor, 2014).

<sup>&</sup>lt;sup>19</sup>Appendix E describes the solution method.

<sup>&</sup>lt;sup>20</sup>The likelihood ratio is defined as follows. For each outcome  $i \in \{e, g, b, n\}$  define  $p_1^i, p_2^i$  as the probability of the outcome given high and low effort levels, respectively. Then  $l^i = \frac{p_1^i - p_2^i}{p_1^i}$ .

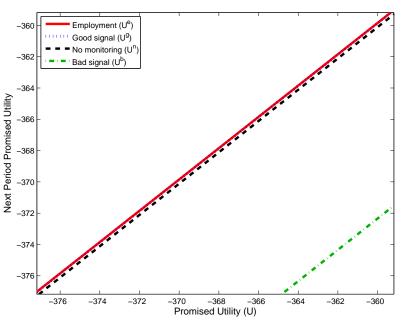


Figure 2: Conditional mapping of promised utility

Notes: The mapping of promised utility from the current period to the next period, conditional on the four possible outcomes: employment, unmonitored unemployment, monitored unemployment with a good signal, and monitored unemployment with a bad signal. The values for employment and monitoring with a good signal are above the diagonal (the diagonal itself is not illustrated) with the one for the good signal lower but indistinguishable form the one for employment. The value for unmonitored unemployment is slightly below the diagonal. Finally, the value for monitored unemployment with a bad signal is further away below the diagonal.

with a bad signal the worker experiences a relatively large decrease in promised utility.

The values of  $U^n, U^g$ , and  $U^b$ , are jointly determined by the following condition, based on the three first-order conditions:  $V'(U^n) = \theta_h V'(U^g) + (1 - \theta_h) V'(U^b)$ . Up to a linear approximation for the derivative of the planner's value function, the ratio of the sanction over the prize in continuation values can be written as:

$$\frac{U^n - U^b}{U^g - U^n} = \frac{\theta_h}{1 - \theta_h} \tag{9}$$

This condition, together with the calibration of  $\theta_h$  at 0.98, implies that the sanction level is significantly higher than the reward (the value of the ratio is 49). This result is consistent with the absence of prizes in the actual monitoring scheme, as prizes are relatively small. More generally, in the optimal contract the prize and the sanction balance each other. For high  $\theta_h$  signals, monitoring allocates modest prizes with a high probability and severe sanctions with a low probability. For

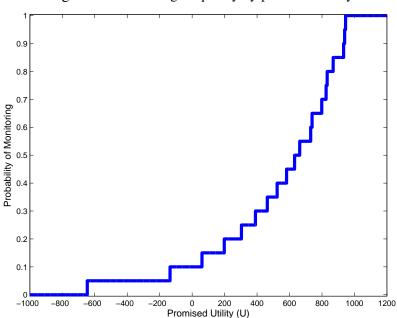


Figure 3: Monitoring frequency by promised utility

Notes: As the generosity of the welfare system increases, the monitoring frequency increases and the relative consumption sanction (see Fig. 4) decreases.

low  $\theta_h$  signals, monitoring allocates high prizes with a low probability and modest sanctions with a high probability.

I now move to discuss monitoring frequency and utility spread decisions. Intuitively, the planner can use each of those two instruments to support the incentive-compatibility constraint: increasing the monitoring frequency increases the *probability* of the threat; increasing the spread increases the *magnitude* of the threat. This is shown formally in Claims 2 and 3 in Appendix A.

Figure 3 shows monitoring frequency by promised utility.<sup>21</sup> Monitoring frequency increases monotonically across the promised-utility support. Notice that for low enough levels of U no monitoring takes place.

As for the spreads, observe in Figure 2 that  $U^b$  is much lower than the other three future utilities. Given the calibration of  $\theta_h$ , this is consistent with condition (9) above. To demonstrate the dynamics of the spreads, I focus on the level of  $U^b$  relative to  $U^n$ . The dynamics of the remaining spreads are identical.

 $<sup>^{21}</sup>$ The monitoring probability here is set in steps of 5%. Increasing the resolution of monitoring has no significant effect, either qualitative or quantitative, on the results. The span of promised utility in this figure ranges from equivalent consumption levels of \$2 to \$6,000 per month. This large span is used for demonstrating the contract's qualitative characteristics.

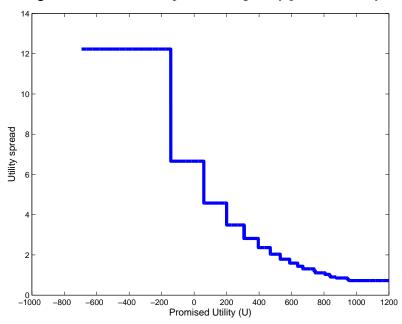


Figure 4: The sanction upon a bad signal by promised utility

Notes: As the generosity of the welfare system increases, the relative consumption sanction decreases When the monitoring frequency is constant (see Fig. 3), so is the sanction.

Figure 4 shows the difference  $U^n - U^b$  by promised utility for levels of promised utility at which  $\mu > 0.^{22}$  Contrasting Figures 3 and 4 regarding the monitoring and sanction reveals that as the generosity of the welfare system increases, the planner monitors the unemployed more frequently but imposes more moderate sanctions.

What is the intuition for the interplay between the monitoring frequency and the spread? The key to understanding this is the cost of spreading out utilities. For the case of log, that cost increases with promised utility (see Appendix C). Therefore, it is more costly to compensate workers when U is higher. In contrast to the cost of spreading utilities, the monitoring cost  $\kappa$  is independent of promised utility. Hence, as promised utility increases, the planner shifts the composition of the two monitoring components by increasing the monitoring frequency and decreasing the spreads.<sup>23</sup>

<sup>&</sup>lt;sup>22</sup>The equivalent permanent consumption sanction is  $exp((\Delta U) * (1 - \beta))$ . Therefore, a spread of 10 utilities in Figure 4 is equivalent to about a a permanent reduction in consumption of 4%.

