Macro Theory B

Search, Matching and Unemployment

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1 Math preliminaries

1.1 Expectancy

Assume that p is a random variable with CDF: $F(P) = Pr(p \leq P)$. We assume that $F(0) = 0$ and that there exists a B s.t: $F(B) = 1$.

1. $E(p) = \int_0^B p \cdot dF(p) := \int_0^B p \cdot f(p)dp$, where $f(p) = F'(p)$ is the Pdf of p.

2. $\int_0^B 1 \cdot dF(p) = 1$. This can be seen either from the expectency formula ($p = 1$) or from the fact that $\int_0^B 1 \cdot dF(p) = \int_0^B 1 \cdot f(p)dp = \int_0^B Pr(p = P)dp = 1$ (integral over all probabilities).

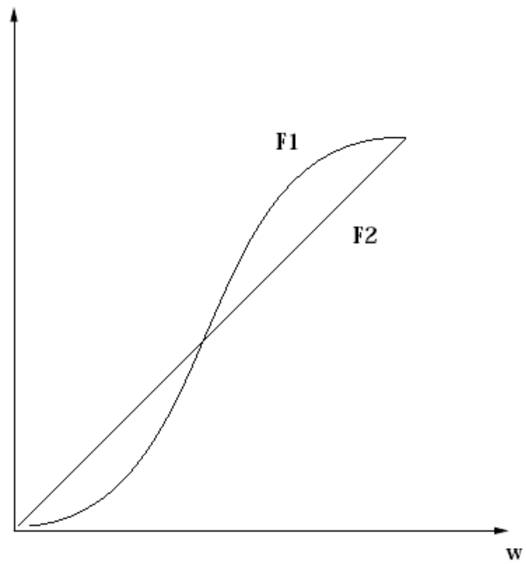
3. $E(p) = B - \int_0^B F(p)dp \rightarrow \int_0^B F(p)dp = B - E(p)$. This is obtained using integration by parts on the expression $\int_0^B [1 - F(p)]dp$.

1.2 Mean preserving spreads

This is a convenient way to characterize the riskiness of two distributions with the same mean (and with the same support, in our case $[0, B]$. A condition from two distrbutions F_1, F_2 to be mean preserving can be gained from 3 in the last section: $\int_0^B F_1(p)dp = \int_0^B F_2(p)dp$.

Assume there are two different distributions with the same means and same support F_1,F_2. We want to look at the case where F_2 is riskier than F_1. Look at this diagram (F_1 here looks like the normal distribution and F_2 is the 45 degree line).

*This set of notes was prepared by Ido Shlomo, an MA student in the course in 2014.



We can see that they satisfy:

$$\int_0^y F_2 dp \geq \int_0^y F_1 dp \, \forall y \in [0, B]$$

This is apparent, for example, by seeing that before the intersection the under F_2 is obviously greater, and after the intersecting the integral of F_2 is always carrying the 'extra' space between the curves from before the intersection. This space is only fully gained by F_1 at the point B.
Looking at $F_1' = f_1, F_2' = f_2$, we can see that at the edges of the box $f_2 > f_1$, and this means that F_2 gives higher probabilities to the edges than does F_1.This is was makes F_2 riskier than F_1. If the agent is risk averse, than F_1 dominates F_2.

2 McCall's model of intertemporal job search

At every period an umemployed worker randomly selects a job offer with wage w iid from a distribution $F(w)$ with support $[0, B]$. The worker's utility function is $U = \sum_{t=0}^{\infty} \beta^t y_t$. This is a *stationary problem*, meaning that at every time the problem facing the worker is the same (until he accepts a job, at which point the model ends).

Assumptions:

- Once the worker is accepts a job he is is employed forever.
- The worker can't quit is job.
- The worker can't go back to a previously offered wage w.

- The value of leisure is c. This represents unemployment wage etc.

Important assumptions:

- The distribution $F(w)$ is known to all. Either way, there is already incentive for the worker to decline an offer - that he may get a better one next period. If $F(w)$ were unknown, there would be another incentive - to wait and learn about $F(w)$.
- w is revealing. This means that the worth of the job is totally encompassed by w (like the type of boss, the atmosphere, etc).

The maximization problem of the worker then is:

$$v(w) = \underset{accept,reject}{max}\{\frac{w}{1-\beta},c + \beta \int_0^B v(w')dF(w')\} \tag{6.3.1}$$

- w is the wage offer given at the current time.
- $\frac{w}{1-\beta}$ is the present discounted value of getting w for infinity (the result from *accept*).
- $\int_0^B v(w')dF(w') = E(v(w'))$. In effect, the *reject* result is simply $c + \beta E(v(w'))$ - meaning recieve c this period and get value the expected value of$v(w')$ next period (since the worker faces the same problem every period).

