Macro Theory B

PS 2: Dynamic optimization - Value Function Iteration

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Consider the following model presented in class.

$$Max_{\{c_t,k_{t+1}\}_{t=0}^{\infty}} = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1,$$

$$\text{subject to} \qquad c_t + k_{t+1} \leq f(k_t) + (1-\delta)k_t,$$

$$k_0 \text{ given},$$

$$\lim_{T\to\infty} \beta^T u_1(c_T)k_T = 0.$$

Assume $f(k) = Ak^{\alpha}$, $u(c) = \log(c)$, $0 < \delta \leq 1, 0 < \alpha < 1, 0 < \beta < 1$.

This problem can be written as:

$$V(k) = \max_{k'} \left\{ u\left(f(k) + (1-\delta)k - k'\right) + \beta V(k') \right\},$$

where the transversality condition is verified ex-post.

1. Write a (matlab or other) code for finding the optimal solution using Value Function Iteration (VFI). You will need to construct a(n equi-distance) size-N grid over the state k, and use an initial guess on $V(k)$ over that grid.

2. Assume that the policy function $k' = g(k)$ takes values only on the grid of k. Here, you will need to calculate for each value of k the maximum value of $V(k)$ given the previous guess of the value function over the N possible values of k'.

3. Now further assume that $A = 1, \alpha = 0.3, \delta = 0.1, \beta = 0.95$ and use the code to find the optimal (numerical) solution to the problem. you will need to assume a convergence threshold ε for the maximum difference of $V(k)$ over two consecutive iterations.

4. Plot the policy functions $k\prime(k), c(k)$ and the value function $V(k)$.

5. Increase A to 2 and repeat steps (4) and (5) above. Plot the policy functions and the value function for the two cases and discuss the results.