<sup>&</sup>lt;sup>23</sup>Figures 3 and 4 demonstrate an additional feature of the two-period optimal contract (see Appendix B). When the monitoring frequency is constant, the sanction remains constant as well. This occurs for  $\mu = 1$  and whenever the monitoring frequency is constant.

## 4.3 The value of monitoring

How effective is monitoring in reducing the cost associated with moral hazard? To answer this question, we first need to establish a metric by identifying the relevant planner's values to be used. At one extreme, the planner's value under the first-best allocation, assuming that moral hazard is absent, is the highest value for delivering a given expected utility to the worker. At the other extreme, the planner's value under optimal unemployment insurance, where moral hazard exists, is a special case of monitoring with a monitoring frequency of 0. The planner's value from the optimal monitoring policy therefore has to lie between those two values.

To study the effectiveness of monitoring relative to unemployment insurance, I define  $\nu = \frac{V^{MON} - V^{OUI}}{V^{FB} - V^{OUI}}$ , where  $V^{MON}$ ,  $V^{OUI}$  and  $V^{FB}$  are the planner's values for optimal unemployment insurance with monitoring, optimal unemployment insurance, and the first best, respectively.<sup>24</sup> The difference  $V^{FB} - V^{OUI}$  is the moral hazard cost if no monitoring was available. The metric  $\nu$  is, therefore, the percentage of that cost that can be saved by including monitoring.

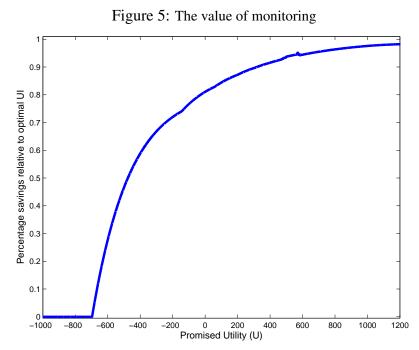
Figure 5 shows the value of monitoring over the support of promised utility. Monitoring is relatively more effective at high levels of promised utility because for log-utility preferences the cost of spreading out utilities increases with promised utility. At sufficiently low levels of promised utility, where monitoring is not used, the optimal monitoring policy coincides with the optimal unemployment insurance policy and the value is zero. At the other extreme of promised utility, savings strictly increase even though the monitoring frequency is constant because the cost of spreads continues to increase.

Since the effectiveness of monitoring varies significantly across states, I report the savings at the level of promised utility that balances the government's budget. The balanced budget point is  $U_0^*$ , such that  $V(U_0^*) = 0$ . This is the level of promised utility to ensure that tax revenues exactly cover the costs of benefits, wage subsidies and monitoring. This point is unique because V(U) is strictly monotone in U.<sup>25</sup> At  $U_0^*$  for the model with no monitoring, the addition of monitoring saves 61% of the moral hazard cost.<sup>26</sup>

<sup>&</sup>lt;sup>24</sup>The model with no monitoring is closely related to the model used in Hopenhayn and Nicolini (1997). The main difference is that in Hopenhayn and Nicolini the job-search effort level is continuous and not discrete. For the sake of consistency I use a discrete level of effort in both models.

<sup>&</sup>lt;sup>25</sup>Auray, Fuller, and Lkhagvasuren (2015) estimate that the fraction of eligible workers who collect benefits (the take-up rate) is 77%. Including this fraction in the parameterization has only a very small effect on the results.

<sup>&</sup>lt;sup>26</sup>In absolute values the potential savings of the difference between the value of the first best and that of optimal UI without monitoring is very low - less than \$10 per worker. This reflects the observation of Hopenhayn and Nicolini (1997) that a contract that is conditioned on the complete history of the worker, and uses the tax level as an instrument, gets very close to the first best. Since it is impractical to fully implement complete-histories contracts, the potential value of monitoring is probably higher than that absolute value. In any case, the role of the optimal contract is to



Notes: The value of monitoring as the fraction of moral-hazard cost associated with optimal unemployment insurance that is saved when the monitoring technology is available.

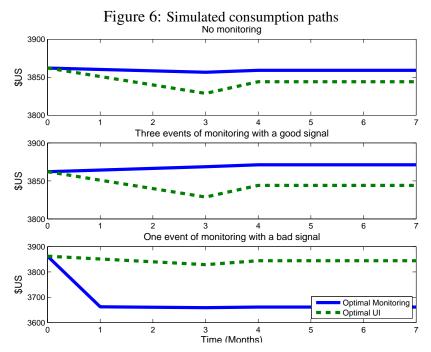
At  $U_0^*$  the monitoring frequency is 5% and the relative consumption sanction, which is approximately a permanent decrease in consumption, is 5%. As mentioned above, and shown in figure 2, changes in the other two spreads relative to unmonitored unemployment ( $U^e - U^n$  and  $U^g - U^n$ ) are much smaller than the spread upon a bad signal.

### 4.3.1 What makes monitoring effective?

In optimal unemployment insurance, the planner is required to spread out future utilities to create the job-search incentives. This action is costly because the worker is risk averse. Thus, the planner's cost due to monitoring can be reduced only by consumption smoothing. To demonstrate this I simulate the consumption paths for both the optimal unemployment insurance and the monitoring models.

Figure 6 shows three examples of consumption paths (in US\$) according to the two policies. In each example, the worker starts off unemployed with a promised utility level of  $U_0^*$ , stays unemployed for 3 periods, and then finds a job. In the top panel, there is no monitoring. In the middle

highlight the key economic forces that are relevant for the design of the policy.



Notes: Simulated consumption paths according to optimal monitoring and optimal unemployment insurance policies. The consumption paths for the unemployment insurance policy are identical. The consumption paths for the monitoring policy depend on whether monitoring was applied and on the signal's result. The vertical axis in the top two panels varies spans \$100. In the bottom one it spans \$300.

panel, monitoring is applied in periods 1, 2 and 3, resulting in a good signal in all three cases. In the bottom panel, monitoring is applied once in period 1, resulting in a bad signal.

In the absence of additional signals, the consumption paths for optimal unemployment insurance for these three cases are identical and are presented here as a reference. Consumption in this model first decreases monotonically and then increases when the worker finds a job. These shifts in consumption are required for creating the necessary incentives for unemployed workers to expend high effort given that only two possible outcomes are possible.

