Macro Theory B

PS 6: Incomplete Markets Model

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In this problem set you will solve a simple variation of the income fluctuations problem. In this problem set we focus on the solution to the (stochastic) consumer's problem by imposing the solution to be on a grid.

1 The economy

The economy is described as follows.

- A continuum of size 1 of infinitely-lived consumers.
- Consumers maximize the discounted expected utility: $\sum_{t=0}^{\infty} \beta^t \log(c_t)$, with $\beta = 0.95$.
- Consumers efficiency units (income levels) are denoted by $\varepsilon \in \epsilon = \{\varepsilon_L, \varepsilon_H\}$. Assume $\varepsilon_L = 1, \varepsilon_H = 3$.
- The borrowing limit is 0, i.e. no borrowing is allowed.
- The transition matrix for efficiency units $\pi(\varepsilon'|\varepsilon)$ is:

	ε_L	ε_H
ε_L	0.6	0.4
ε_H	0.4	0.6

2 Solution concept

1. Consider the consumer's problem:

$$V(a,\varepsilon) = \max_{c,a' \in A} \left\{ u(c) + \beta \sum_{\varepsilon' \in \epsilon} \pi\left(\varepsilon' | \varepsilon\right) V(a', \varepsilon') \right\} \tag{1}$$
$$s.t: \quad c + a' = (1+r)a + w\varepsilon$$
$$a' \geq 0,$$

   where the consumer take prices $\{w, r\}$ as given, and A represents a grid over asset holdings. Since we solve this problem in partial equilibrium at this stage, assume that $r = 0.02$ and that $w = 1$. Also assume that $A = \{0, 0.2, 0.4, 0.6, 0.8, 1, 1.2....100\}$, i.e., assets take 501 values.[1]

2. With the specification above, this problem can be written as two Bellman equations:

$$V(a,\varepsilon_L) = \max_{a' \in A} \left\{ \log((1+r)a + w\varepsilon_L - a') + \beta\pi\left(\varepsilon_L|\varepsilon_L\right) V(a', \varepsilon_L) + \beta\pi\left(\varepsilon_H|\varepsilon_L\right) V(a', \varepsilon_H) \right\} \tag{2}$$

$$V(a,\varepsilon_H) = \max_{a' \in A} \left\{ \log((1+r)a + w\varepsilon_H - a') + \beta\pi\left(\varepsilon_L|\varepsilon_H\right) V(a', \varepsilon_L) + \beta\pi\left(\varepsilon_H|\varepsilon_H\right) V(a', \varepsilon_H) \right\} \tag{3}$$

   and note that the budget constraint was incorporated into the problem. Also note that the borrowing constraint was omitted simply because the minimum level of a' on the grid is 0.

[1] At $a = 100$ the consumer can receive $ra = 0.02 * 100 = 2$, which is the difference between the high and low earnings $(w\varepsilon_H - w\varepsilon_L)$. Given that this level of assets insures him completely against low income shocks and that $\beta R < 1$, the consumer will never want to go beyond that level of assets.

3 Solution method

1. Guess an initial value for the value functions (2) and (3). Each of those two is a vector of 501 values that maps the state of 501 values of a and a single value of ε to a value V. A reasonable initial guess is:

$$V(a,\varepsilon) = \frac{u(ra + \varepsilon w)}{1-\beta}.$$

2. Use the current guess in (2) and (3) to get $a'(a,\varepsilon)$. This means that for each of the 501 values of a, you should consider all 501 future values of a' and choose the one that maximizes the value. Note that you will need the guesses of both $V(a,\varepsilon_L)$ and $V(a,\varepsilon_H)$ to calculate the continuation value because of the stochastic nature of ε.

3. The optimal choice of a' for every (a,ε) defines a new guess for $V(a,\varepsilon)$. This is your new guess for V that will be used in the next iteration.

4. Continue till the maximum distance between the two consecutive guesses of V is smaller than $\mu = 0.01$.