Macro Theory B

PS 4: An island model (Lucas Prescott - 1974)

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1 A two-islands economy

Consider the following simple islands economy. There are only two islands and there are only *three* periods. All workers discount the future ate rate β. It takes two periods to travel between islands, so workers leaving an island at the beginning of period 1 will only arrive at their target island at the beginning of period 3.

The initial measure of workers in each island at the beginning of period 1 is $\left\{\frac{1}{2}, \frac{1}{2}\right\}$. Productivity is $\theta \in \{\theta_L, \theta_H\}$, $\theta_L < \theta_H$, and the production function in each island is $\theta f(n)$ with the standard assumptions on the production function. A worker recieves a wage equal to her marginal productivity. The first period's productivity is revealed at the beginning of the period and it will remain for two periods. I.e., productivity in period 2 is the same as the productivity in period 1. The productivity of period 3 is revealed at the beginning of period 2, allowing those who left at the beginning of period 1 to the other island to change course and come back to their original island in time to work at period 3, if they want to. The probability that an island's productivity remains the same between period 2 and period 3 is $\pi > \frac{1}{2}$.

Assume that at the beginning of period 1, the productivity of island 1 is θ_L and the productivity of island 2 is θ_H.

Denote by V_i^j the value of a worker in island i in period j. Also denote by x the number of workers leaving an island for the other island.

1. Write down the value functions for a worker at each island in period 1 and in period 3.

2. Write down the condition required in order to have some workers leave their island at the beginning of period 1 (i.e. $x > 0$).

3. Assuming that the condition in the previous section is met, write down the equilibrium condition from which the value of x can be found.

2 An illustrative example (exercise 26.1 in LS)

Let the island economy in this chapter have a productivity shock that takes on two possible values, θ_L, θ_H with $0 < \theta_L < \theta_H$. An island's productivity remains constant from one period to another with probability $\pi \in (.5, 1)$, and its productivity changes to the other possible value with probability $1 - \pi$. These symmetric transition probabilities imply a stationary distribution where half of the islands experience a given θ at any point in time. Let $\hat{x}$ be the economy's labor supply (as an average per market).

a. If there exists a stationary equilibrium with labor movements, argue that an island's labor force has two possible values, $\{x_1, x_2\}$ with $0 < x_1 < x_2$.

b. In a stationary equilibrium with labor movements, construct a matrix Γ with the transition probabilities between states (θ, x), and explain what the employment level is in different states.

c. In a stationary equilibrium with labor movements, we observe only four values of the value function $v(\theta, x)$ where $\theta \in \{\theta_L, \theta_H\}$ and $x \in \{x_1, x_2\}$. Argue that the value function takes on the same value for two of these four states.

d. Show that the condition for the existence of a stationary equilibrium with labor movements is

$$\beta(2\pi - 1)\theta_H > \theta_L \tag{1}$$

and, if this condition is satisfied, an implicit expression for the equilibrium value of x_2 is

$$[\theta_L + \beta(1-\pi)\theta_H]f'(2\hat{x} - x_2) = \beta\pi\theta_H f'(x_2) \tag{2}$$