In contrast, consumption in the monitoring model, in the top two panels varies very little. It is only when the worker is sanctioned in the bottom panel (plotted with a wider vertical span) that consumption moves a lot. Sanctions, however, are a rare event; they happen only when both monitoring and a bad signal occur. At the balanced budget point the unconditional probability of a sanction,  $\mu * (1 - \theta_h)$ , is a very low one of 0.10%.

To quantify the consumption-smoothing effect of monitoring, I simulate the model for 5,000 workers over 60 periods. Using those simulations I find that monitoring reduces log-variance of

Table 2: Sensitivity analysis with respect to the monitoring technology parameters

A. Se	ensitivity	analy	sis of $\nu$	for the	e value of $\theta_k$
$ heta_h$	0.90	0.95	0.98	0.99	1.00
ν	0.25	0.44	0.61	0.69	1.00

<b>B.</b> Se	ensitivity	/ analy	sis of $\nu$	for the	e value of $\theta_l$
$\theta_l$	0.0	0.08	0.16	0.32	0.64
ν	0.63	0.61	0.58	0.51	0.20

<b>C.</b> Sensitivity analysis of $\nu$ for the value of $\kappa$					
$\kappa$ (\$)	0	19	37	75	150
ν	0.99	0.71	0.61	0.40	0.00

consumption by 46%. I conclude that because of the additional information regarding the jobsearch effort, monitoring allows the planner to smooth unemployed workers' consumption.

#### 4.4 Robustness

This section includes two parts. First, I show to what extent the value of monitoring is robust to the parameters used in the quantitative analysis. Second, I study how different levels of risk aversion affect the optimal contract.

#### 4.4.1 Sensitivity analysis

The comparison above between the models with and without monitoring relies on the effectiveness of monitoring. This in turn relies on the three parameters of monitoring technology: the probabilities of a good signal given high and low job-search efforts  $\theta_h$  and  $\theta_l$ , respectively; and the cost per unit of monitoring  $\kappa$ . In order to examine the sensitivity of the savings to these three parameters I analyze the response of savings at the balanced budget point to various values of the parameters.

Equation (7) is useful when choosing values for the signal's precision. This equation, together with the condition that  $\theta_h > \theta_l$ , implies that the *lower* bound of  $\theta_h$  and the *upper* bound of  $\theta_l$  are both equal to  $0.84.^{27}$ 

<sup>&</sup>lt;sup>27</sup>To get the lower bound on  $\theta_h$ , isolate  $\theta_l$  in Equation (3) and impose that the resulting term is less than  $\theta_h$ . To get the upper bound on  $\theta_l$ , isolate  $\theta_h$  in Equation (3) and impose that the resulting term is greater than  $\theta_l$ .

The probability of a good signal given the high job-search effort  $\theta_h$  determines the precision of the information extracted by applying monitoring. As  $\theta_h$  increases, the planner receives more accurate information on the level of the worker's job search. In the extreme case when  $\theta_h = 1$ , it is possible to get arbitrarily close to the first-best allocation by using a combination of a very low monitoring frequency (that costs very little) and an extremely severe punishment that is never applied. Panel A in Table 2 presents the savings at the balanced budget point for various levels of  $\theta_h$ . Holding  $\theta_l$  and  $\kappa$  fixed, as  $\theta_h$  increases beyond the benchmark value, the efficiency of monitoring increases as expected. As  $\theta_h$  decreases, the savings level decreases sharply.

As for  $\theta_l$ , if overpayment ( $z_1$  in equation (5)) is understated then the value of  $\theta_l$  is underestimated. The sensitivity analysis of  $\theta_l$  in panel B of Table 2 shows that as  $\theta_l$  gets closer to  $\theta_h$ , savings decrease significantly.

Panel C of Table 2 shows the savings for various values of the monitoring  $\cot \kappa$ . First, note that when  $\kappa = 0$ , the first best is not achieved because free monitoring provides imperfect information. Thus, he still needs to condition the promised utility on outcomes that will be realized in equilibrium, which is costly. Second, as the cost of monitoring increases, the planner uses monitoring less frequently and the level of savings decreases. When  $\kappa = 150$  – a monitoring cost that is more than four times higher than the benchmark calibration – savings goes to 0%.

### 4.4.2 General preferences

In this section I study to what extent the main result, that monitoring frequency increases and the spread decreases with promised utility, depends on the assumption of log preferences. To this end I use the class of constant relative risk aversion (CRRA) preferences:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \sigma \neq 1\\ \log(c) & \sigma = 1, \end{cases}$$
(10)

where  $\sigma$  is the coefficient of constant relative risk aversion.

Using this class of preferences I run the model with many instances of the coefficient  $\sigma$ .<sup>28</sup> Figures 7 and 8 show the emerging pattern for the monitoring frequency and the utility spread for three representative cases of the coefficient of risk aversion in CRRA: 0.25, 0.50, 0.75. For values

<sup>&</sup>lt;sup>28</sup>The disutility from job-search and work effort is parameterized to reflect (at the balanced-budget point) the same opportunity cost as the value of e = 0.67 in log-utility. This is done by finding the value that makes a worker indifferent between working and consuming  $c^h$  and not working and consuming  $c^u$  in the following static condition:  $u(c^e) - e = u(c^u)$ .

