Macro Theory B

PS 7: Incomplete Markets Model (part II)

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In this problem set you will solve a simple variation of the general equilibrium model presented in class. We solve this problem in two separate problem sets. In the previous problem set you found the solution to the (stochastic) consumer's problem by imposing the solution to be on a grid. Here you will use this solution in a nested loop that aims at finding the endogenous interest rate that clears the asset market. I include the details from the previous problem set for completeness. Notice that I updated the grid for A in order to concentrate on the levels of assets of interest.

1 The economy

The economy is described as follows.

- A continuum of size 1 of infinitely-lived consumers.
- Consumers maximize the discounted expected utility: $\sum_{t=0}^{\infty} \beta^t \log(c_t)$, with $\beta = 0.95$.

- Consumers efficiency units are denoted by $\varepsilon \in \epsilon = \{\varepsilon_L, \varepsilon_H\}$. Assume $\varepsilon_L = 1, \varepsilon_H = 3$.

- The borrowing limit is 0, i.e. no borrowing is allowed.

- The transition matrix for efficiency units $\pi(\varepsilon'|\varepsilon)$ is:

	ε_L	ε_H
ε_L	0.6	0.4
ε_H	0.4	0.6

- The firm's production function is $F(K, L) = K^{0.34}L^{0.66}$, where K is total assets and L is total efficiency units.

- Capital depreciates at rate $\delta = 0.05$.

2 Solution concept

1. Consider the consumer's problem:

$$V(a, \varepsilon) = \max_{a' \in A} \left\{ u(c) + \beta \sum_{\varepsilon' \in \epsilon} \pi\left(\varepsilon'|\varepsilon\right) V(a', \varepsilon') \right\} \tag{1}$$
$$s.t: \quad c + a' = (1 + r)a + w\varepsilon$$
$$a' \geq 0,$$

where the consumer take prices $\{w, r\}$ as given, and A represents a grid over asset holdings.Assume that $A = \{0, 0.02, 0.04, 0.06, 0.08, 0.1, ...20\}$, i.e., assets take 1001 values.[1]

[1] At $a = 100$ the consumer can receive $ra = 0.02 * 100 = 2$, which is the difference between the high and low earnings $(w\varepsilon_H - w\varepsilon_L)$. Given that this level of assets insures him completely against low income shocks and that $\beta R < 1$, the consumer will never want to go beyond that level of assets.

2. With the specification above, this problem can be written as two Bellman equations:

$$V(a,\varepsilon_L) = \max_{a'\in A}\left\{\log((1+r)a + w\varepsilon_L - a') + \beta\pi\left(\varepsilon_L|\varepsilon_L\right)V(a',\varepsilon_L) + \beta\pi\left(\varepsilon_H|\varepsilon_L\right)V(a',\varepsilon_H)\right\} \quad (2)$$

$$V(a,\varepsilon_H) = \max_{a'\in A}\left\{\log((1+r)a + w\varepsilon_H - a') + \beta\pi\left(\varepsilon_L|\varepsilon_H\right)V(a',\varepsilon_L) + \beta\pi\left(\varepsilon_H|\varepsilon_H\right)V(a',\varepsilon_H)\right\} \quad (3)$$

and note that the budget constraint was incorporated into the problem. Also note that the borrowing constraint was omitted simply because the minimum level of a' on the grid is 0.

3 Solution method

1. Guess an initial value for the value functions (2) and (3). Each of those two is a vector of 1001 values that maps the state of 1001 values of a and a single value of ε to a value V. A reasonable initial guess is:

$$V(a,\varepsilon) = \frac{u\left(ra + \varepsilon w\right)}{1-\beta}.$$

2. Use the current guess in (2) and (3) to get $a'\left(a,\varepsilon\right)$. This means that for each of the 1001 values of a, you should consider all 1001 future values of a' and choose the one that maximizes the value. Note that you will need the guesses of both $V(a,\varepsilon_L)$ and $V(a,\varepsilon_H)$ to calculate the continuation value because of the stochastic nature of ε.

3. The optimal choice of a' for every (a,ε) defines a new guess for $V\left(a,\varepsilon\right)$. This is your new guess for V that will be used in the next iteration.

4. Continue till the maximum distance between the two consecutive guesses of V is

smaller than $\mu = 0.01$.

4 Endogenous prices

So far we assumed that prices $\{w, r\}$ are exogenous. We now close the model's solution by finding the endogenous prices that clear the markets. The concept is to guess r, and compute both the demand and the supply of assets given that r. Then adjust r depending on whether the previous guess generated excess demand or excess supply of assets.

Proceed according to the algorithm provided in the notes.

Please provide the Matlab code, the resulting equilibrium levels of r and w and a plot of the decision vectors of next level assets.

Mandatory Reading

Heathcote, J., Storesletten, K., Violante, G.L., 2009. Quantitative Macroeconomics with Heterogeneous Households. Annual Review of Economics 1, 319-354.