Macro Theory B

Self Insurance

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1 Prudence

Here we make an assumption on the properties of $u(c)$: That $u''' > 0$ (u' is convex). This means that given a stochastic process with mean $\bar{y}$, if we increase its variance (i.e increase the measure of uncertainty) but maintain the mean, the individual will still want to increase his savings. In other words, inducing a mean preserving transformation on y_t affects the individual's savings.

Under PIH a mean preserving transformation wouldn't have any effect since we saw that PIH satisfies certainy equivalence: We can see that c_t is only dependent on the expectancy of y_t, which doesn't change under a mean preserving transformation.

Assumptions:

- We assume for simplicity that there are only two periods $T = 2$.
- $u'(c) > 0$
- Parameters: β, r
- c_0, c_1: Consumption in both periods.
- y_0: Exegenous deterministic income at time $t = 0$.
- $\tilde{y}_1$: Exogenous stochastic income at time $t = 1$.
- a_1: Asset bought at time $t = 0$ and realized at time $t = 1$. This is the same type of asset as before, i.e non-contigent on the realized state (yields the same payoff in every possible state).

The HH problem is:

$$\begin{aligned} max_{c_0,c_1,a_1} & \quad [u(c_0) + \beta u(c_1)] \\ s.t: & \\ c_1 &= \tilde{y}_1 + a_1(1+r) \\ c_0 + a_1 &= y_0 \end{aligned}$$

Reformulated (we denote $R = (1+r)$)

$$max_{a_1}[u(y_0 - a_1) + \beta u(\tilde{y}_1 + a_1 R)]$$

FOC w.r.t a_1:

$$u'(y_0 - a_1) = \beta R \cdot E(u'(\tilde{y}_1 + a_1 R)) \tag{8}$$

The solution to the FOC yields the equilibrium a_1^*.

1.1 Analysis

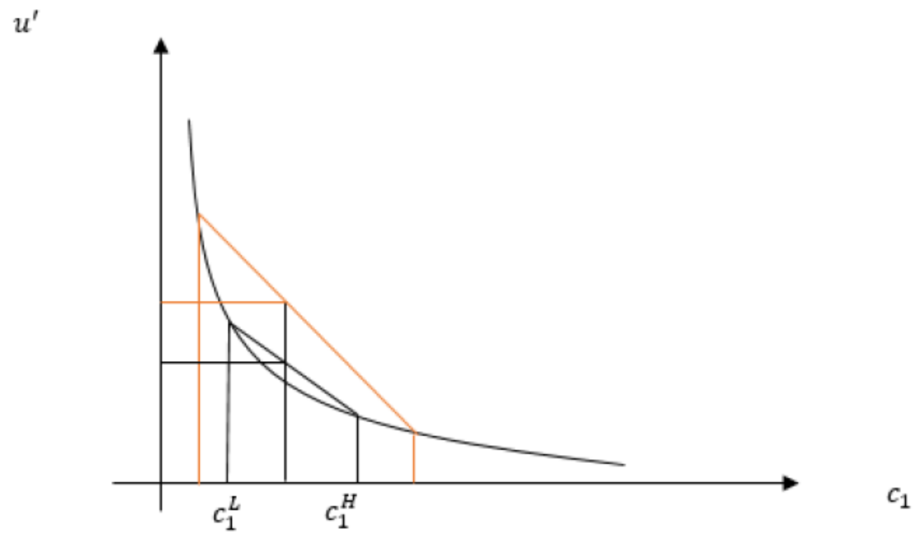
Lets assume that there are two possible values for the second period income

$$\tilde{y}_1 = \begin{cases} y_L, & p \\ y_H, & 1-p \end{cases}$$

And then we get that:

$$E(u'(\tilde{y}_1 + a_1 R)) = E(u'(C_1)) = p \cdot u'(C_1^L) + (1-p)u'(C_1^H)$$

We look at this graphically:



The downard sloping black curve is u', which we know is convex by assumption of Prudence. We can see that the RHS in (8) is equivalent to $\beta R \cdot E(u'(C_1))$. If we increase the uncertainty of $\tilde{y}_1$but preserve the means (i.e increase the distance between C_1^L and C_1^H as shown in the orange lines, but in such a way that yields the same mean), then we get a *higher value* for the RHS of (8). From (8) we then get that $u'(c_0) \uparrow$ which means that $c_0 \downarrow$ which in turn by the RC increases savings $a_1 \uparrow$ (since y_0 is unchanged).

Summarizing:

$$Uncertainty \uparrow \to E(u'(C_1)) \uparrow \to RHS \uparrow \to LHS \uparrow \to u'(c_0) \uparrow \to c_0 \downarrow \to a_1 \uparrow$$

So we see that applying a mean preserving transformation (increasing the risk) changes the level of savings (increases savings). The intuition for this is that a greater level of uncertainty makes the Prudent indvididual more sensitive to the risks, even if the expected payoff is the same.

2 Measures of Risk Aversion

DEFINITION: The Arrow-Pratt measure of absolute risk aversion (ARA) for a given utility function $u(c)$:

$$\alpha := -\frac{u''(c)}{u'(c)}$$

DEFINITION: The Arrow-Pratt measure of realtive risk aversion (RRA) for a given utility function $u(c)$:

$$\alpha_R := c \cdot \alpha = -\frac{c \cdot u''(c)}{u'(c)}$$

Intuitively, this measures the percent of the *subjective income* that is being risked.