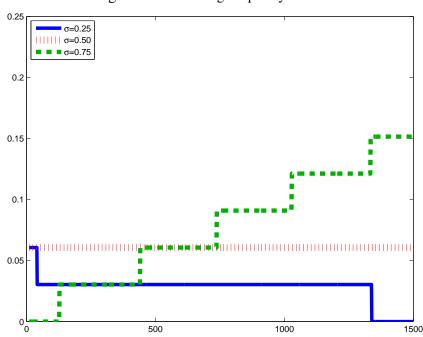


Figure 7: Monitoring frequency CRRA

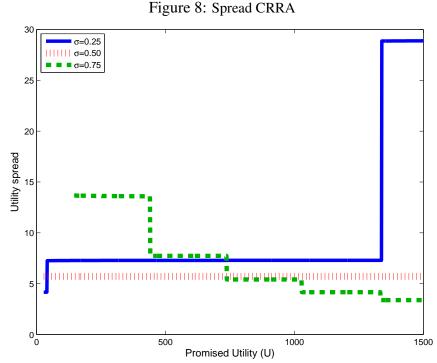
Notes: The monitoring probability for various values of the coefficient of risk aversion,  $\sigma$ .

of risk aversion higher than 0.50 we get a qualitatively similar result to that of log utility: the monitoring frequency increases and the spread decreases with promised utility. For values of risk aversion lower than 0.50 the opposite occurs. Finally, for the knife-edge case of a coefficient of constant risk aversion of 0.50 the monitoring frequency and the sanction are both constant.

The intuition for this result goes back to the observation shown in Appendix C that the cost of spreading out utilities increases with promised utility if and only if the coefficient of risk aversion is greater than 0.5. Thus, the same driving force for the result for log, i.e., the relative cost of spreading out utilities relative to the cost of monitoring determines the mix of the two instruments for each level of promised utility.

## **5** Concluding remarks

In this paper I added a monitoring technology to the optimal unemployment insurance framework. The introduction of monitoring into the model follows the practice of monitoring in the US: a caseworker randomly verifies the job-search activity of the unemployed worker (in the form of employment contacts) and sanctions the worker if the effort seems unsatisfactory. The model allows



Notes: The utility spread for various values of the coefficient of risk aversion,  $\sigma$ .

characterization of the optimal contract. In addition I evaluate the gain from using the monitoring technology, compared both to the optimal contract without monitoring and the actual policy in the US.

Allowing the signal to be imperfect in this analysis has important advantages both qualitatively and quantitatively. Qualitatively, the optimal contract includes three realistic features: a non trivial decision of the monitoring probability; endogenously limited sanctions and rewards; and application of sanctions in equilibrium. Quantitatively, this technology permits an assessment of the optimal monitoring technology and its value.

In this paper I used the standard assumption in optimal unemployment insurance that the hazard rate  $(\pi_h)$  is constant. This assumption allows a clear characterization of the contract and a straightforward evaluation of the gain from monitoring. Under this assumption the monitoring frequency and the sanction is fairly constant along the unemployment spell. In future research, the model can be extended to allow for negative duration dependence (as in Pavoni, 2009) in order to study the dynamics of monitoring frequency and sanctions over the unemployment spell.

One limitation of the framework used in this paper is that the model's tractability depends on the assumption that the planner controls the worker's consumption, that is, that no savings are allowed

on the worker's side. As pointed out by Abdulkadiroglu, Kuruscu, and Şahin (2002) and Shimer and Werning (2008), allowing workers to accumulate unobservable savings may significantly affect the results. The recursive contract framework strength is in demonstrating the main trade-offs for *the* optimal contract when a costly imperfect signal is available. It appears that as long as differentiation of future payments is necessary, monitoring could be effective in reducing the need for costly spreads.

Another limitation of this framework is that the sanctions are unjustified. This occurs because the incentive-compatibility constraint in the model holds. Nevertheless, the sanctions need to be placed in the contract to keep the workers' incentives in place. Note that the same concept of unjustified punishment holds in optimal unemployment insurance. There, conditional on unemployment, the worker experiences benefit cuts even though the planner is aware that the worker puts forth the recommended effort. A more realistic model would include unobserved heterogeneity in disutility from job search and from work. Under such circumstances, the sanctions in equilibrium would be partially justified.

According to Grubb (2000), there are significant differences across countries in all the policy's main characteristics. For example, in Australia, a moderate sanction of 18% of the benefits level is applied for a duration of six months, which is considerably longer that the one week denial of benefits in the US. At the same time, Australia's annual sanction rate, standing at 1.2%, is relatively low when compared with the 33% in the US. An extended model could reveal whether the variation in policies follows labor market characteristics or some inefficiencies.

Finally, the model can be used to study other applications of monitoring with costly signals, such as in the contexts of crime and punishment and labor contracts. Notice that the model can also be used in contexts where the probability of a good signal given high effort is very low (e.g., publishing in a very prestige journal). In such an environment the model's policy prescription is to provide the agent with a large prize (with a low probability) if a good signal is realized and a small sanction (with a low probability) if a bad signal is realized.

# Appendices

## A Proofs (infinite-horizon model)

Claim 1 The PK and the IC constraints are binding at the optimum.

**Proof.** Assume by negation that the PK constraint is not binding at the optimum. The solution can be improved by decreasing, for example,  $U^b$  by  $\varepsilon$ , since the IC constraint is slack (because  $\theta_h > \theta_l$ ). Hence, the PK constraint binds.

Assume by negation that the IC constraint is not binding at the optimum. Decrease  $U^e$  by  $\varepsilon$ , and increase  $U^b$  by  $\frac{\varepsilon \pi}{(1-\pi)\mu(1-\theta_h)}$ , such that V(U) remains unchanged. The LHS of the PK constraint increases by  $\frac{\varepsilon \pi}{(1-\pi)\mu(1-\theta_h)}(1-\pi)\mu(1-\theta_h)u(U^b) - \pi\varepsilon u(U^e)$ , which is positive since  $U^e > U^b$  (from the likelihood ratios). Since u is concave, the PK constraint becomes slack. Hence, the IC constraint binds at the optimum.