Remarks:

- $\frac{w}{1-\beta}$is increasing in w.
- $c + \beta \int_0^B v(w')dF(w')$ is not dependent on w and is therefore a constant.

We therefore get the following dynamic:.

We denote by $\bar{w}$ the intersection:

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^B v(w')dF(w')$$

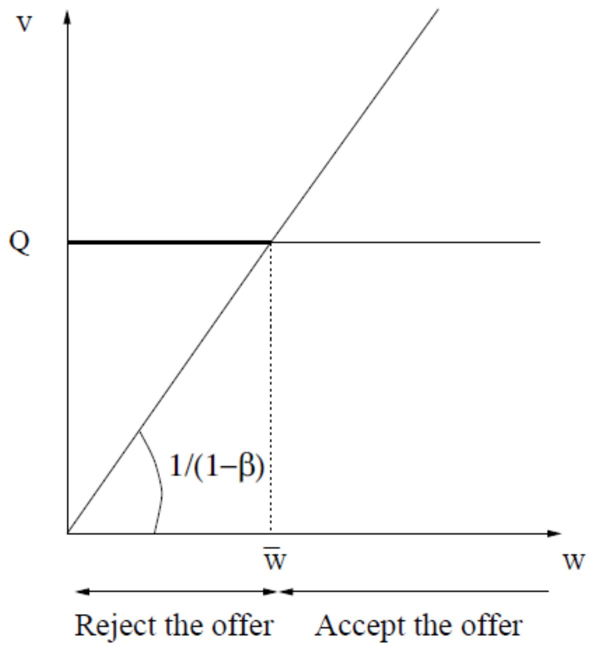


Figure 6.3.1: The function $v(w) = \max\{w/(1-\beta), c + \beta\int_0^B v(w')dF(w')\}$. The reservation wage $\overline{w} = (1-\beta)[c + \beta\int_0^B v(w')dF(w')]$.

From this equation we can derive the *reservation wage*, meaning the wage below which no worker will accept the offer: $\bar{w} = (1-\beta)(c+\beta\int_0^B v(w')dF(w'))$ From this we can figure out what $v(w)$ is:

$$v(w) = \begin{cases} \frac{\bar{w}}{1-\beta} = c + \beta\int_0^B v(w')dF(w') & w \leq \bar{w} \\ \frac{w}{1-\beta} & w \geq \bar{w} \end{cases} \tag{6.3.2}$$

Looking at (6.3.2), we can split the integral into two intervals:
$\frac{\bar{w}}{1-\beta} = c + \beta\int_0^{\bar{w}} v(w')dF(w') + \beta\int_{\bar{w}}^B v(w')dF(w')$
And then using (6.3.1) replace $v(w')$ with the corresponding value:
$\frac{\bar{w}}{1-\beta} = c + \beta\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta}dF(w') + \beta\int_{\bar{w}}^B \frac{w'}{1-\beta}dF(w')$
We multiply the LHS by $1 = \int_0^B dF(w')$ and get:
$\frac{\bar{w}}{1-\beta}\int_0^{\bar{w}} dF(w') + \frac{\bar{w}}{1-\beta}\int_{\bar{w}}^B dF(w') = c + \beta\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta}dF(w') + \beta\int_{\bar{w}}^B \frac{w'}{1-\beta}dF(w')$
We then conbine integrals with matching intervals and get:
$\bar{w}\int_0^{\bar{w}} dF(w') - c = \frac{1}{1-\beta}\int_{\bar{w}}^B (\beta w' - \bar{w})dF(w')$
Adding $\bar{w}\int_{\bar{w}}^B dF(w')$ to both sides gives:

$$(\bar{w} - c) = \frac{\beta}{1-\beta}\int_{\bar{w}}^B (w' - \bar{w})dF(w') \tag{6.3.3}$$

Remarks:

- $\bar{w} - c$ is the cost of searching again given the fact that I drew $\bar{w}$ (i.e what I have to give up if I reject a job offer of value $\bar{w}$).
- RHS=$\frac{\beta}{1-\beta}\int_{\bar{w}}^{B}(w' - \bar{w})dF(w') = \frac{\beta}{1-\beta}E(w' - \bar{w})$: means how much I gain if I search for a new job gien the fact that I drew $\bar{w}$. Notice the interval begins only at $\bar{w}$ since that for any $w < \bar{w}$ I can choose to take c instead. If I choose some $w' \geq \bar{w}$ in the next period (hence the β) I get $w'$for infinity (hence the $\frac{1}{1-\beta}$).