We look at different types of ARAs and RRAs and the utility functions that yield them:

- CRRA: Constant relative risk aversion ($\alpha'_R = 0$)

  $$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma}, & \sigma \neq 1 \\ log(c), & \sigma = 1 \end{cases}$$

  Here we get

  $$\alpha_R = \begin{cases} -\frac{-\sigma c^{-\sigma-1}}{c^{-\sigma}} \cdot c = \frac{\sigma}{c} \cdot c = \sigma, & \sigma \neq 1 \\ -\frac{\frac{-1}{c^2}}{\frac{1}{c}} \cdot c = \frac{1 \cdot c}{c} = 1, & \sigma = 1 \end{cases} = \sigma \equiv const \rightarrow \alpha'_R = 0$$

  This means that the level of relative risk aversion is independent of the level of consumption, i.e the level of income. We note that σ is considered to be the coefficient of risk aversion in the utility function, and we can see that when $\sigma \uparrow \rightarrow \alpha_R \uparrow$.

- DARA: Decreasing absolute risk aversion ($\alpha' < 0$)
  This means that the level of absolute risk aversion is dependent on the level of consumption, i.e the level of income. Specifically, greater income implies lower risk aversion.
  We are interested in this case because it reflects the data, so we want to derive a condition for it:

  $$0 > \alpha' = \frac{-u''' \cdot u' - (u'')^2}{(u')^2} \Leftrightarrow u''' > \frac{(u'')^2}{u'} > 0 \Leftrightarrow u''' > 0$$

  This is exactly the condition satisfied by Prudence, i.e Prudence$\rightarrow$DARA. Moreover, DARA=CRRA+$\alpha' < 0$.

- CARA: Constant absolute risk aversion ($a' = 0$)

$$u(c) = e^{-k \cdot c}$$

Here we get:

$$\alpha = -\frac{e^{-k \cdot c} \cdot k^2}{e^{-k \cdot c} \cdot -k} = k \equiv const \to \alpha' = 0$$

DEFINITION: Intertemporal Elasticity of Substution (IES) for any given utility function $u(c)$:

$$IES = -\frac{\partial c}{\partial u'} \cdot \frac{u'}{c}$$

This measures how much the individual cares about changes in the consumption between today an tomorrow (as opposed to ARA and RRA which measure how much he cares about risking a certain amount, given the fact that he is at a certain level of consumption c).

Looking at CRRA, we see that:

$$IES|_{CRRA} = -\frac{1}{\frac{\partial u'}{\partial c}} \cdot \frac{u'}{c} = -\frac{1}{\frac{\partial c^{-\sigma}}{\partial c}} \cdot \frac{c^{-\sigma}}{c} = -\frac{1}{\sigma c^{-\sigma-1}} \cdot \frac{c^{-\sigma}}{c} = \frac{1}{\sigma}$$

This means that $\sigma \uparrow \to IES \downarrow \to -\Delta c \approx more\, \Delta u' \to -\Delta c \approx more\, -\Delta u$, i.e a negative change in c has a *greater* negative effect on utility than before.
So, σ has a double effect, not only does $\sigma \uparrow \to \alpha_R \uparrow$ but also $\sigma \uparrow \to$greater aversion from change in consumption.

In theory, we wan't utility functions in which σ doesn't have this double effect. Specifically, we wish to be able to increase α_R without affecting IES, since this is what the data shows. There are such functions.

Note: The general definition of IES is:

$$IES = \frac{d\, log(\frac{c_{t+1}}{c_t})}{dr}$$

We usually assume 'separability' of the utility function, i.e. $U(\{c_t\}_{t=0}^{\infty}) = \sum \beta^t u(c_t) \to_{FOC} u'(c) = \beta R u'(c')$. Taking the log and a linear expansion around $R = 1$ we get:

$$\begin{aligned}
log(u'(c)) &= log(\beta) + log(R) + log(u'(c')) \approx log(\beta) + R - 1 + log(u'(c')) \\
&= log(\beta) + r + log(u'(c')) \\
\to r &= log(u'(c)) - log(u'(c')) - log(\beta) = log(\frac{u'(c)}{u'(c')}) - log(\beta) \\
&= -log(\frac{u'(c')}{u'(c)}) - log(\beta) \\
\to dr &= -d\, log(\frac{u'(c')}{u'(c)})
\end{aligned}$$

Now we use the general definition of IES to get the definition which we used:

$$
\begin{aligned}
IES &= \frac{d\,log(\frac{c_{t+1}}{c_t})}{dr} = \frac{d\,log(\frac{c'}{c})}{-d\,log(\frac{u'(c')}{u'(c)})} = -\frac{d\,\frac{c'}{c}}{\frac{c'}{c}} \cdot \frac{(\frac{u'(c')}{u'(c)})}{d\,(\frac{u'(c')}{u'(c)})} \\
&=_{continuous\,time} -\frac{dc}{c} \cdot \frac{u'(c)}{d\,u'(c)} = -\frac{\partial c}{\partial u'} \cdot \frac{u'}{c}
\end{aligned}
$$