**Claim 2** Increasing the monitoring frequency (locally) relaxes the incentive-compatibility (IC) constraint.

**Proof.** Rewrite the IC constraint as follows:

$$-\frac{e}{\beta} + \pi U^{e} + (1 - \pi) \left[ (1 - \mu) U^{n} + \mu \left( \theta_{h} U^{g} + (1 - \theta_{h}) U^{b} \right) \right]$$

$$\geq (1 - \mu) U^{n} + \mu \left( \theta_{l} U^{g} + (1 - \theta_{l}) U^{b} \right).$$
(11)

Rearrange:

$$\pi U^{e} - \pi (1-\mu) U^{n} + \mu \left[ (1-\pi) \theta_{h} - \theta_{l} \right] U^{g} + \mu \left[ (1-\pi) (1-\theta_{h}) - (1-\theta_{l}) \right] U^{b} \ge \frac{e}{\beta}.$$
 (12)

The partial derivative of the LHS of (12) w.r.t.  $\mu$  is:

$$\pi U^{n} + \left[ (1 - \pi) \,\theta_{h} - \theta_{l} \right] U^{g} + \left[ (1 - \pi) \, (1 - \theta_{h}) - (1 - \theta_{l}) \right] U^{b}.$$
(13)

Rewrite as follows:

$$\pi U^{n} + \left[ (1 - \pi) \,\theta_{h} - \theta_{l} \right] U^{g} + \left[ (1 - \pi) \, (1 - \theta_{h}) - (1 - \theta_{l}) \right] U^{b}. \tag{14}$$

I will now show that this term is strictly greater than 0; hence, increasing the monitoring frequency relaxes the IC constraint.

$$\pi U^{n} + \left[ (1-\pi) \theta_{h} - \theta_{l} \right] U^{g} + \left[ (1-\pi) (1-\theta_{h}) - (1-\theta_{l}) \right] U^{b} > 0 \iff \\ \left[ (1-\pi) \theta_{h} - \theta_{l} \right] (U^{g} - U^{n}) + \left[ (1-\pi) (1-\theta_{h}) - (1-\theta_{l}) \right] \left( U^{b} - U^{n} \right) > 0 \iff \\ \left[ (1-\pi) \theta_{h} - \theta_{l} \right] (U^{g} - U^{n}) > \left[ (1-\pi) (1-\theta_{h}) - (1-\theta_{l}) \right] \left( U^{b} - U^{n} \right) \iff \\ \frac{U^{g} - U^{n}}{U^{b} - U^{n}} > \frac{(1-\pi) (1-\theta_{h}) - (1-\theta_{l})}{(1-\pi) \theta_{h} - \theta_{l}} \tag{15}$$

According to (9)  $\frac{U^g - U^n}{U^b - U^n} = \frac{1 - \theta_h}{\theta_h}$ . Therefore,

$$\frac{1 - \theta_h}{\theta_h} > \frac{(1 - \pi) (1 - \theta_h) - (1 - \theta_l)}{(1 - \pi) \theta_h - \theta_l}.$$
(16)

This inequality holds  $iff \ \theta_h > \theta_l$ .

### Claim 3 The spread between promised utilities relaxes the IC constraint.

**Proof.** As an example this is shown for increasing the g, b spread. The same exercise can be used for any pair of outcomes. Consider an increase of  $\epsilon$  in the level of  $U^g$  and a decrease of  $\frac{\epsilon \theta_h}{1-\theta_h}$  in  $U^b$ . These two changes improve the IC constraint leaving the promise-keeping (PK) constraint unchanged.

## **B Proofs (the two-period model)**

Recall that the problem of the planner for an unemployed worker (3) is:

$$V(U) = \max_{\substack{c,c^{e},c^{n},c^{g},c^{b},\mu \\ s.t.:}} -c - \pi (c^{e} - w) - (1 - \pi) \{(1 - \mu) c^{n} + \mu \left[\theta_{h}c^{g} + (1 - \theta_{h}) c^{b}\right] + \kappa \mu \}$$
(17)

$$U = u(c) - e + \pi u(c^{e}) + (1 - \pi) \left[ (1 - \mu) u(c^{n}) + \mu \left( \theta_{h} u(c^{g}) + (1 - \theta_{h}) u(c^{b}) \right) \right]$$
(18)

$$U \geq u(c) + \left[ (1-\mu) u(c^{n}) + \mu \left( \theta_{l} u(c^{g}) + (1-\theta_{l}) u(c^{b}) \right) \right].$$
(19)

For the exposition of the proofs, it is useful to set the problem in two steps. In the first step we solve for  $M(U,\mu)$ , which is identical to (3) except that the monitoring frequency is given exogenously. In the second step we solve for  $V(U) = \max_{\mu} M(U,\mu)$ . The Lagrangian of  $M(U,\mu)$ is

$$-c - \pi (c^{e} - w) - (1 - \pi) \left\{ (1 - \mu) c^{n} + \mu \left[ \theta_{h} c^{g} + (1 - \theta_{h}) c^{b} \right] - \kappa \mu \right\} + \lambda_{1} \left\{ U - u (c) + e - \pi u (c^{e}) - (1 - \pi) \left[ (1 - \mu) u (c^{n}) + \mu \left( \theta_{h} u (c^{g}) + (1 - \theta_{h}) u (c^{b}) \right) \right] \right\} + \lambda_{2} \left\{ U - u (c) - (1 - \mu) u (c^{n}) - \mu \left( \theta_{l} u (c^{g}) + (1 - \theta_{l}) u (c^{b}) \right) \right\},$$
(20)

where  $\lambda_1$  and  $\lambda_2$  are the Lagrange multipliers of the promise-keeping and incentive-compatibility constraints, respectively. Note that both  $\lambda_1$  and  $\lambda_2$  are negative.

**Lemma 1** In the optimal solution,  $c^e > c^g > c^n > c^b$ .

**Proof.** This follows directly from the likelihood ratios with  $l^e > l^g > l^n > l^b$ , where:  $l^e = 1$ ,  $l^g = \frac{(1-\pi)\theta_h - \theta_l}{(1-\pi)\theta_h}$ ,  $l^n = -\frac{\pi}{1-\pi}$ ,  $l^b = \frac{(1-\pi)(1-\theta_h) - (1-\theta_l)}{(1-\pi)(1-\theta_h)}$ .

**Claim 1** In the optimal solution current consumption is equal to an average of future consumption levels, weighted by the probabilities given high effort for outcomes  $i \in \{e, n, g, b\}$ .