We can choose to look at both sides as function of w instead of $\bar{w}$. We then get:

$$RHS = h(w) = \frac{\beta}{1-\beta}\int_{w}^{B}(w' - w)dF(w') \qquad (6.3.4)$$

Properties of $h(w)$:

- $h(0) = \frac{\beta}{1-\beta}\int_{0}^{B}(w' - 0)dF(w') = \frac{\beta}{1-\beta}E(w)$
- $h(B) = 0$
- $h'(w) = -\frac{\beta}{1-\beta}[1 - F(w)] < 0$
- $h''(w) = \frac{\beta}{1-\beta}F'(w) > 0$

For computing $h'(w)$ we use Leibniz's rule on equation (6.3.4). The rule is:
Let $\phi(t) = \int_{\alpha(t)}^{\beta(t)} f(x,t)dx$ for $t \in [c,d]$. Assume that f and f_t are continuous and that α, β are differentiable on $[c,d]$. Then the rule asserts that $\phi(t)$ is differentiable on $[c,d]$ and

$$\phi'(t) = f[\beta(t),t]\beta'(t) - f[\alpha(t),t]\alpha'(t) + \int_{\alpha(t)}^{\beta(t)} f_t(x,t)dx$$

To apply this formula to $h(w)$ we make the following replacments:

ϕ	t	f	x	$\alpha(t)$	$\alpha'(t)$	$\beta(t)$	$\beta'(t)$	f_t
h	w	$w' - w$	w'	w	1	B	0	-1

So we get:
$h'(w) = (B - w)\cdot 0 - (w - w')\cdot 1 + \frac{\beta}{1-\beta}\int_{w}^{B}(-1)dF(w') = -\frac{\beta}{1-\beta}\int_{w}^{B} dF(w')$
$= -\frac{\beta}{1-\beta}\int_{w}^{B} f(w')dw' = -\frac{\beta}{1-\beta}[F(B) - F(w)] = -\frac{\beta}{1-\beta}[1 - F(w)] < 0$

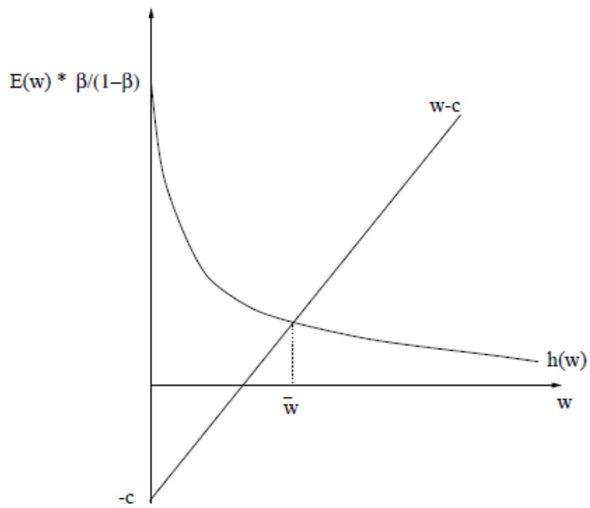


Figure 6.3.2: The reservation wage, $\overline{w}$, that satisfies $\overline{w} - c = [\beta/(1-\beta)] \int_{\overline{w}}^{B} (w' - \overline{w}) dF(w') \equiv h(\overline{w})$.

2.1 Comparitive statics

2.1.0.1 Moving c

Assume that $c \uparrow$. We get that $\bar{w} \uparrow$.

2.1.0.2 Moving E(w)

Assume that $E(w) \uparrow$ .We get that $\bar{w} \uparrow$.

2.1.0.3 Moving β

Assume that $\beta \uparrow$. We get that $\bar{w} \uparrow$.

