Macro Theory B

Search and Matching - aka DMP
An Analysis of equilibrium conditions

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*This set of notes was prepared by Ido Shlomo, an MA student in the course in 2014.

1 Graphical solution

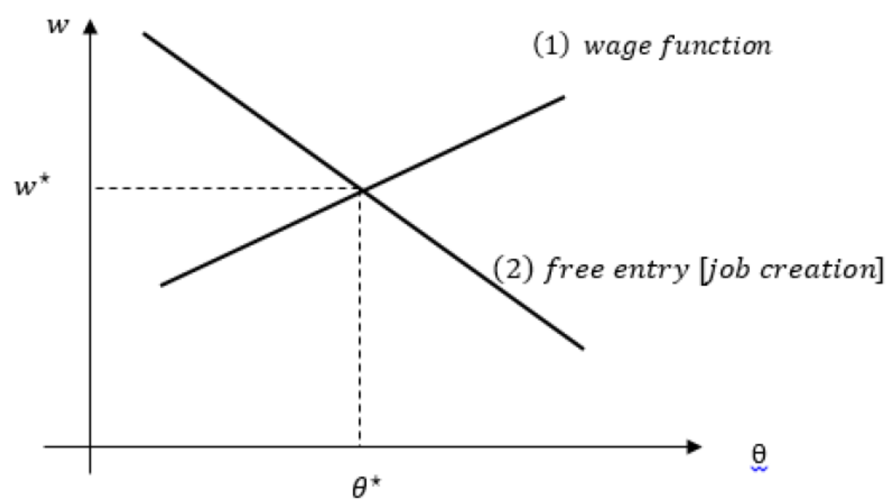
1.1 w and θ:

Recalling steps 11 and 12 (from previous notes):

- $w = z + \phi(y - z + c \cdot \theta)$ (1)
  We denote the equation the *wage function*. In this equation w is increasing in θ. This is because when the ratio between vacancies and job seekers increases the worker's position (not ϕ which is exogenous) is strengthened and so he will demand a higher wage.

- $w = y - \frac{c[r+s]}{q(\theta)}$ (2)
  We denote this equation the *entry function*. In this equation θ figures inside $q(\theta) = q(\theta)$. As θ increases $q(\theta)$ decreases (more vacancies compared to less job seekers means a lower probability for matches) and so the entry function decreases. This is intuitive because in equilibrium $V = 0$ and so when $q(\theta)$ goes down V would become negative unless the firms compensated in some way, i.e lowering the wage.

Graphically:



We can see from the graph that there is a single unique solution θ^*, and therefore also for the wage w^*.

1.2 u and v:

From the decentralized solution we can get the following expression for the next-period unemployment:

$$u' = u(1 - \theta q(\theta)) + (1 - u)s$$

u: The current unemployment.

$-u\theta q(\theta)$: The flow out of unemployment.

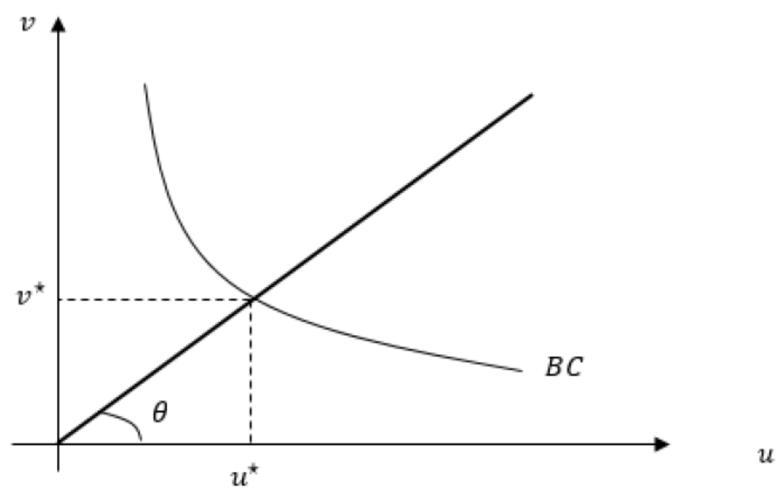
$(1-u)s$: This is the flow into unemployment.

The full equation means that u' is simply the net result of these flows: u-[outflow]+[inflow]. In the steady state equilibrium it must hold that $u' = u$ and so we get

$$u = \frac{s}{s + \theta q(\theta)(\theta)} \tag{3}$$

This gives us a connection between u, v. Specifically, if $v \uparrow$ then $\theta \uparrow$ and then $\theta q(\theta) \uparrow$ since the more vacancies we have per worker then the more matchings the worker can get. This means that $u \downarrow$, and so we get an inverse connection between v, u which is portrayed by the Beveridge Curve.