**Proof.** The first-order conditions of (20) with respect to the choices of future consumption levels  $c^i$ ,  $i \in \{e, n, g, b\}$  are:<sup>29</sup>

$$c^i = -\lambda_1 - \lambda_2 l^i \tag{21}$$

<sup>&</sup>lt;sup>29</sup>For brevity of notation, I omit the state  $\{U, \mu\}$  from  $\{c^i, \lambda_1, \lambda_2\}$  although these are functions of the state.

Averaging (21) over outcomes *i* with the respective high-effort probabilities yields that  $\lambda_1 = -\pi c^e - (1 - \pi) \{(1 - \mu) c^n + \mu \left[\theta_h c^g + (1 - \theta_h) c^b\right]\}$ . Using the first-order condition of (20) with respect to the choice of current consumption yields  $\lambda_1 = -c$ , implying that:

$$\pi c^{e} + (1 - \pi) \left\{ (1 - \mu) c^{n} + \mu \left[ \theta_{h} c^{g} + (1 - \theta_{h}) c^{b} \right] \right\} = c = -\lambda_{1}.$$
(22)

**Lemma 2** In the optimal solution consumption upon the state n is equal to an average of consumption levels upon the signal, weighted by the probability of a good signal given a high effort.

$$c^n = \theta_h c^g + (1 - \theta_h) c^b \tag{23}$$

**Proof.** Immediate from using (21) for  $c^g, c^n$ , and  $c^b$ .

**Proposition 1** *The optimal monitoring frequency increases with U.* 

**Proof.** Assume that  $M(U,\mu)$  is twice differentiable. By theorem 2.2(1) in Athey, Milgrom, and Roberts (1998) M has increasing differences if and only if for all  $(U,\mu)$ ,  $\frac{\partial^2 M(U,\mu)}{\partial U \partial \mu} \ge 0$ .

 $\lambda_1$  is the shadow price of the promised utility constraint  $\Rightarrow \frac{\partial M(U,\mu)}{\partial U} = \lambda_1$ . Then:

$$\frac{\partial^2 M\left(U,\mu\right)}{\partial U \partial \mu} = \frac{\partial \left\{\frac{\partial M\left(U,\mu\right)}{\partial U}\right\}}{\partial \mu} = \frac{\partial \lambda_1}{\partial \mu}$$
(24)

From Lemma 2:

$$\lambda_{1} = \frac{-\pi c^{e} - (1 - \pi) \left\{ (1 - \mu) c^{n} + \mu \left[ \theta_{h} c^{g} + (1 - \theta_{h}) c^{b} \right] \right\} - c}{2}$$

$$\Rightarrow \frac{\partial^{2} M (U, \mu)}{\partial U \partial \mu} = \frac{\partial \left\{ \frac{-\pi c^{e} - (1 - \pi) \left\{ (1 - \mu) c^{n} + \mu \left[ \theta_{h} c^{g} + (1 - \theta_{h}) c^{b} \right] \right\} - c}{2} \right\}}{\partial \mu}.$$
(25)

The numerator's argument,  $\frac{-\pi c^e - (1-\pi)\left\{(1-\mu)c^n + \mu\left[\theta_h c^g + (1-\theta_h)c^b\right]\right\} - c}{2}$ , is the total cost for the planner providing U excluding the monitoring cost. That cost cannot increase when monitoring in-

creases because the additional information can be ignored.<sup>30</sup> That is

$$\frac{\partial \{\frac{-\pi c^e - (1-\pi) \{(1-\mu)c^n + \mu \left[\theta_h c^g + (1-\theta_h)c^b\right]\} - c}{2}\}}{\partial \mu} \ge 0.$$
(26)

 $\Rightarrow M$  has increasing differences.

The proof is completed by Theorem 2.3 in Athey, Milgrom, and Roberts (1998):  $\mu$  is nondecreasing in U if f M has increasing differences.<sup>31</sup>

### **Lemma 3** When either $\mu$ , U, or both, change, all the spreads move in the same direction.

**Proof.** Let  $\{c_i^e, c_i^n, c_i^g, c_i^b\}$ ,  $\{c_j^e, c_j^n, c_j^g, c_j^b\}$  be the optimal consumption levels for  $\{U_i, \mu_i\}$ ,  $\{U_j, \mu_j\}$ , respectively. Suppose that the ratio of consumption levels between two outcomes-for example  $\{e, n\}$ -differs between  $\{i, j\}$ . Assume without loss of generality that  $\frac{c_i^e}{c_i^n} \ge \frac{c_j^e}{c_j^n}$ , i.e., that the  $\{e, n\}$  spread is larger when  $\{U, \mu\} = \{U_i, \mu_i\}$ . By  $(21) \Rightarrow \frac{c^e}{c^n} = \frac{\lambda_1 + \lambda_2 l^e}{\lambda_1 + \lambda_2 l^n}$ . Denote the lagrange multipliers of (20) by  $\{\lambda_1^x, \lambda_2^x\}$  for  $x \in \{i, j\}$ . Then by (21) and the assumption above  $\frac{c_i^e}{c_i^n} \ge \frac{c_j^e}{c_j^n} \Rightarrow \frac{\lambda_1^i + \lambda_2^j l^e}{\lambda_1^i + \lambda_2^j l^n} \ge \frac{\lambda_1^j + \lambda_2^j l^e}{\lambda_1^i + \lambda_2^j l^n}$ . Cross multiply, rearrange and use  $c_i^e > c_i^n, c_j^e > c_j^n$  (Lemma 1) to get that:  $\lambda_2^i \lambda_1^j - \lambda_1^i \lambda_2^j \ge 0$ .