2.1.0.4 Mean preserving spreads

We want to know how the movement from a distribution F_1 to a more risky distribution F_2 affects $\bar{w}$. We work on equation (6.3.3) and complete the intergral expression on the whole interval (add and remove it):

$$\begin{aligned} \bar{w} - c = \quad & \tfrac{\beta}{1-\beta} \textstyle\int_{\bar{w}}^{B} (w' - \bar{w}) dF(w') + \tfrac{\beta}{1-\beta} \int_{0}^{\bar{w}} (w' - \bar{w}) dF(w') - \tfrac{\beta}{1-\beta} \int_{0}^{\bar{w}} (w' - \bar{w}) dF(w') = \\ & = \tfrac{\beta}{1-\beta} E(w) - \tfrac{\beta}{1-\beta} \bar{w} - \tfrac{\beta}{1-\beta} \textstyle\int_{0}^{\bar{w}} (w' - \bar{w}) dF(w') \end{aligned}$$

eventually we get:

$$\bar{w} - (1-\beta)c = \beta E(w) - \beta \int_{0}^{\bar{w}} (w' - \bar{w}) dF(w')$$

After integration by parts we get:

$$\bar{w} - c = \beta(E(w) - c) + \beta \int_0^{\bar{w}} F(w')dw' \tag{6.3.5}$$

Now we define

$$g(s) = \int_0^s F(p)dp \tag{6.3.6}$$

Note that $g(0) = 0$, $g'(s) = \beta F(s) > 0$.

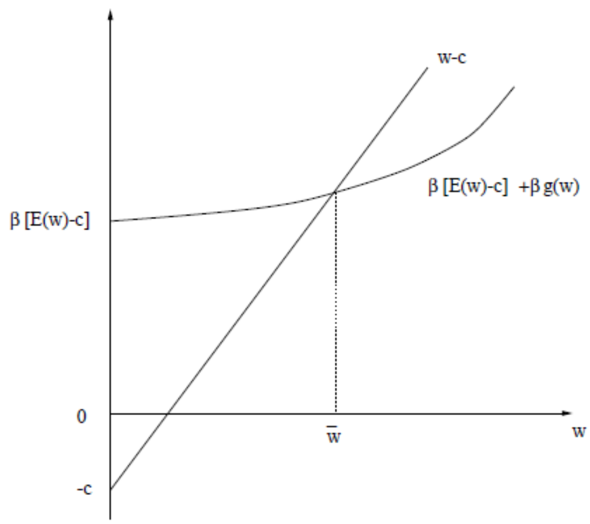


Figure 6.3.3: The reservation wage, $\overline{w}$, that satisfies $\overline{w} - c = \beta(Ew - c) + \beta \int_0^{\overline{w}} F(w')dw' \equiv \beta(Ew - c) + \beta g(\overline{w})$.

We look at: $g_1(s)$, $g_2(s)$. Since we assume that $F_2(s)$ is riskier than $F_1(s)$ then result is that, we can see (via the math preliminaries) that $g_2(w) > g_1(w)$.

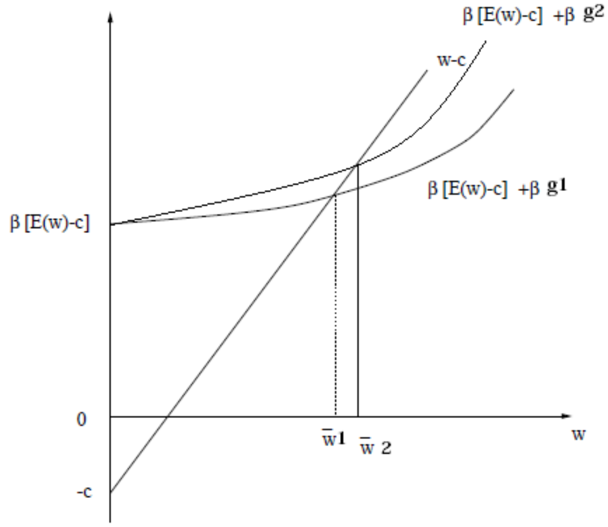


Figure 6.3.3: The reservation wage, $\overline{w}$, that satisfies $\overline{w}-c = \beta(Ew-c)+\beta\int_0^{\overline{w}} F(w')dw' \equiv \beta(Ew-c)+\beta g(\overline{w})$.

We can see that the result is $\bar{w}_2 > \bar{w}_1$, which means that when the distribution is riskier the player actually raises his reservation wage. This is counter intuitive, since we usually assume that a riskier distribution leads to worse results, but here this happens because: Under F_1 the agent ignored any $w < \bar{w}_1$. Under F_2, there is a higher probability in the ends ("tails") of $[0, B]$. So now the agent has more chance of being better off, $w > \bar{w}_1$, and being worse off, $w < \bar{w}_1$. But, since he ignores the latter part anyway, he has only gained, since he can be in the first part with greater pobability. And so he raises his reservation wage to $\bar{w}_2$.