Graphically:



We place see that BC intersects the θ^* line which we found previously, and then we get u^*, v^*.

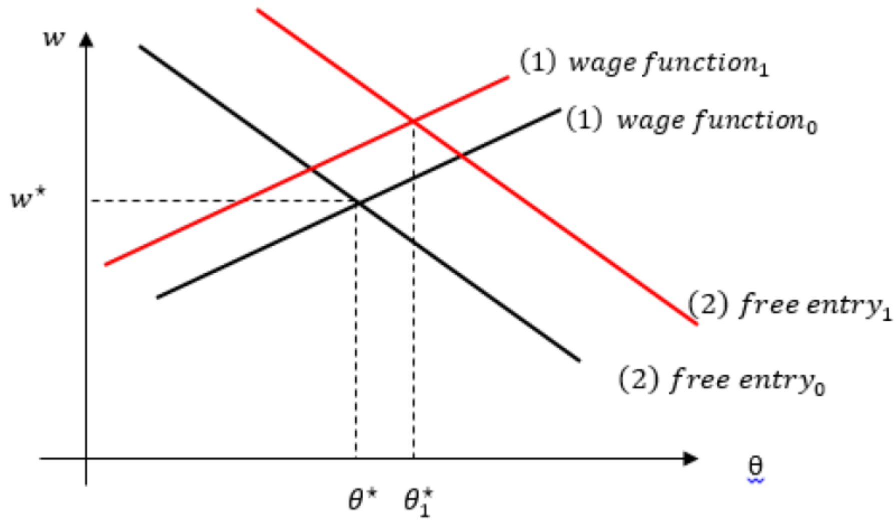
1.3 Comparative statics

- Wage function: $w = z + \phi(y - z + c \cdot \theta)$
- Entry function: $w = y - \frac{c[r+s]}{q(\theta)}$

1.3.1 Rise in y

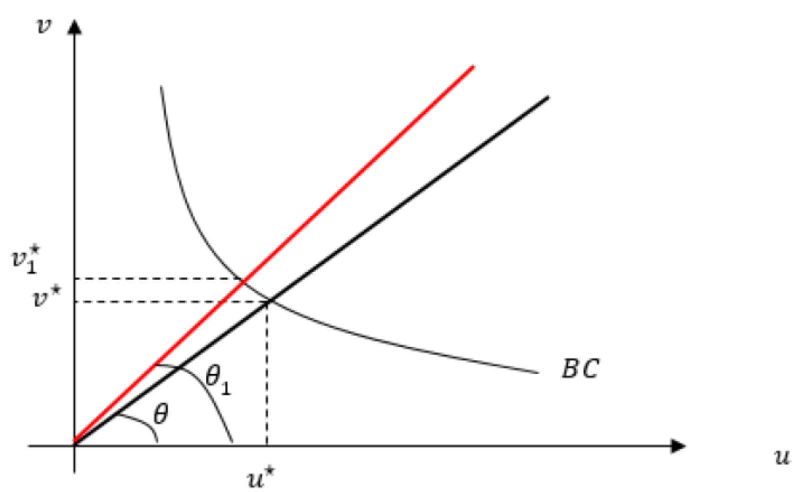
If $y \uparrow$ then $w \uparrow$ according to both (1) and (2) (notice that it rises more shaply in the second one). The result is that both $\theta \uparrow$ and $w \uparrow$.

Graphically:



Now, because $\theta \uparrow$in (3) we get a new intersection with the BC curve, resulting in $v \uparrow$ and $u \downarrow$.

Graphically:



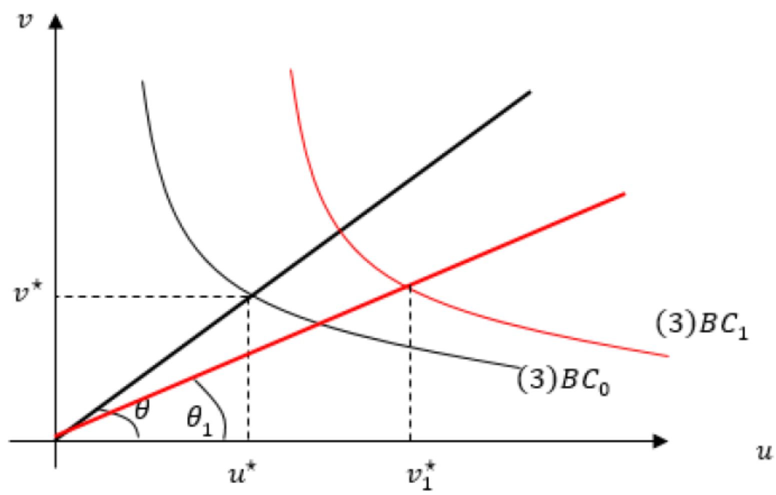
1.3.2 Rise in z:

We get that $w \uparrow$according to (1), and unsure about (2). The result is that $\theta \downarrow$. From (3) we get that $u \uparrow$ and $v \downarrow$.