Repeating the same steps in reverse order for any pair  $\{k, l\} \in \{e, n, g, b\}$  gives that:  $\frac{c_i^k}{c_i^l} \ge \frac{c_j^k}{c_j^l}$ , with strict inequality if  $\frac{c_i^e}{c_i^l} > \frac{c_j^e}{c_j^l}$ . For example, choose  $\{g, b\}$ .

$$\lambda_{2}^{i}\lambda_{1}^{j} - \lambda_{1}^{i}\lambda_{2}^{j} \geq 0 \Rightarrow$$

$$\left(l^{g} - l^{b}\right)\left(\lambda_{2}^{i}\lambda_{1}^{j} - \lambda_{1}^{i}\lambda_{2}^{j}\right) \geq 0 \Rightarrow$$

$$l^{g}\left(\lambda_{2}^{i}\lambda_{1}^{j} - \lambda_{1}^{i}\lambda_{2}^{j}\right) + \lambda_{1}^{i}\lambda_{1}^{j} + \lambda_{2}^{i}\lambda_{2}^{j}l^{g}l^{b} \geq l^{b}\left(\lambda_{2}^{i}\lambda_{1}^{j} - \lambda_{1}^{i}\lambda_{2}^{j}\right) + \lambda_{1}^{i}\lambda_{1}^{j} + \lambda_{2}^{i}\lambda_{2}^{j}l^{g}l^{b} \Rightarrow$$

$$\frac{-\lambda_{1}^{i} - \lambda_{2}^{i}l^{g}}{-\lambda_{1}^{i} - \lambda_{2}^{i}l^{b}} \geq \frac{-\lambda_{1}^{j} - \lambda_{2}^{j}l^{g}}{-\lambda_{1}^{j} - \lambda_{2}^{j}l^{b}} \Rightarrow$$

$$\frac{c_{i}^{g}}{c_{i}^{b}} \geq \frac{c_{j}^{g}}{c_{j}^{b}}.$$

$$(27)$$

<sup>&</sup>lt;sup>30</sup>Formally, denote the initial monitoring probability by  $\mu_1$  and the new higher probability by  $\mu_2 > \mu_1$ . The planner can ignore the additional information by using a lottery on the monitoring application with weight  $\delta = \frac{\mu_1}{\mu_2}$ . When monitoring is applied (with probability  $\mu_2$ ) the planner will use the same allocation given  $\mu_1$  with probability  $\delta$ , and the allocation given that monitoring was not applied with probability  $1 - \delta$ .

<sup>&</sup>lt;sup>31</sup>The conditions for Theorem 2.3 are satisfied. In particular  $\mu \in [0, 1] \in \Re$ , and no restrictions are imposed on the Lagrange of M.

Since  $u(c^e) - u(c^n) = \log(\frac{c^e}{c^n})$ , if one spread decreases, then the rest of the spreads must decrease as well.

### **Proposition 2** The spreads decrease with promised utility.

**Proof.** Rewrite the incentive-compatibility constraint as a linear combination of spreads:

$$\pi[u(c^e) - u(c^g)] + \pi (1 - \mu) [u(c^g) - u(c^n)] + \mu(\pi (1 - \theta_h) + \theta_h - \theta_l) [u(c^g) - u(c^b)] = e.$$
(28)

Using (28), the effect of an increase in the monitoring probability on the left-hand side of the constraint can be written as

$$\pi[u(c^{n}) - u(c^{b}) - \theta_{h}\left(u(c^{g}) - u(c^{b})\right)] + (\theta_{h} - \theta_{l})[u(c^{g}) - u(c^{b})]$$
(29)

The second term is strictly positive because  $\theta_h > \theta_l$  and  $c^g > c^b$ . Therefore, the total effect of the increase in the monitoring frequency on the left-hand side is strictly positive if the first term is non-negative. For log utility, this is equivalent to showing that

$$\log(c^n) > \theta_h \log(c^g) + (1 - \theta_h) \log(c^b) \tag{30}$$

Since, at the optimum,  $c^n = \theta_h c^g + (1 - \theta_h) c^b$  (see Lemma 2 above), the inequality above holds as the logarithmic function is a concave transformation of f(c) = c.

Hence, following an increase in  $\mu$ , at least one spread must decrease to keep the constraint tight; by Lemma 3, all the spreads drop together.<sup>32</sup>

Since we know from Proposition 1 that an increase in U leads to an increase in the monitoring frequency, and since an increase in the monitoring frequency leads to a decrease in the spreads, it follows that an increase in U also leads to a decrease in the spreads.<sup>33</sup>

Note that together with Lemma 3, condition (28) implies that if the monitoring frequency is constant, then all spreads are constant as well. This is so because the coefficients of all spreads in (28) are all positive, and they can only move in the same direction.

 $<sup>^{32}</sup>$ Appendix C shows formally why the two constraints are tight for the infinite-horizon model. The line of proof for the two-period model is the same.

<sup>&</sup>lt;sup>33</sup>Equation (28) also shows that U may affect the spreads only when there is a change in  $\mu$ . This rules out combined effects such as a decrease in the spreads due to the increase in the monitoring probability and the independent increase in the spread following a change in U.

## C The cost of spreading out utilities

I demonstrate the significance of  $\sigma = \frac{1}{2}$  in a simple two-period model with no monitoring and no current compensation. The problem thus becomes:

$$V(U) = \max_{c^{e}, c^{n}} \{ \pi (w - c^{e}) - (1 - \pi)c^{n} \}$$
  
s.t. :  
$$U = -e + \pi u (c^{e}) + (1 - \pi) u (c^{n})$$
  
$$U \ge u (c^{n}).$$

The solution to this problem is  $c^n = u^{-1}(U)$ ,  $c^e = u^{-1}(U + \frac{e}{\pi})$ . The first-best solution is constant consumption across both future states, i.e.,  $c^n = c^e = u^{-1}(U + e)$ . The difference between the first-best and the second-best planner's cost of providing consumption is then:

$$\pi u^{-1} \left( U + \frac{e}{\pi} \right) + (1 - \pi) u^{-1} \left( U \right) - u^{-1} \left( U + e \right), \tag{31}$$

which is equal to the cost of spreading out the future values.

Using CRRA utility and differentiating this cost with respect to promised utility gives:

$$\pi \left\{ (1-\sigma) \left( U + \frac{e}{\pi} \right) \right\}^{\frac{\sigma}{1-\sigma}} + (1-\pi) \left\{ (1-\sigma) U \right\}^{\frac{\sigma}{1-\sigma}} - \left\{ (1-\sigma) \left( U + e \right) \right\}^{\frac{\sigma}{1-\sigma}}.$$
 (32)

The cost of spreading out future utilities increases with U if and only if this derivative is positive. This condition holds if and only is  $\sigma > \frac{1}{2}$ . To see this notice that the first two terms in (32) comprise a lottery, whose expected value (U + e) appears in the third term. The difference between the value of the lottery and the value of the certainty equivalent is positive if and only if f(x) = $\{(1 - \sigma) x\}^{\frac{\sigma}{1 - \sigma}}$  is a strictly convex function. This holds if and only if  $\sigma > \frac{1}{2}$ .

## **D** The monitoring probability in the US

The calibration of the actual monthly monitoring probability in the US,  $\mu^{ACT}$ , is based on the frequency of required reports of employment contacts that the unemployed workers fill in and on the probability that these contacts are verified. While the weekly frequency of reports is fairly consistent across states (O'Leary 2004), the probability of verifying these contacts varies vastly across states: some states (e.g. Pennsylvania) do not monitor at all; some states (e.g. Washington) have a target monitoring frequency of 10%; and some states (e.g. South Dakota) consistently review contacts every four to six weeks (DOL 2003).

For the probability of verifying employment contacts in the US, I use a conservative value of 5% (the lower this probability the lower is  $\theta_h$ ). This value determines, together with a weekly frequency of reports, a monthly monitoring probability ( $\mu^{ACT}$ ) of 20%.<sup>34</sup>

 $<sup>^{34}</sup>$ The unemployed worker submits  $52/12 = 4\frac{1}{3}$  reports a year. The probability of being monitored at least once in a month is:  $1 - 0.95^{4.33} = 0.20$ , where 0.95 is the probability of not being monitored in a given week.

## **E** Computational method

This appendix describes the solution method for the problem of a planner who recommends the high job-search effort during unemployment.<sup>35</sup>

Transform the maximization problem with six decision variables and two constraints into a maximization problem with four decision variables and no constraints. Write the incentive-compatibility constraint as follows:<sup>36</sup>

$$u(c) - e + \beta \pi U^{e} + \beta (1 - \pi) \left[ (1 - \mu) U^{n} + \mu \left( \theta_{h} U^{g} + (1 - \theta_{h}) U^{b} \right) \right]$$
(33)  
=  $u(c) + \beta \left[ (1 - \mu) U^{n} + \mu \left( \theta_{l} U^{g} + (1 - \theta_{l}) U^{b} \right) \right]$ 

and express  $U^e$  in terms of  $U^n, U^g, U^b$  and  $\mu$ :

$$U^{e} = \frac{e}{\beta\pi} + (1-\mu)U^{n} - \frac{\mu}{\pi} \left\{ \left[ (1-\pi)\theta_{h} - \theta_{l} \right]U^{g} + \left[ (1-\pi)(1-\theta_{h}) - (1-\theta_{l}) \right]U^{b} \right\}$$
(34)

Use the promise-keeping constraint to express c in terms of  $U^e, U^n, U^g, U^b$  and  $\mu$ :

$$c = u^{-1} \left\{ U + e - \beta \left[ \pi U^e + (1 - \pi) \left( (1 - \mu) U^n + \mu \left( \theta_h U^g + (1 - \theta_h) U^b \right) \right) \right] \right\}$$
(35)

Use (34) in the right-hand side of (35) to express the consumption level (c) in terms of  $U^n, U^g, U^b$ and  $\mu$ . Substitute this value of c and the value for  $U^e$  from (34) into (2) to receive the maximization problem with four decision variables:  $U^n, U^g, U^b$  and  $\mu$ , with no constraints.

Those four remaining decision variables consist of three continuation values  $(U^n, U^g, U^b)$  and the monitoring frequency  $\mu$ . While the support for the continuation values is the real line, the support for the monitoring frequency is [0, 1]. This closed support presents a computational challenge, which I overcome by distcritizing the support of the monitoring frequency into 101 values and then solve the maximization problem for each of those 101 values.<sup>37</sup>

Thus, the maximization problem is reduced to three continuous variables:  $U^n, U^g, U^b$ . The solution to this problem is based on the three first-order conditions with respect to  $U^n, U^g$ , and  $U^b$  respectively:

<sup>&</sup>lt;sup>35</sup>In absence of asymmetric information, the solution to the employment problem consists of constant benefits for the complete duration of employment.

<sup>&</sup>lt;sup>36</sup>In the optimal solution, the incentive compatibility constraint always holds with equality. This is the case because delivering an expected discounted utility that is higher than the required one, costs more.

<sup>&</sup>lt;sup>37</sup>The sensitivity of the solution is, therefore, 0.005 of monitoring frequency.

$$\begin{aligned} (u^{-1})' \left( c_{-} \arg \right) &+ \pi W' \left( U^{e} \right) + (1 - \pi) V' (U^{n}) = 0 \\ (u^{-1})' \left( c_{-} \arg \right) \left( 1 - \theta_{l} \right) - W' \left( U^{e} \right) \left( (1 - \pi) \,\theta_{h} - \theta_{l} \right) + (1 - \pi) \theta_{h} V' (U^{g}) = 0 \\ (u^{-1})' \left( c_{-} \arg \right) \theta_{l} - W' \left( U^{e} \right) \left( (1 - \pi) \left( 1 - \theta_{h} \right) - (1 - \theta_{l}) \right) + (1 - \pi) \left( 1 - \theta_{h} \right) V' (U^{b}) = 0 \\ \end{aligned}$$
where I have defined for brevity of notation:  

$$c_{-} \arg = U + e - \beta \left[ \pi U^{e} + (1 - \pi) \left( (1 - \mu) U^{n} + \mu \left( \theta_{h} U^{g} + (1 - \theta_{h}) U^{b} \right) \right) \right] \end{aligned}$$

For the quantitative analysis these equations are solved over a grid of  $\{U, \mu\}$  with 200 and 101 values respectively. The value for V(U) is then the maximum value of V for a given U over all possible levels of monitoring probability. The figures are produced using a much bigger span of utility. For those I used 600 grid points for U and 25 points for  $\mu$ .

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