1.3.3 Rise in s:

A rise in s causes $u\uparrow$ (from 3) and therefore $\theta\downarrow$. The rise in u causes the BC to move up and to the right.

Graphically:



We see that $u\uparrow$obviously, but that the effect on v is undetermined.

1.4 Shimer's puzzle

See slides for discussion.

1.5 Analytic discussion

The equilibrium condition for θ is:

$$y - z = \frac{r + s + \phi\theta q(\theta)}{(1-\phi)q(\theta)} \cdot c$$

In class I derived the elasticity of θ with respect to productivity y.

$$\frac{\partial \log \theta}{\partial \log p} = \frac{\partial \theta}{\partial p}\frac{p}{\theta}$$

The total derivative of the equilibrium condition is:

$$dy * (1-\phi) = -\frac{c(r+s)}{q^2} q' d\theta + \phi c d\theta.$$

Using the following matching function $m(u, v) = u^{\alpha} v^{(1-\alpha)}$ you should verify that the elasticity of the matching function w.r.t. u is α, and that by differentiating q w.r.t. θ that $-\frac{q'}{q} = \frac{\alpha}{\theta}$.

Then

$$dy * (1 - \phi) = c\frac{(r+s)\alpha + \phi q\theta}{q\theta} d\theta$$
$$\frac{d\theta}{dy} = \frac{q\theta(1-\phi)}{c[(r+s)\alpha + \phi q\theta]}$$

From the equilibrium condition:

$$\frac{q(1-\phi)}{c} = \frac{r+s+\phi q\theta}{y-z}$$

Using this in the equation above we get that

$$\frac{\partial \log \theta}{\partial \log y} = \frac{\partial \theta}{\partial y}\frac{y}{\theta} = \frac{y}{y-z}\frac{r+s+\phi\theta q}{(r+s)\alpha + \phi q\theta}$$

1.6 Centralized solution

A planner would choose an allocation that maximizes the discounted value of output and leisure minus the vacancy costs. The central planner can choose the number of workers in the next period n_{t+1} and the number of vacancies for the current period v_t.

The central planner's problem is:

$$max_{v_t, n_{t+1}} \sum_{t=0}^{\infty} \beta^t [n_t \cdot y + (1-n_t)z - c \cdot v_t]$$
$$s.t: \quad n_{t+1} = n_t(1-s) + q(\tfrac{v_t}{1-n_t}) \cdot v_t$$

we note that $u_t = 1 - n_t$, and so $q(\theta_t) = q(\frac{v_t}{1-n_t})$.

The lagrangian is:

$$L = max_{v_t, n_{t+1}} \sum_{t=0}^{\infty} \beta^t \{n_t y + (1-n_t)z - cv_t\} + \mu_t [n_t(1-s) + q(\frac{v_t}{1-n_t})v_t - n_{t+1}]$$

Taking the FOCs:

v_t: $\quad -\beta^t c + \mu_t[q'(\frac{v_t}{1-n_t}) \cdot \frac{1}{1-n_t} v_t + q(\frac{v_t}{1-n_t})] = 0$

$\quad \rightarrow -\beta^t c + \mu_t[q'(\theta_t) \cdot \theta_t + q(\theta_t)] = 0$

n_{t+1}: $\quad -\mu_t + \beta^{t+1}(y-z) + \mu_{t+1}[(1-s) + q'(\frac{v_{t+1}}{1-n_{t+1}}) \cdot \frac{v_{t+1}}{(1-n_{t+1})^2} \cdot v_{t+1}] = 0$

$\quad \rightarrow -\mu_t + \beta^{t+1}(y-z) + \mu_{t+1}[(1-s) + q'(\theta_{t+1}) \cdot \theta_{t+1}^2] = 0$

Solving for μ_t from the first equation and plugging into the second gives:

$$y - z = \frac{r+s+\alpha\theta q(\theta)}{(1-\alpha)q(\theta)} \cdot c$$

We use here the fact that $\beta = \frac{1}{1+r}$ and $q(\theta) = \frac{Au^{\alpha}v^{1-\alpha}}{v} = A\theta^{-\alpha}$, $q'(\theta) = -A\alpha\theta^{-\alpha-1}$.
Comparing this to the decentralized solution (step 13) shows that it is efficient only if $\phi = \alpha$ (this is the Hosius condition). If $\phi > \alpha$ (the worker's bargaining strength is greater than his contribution to the surplus α) then equilibrium job supply is too low (and visa versa).