Macro Theory B

Optimal taxation with commitment - LS 3.16

Ofer Setty

The Eitan Berglas School of Economics - Tel Aviv University

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1 Introduction

The problem is deterministic. The government's goal is to maximize households' welfare subject to finance exogenous expenditures through distortional taxation.

The production factors are time-streams of raw labor (l_t) and physical capital (k_t), and the government levies distorting flat-rate taxes. The problem is to determine the optimal sequences for the two tax rates.

2 The model

2.1 The HH's problem

The representative HH's problem is to maximize:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),\ \beta \in (0,1) \tag{15.2.1}$$

We assume that u is increasing, strictly concave, and three times continuously differentiable in c and l. The HH has an initial endowment, each period, which it can split between labor and leisure: $l_t + n_t = 1$.

2.2 The production constraint

The single good is produced with labor n_t and capital k_t. The output $F(k_t, n_t)$ can be consumed by HH (c_t), the government (g_t), or used to augment the capital stock (k_{t+1}). Thus the technology (the method of converting the production into these three parts) is:

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1 - \delta)k_t \tag{15.2.3}$$

where $\delta \in (0, 1)$ is the rate of capital depreciation and $\{g_t\}_{t=0}^{\infty}$ is an *exogenous* sequence of government purchases. F is a standard concave production function with CRTS. By Eueler's theorem and homogeneity of order 1 we get:

$$F(k, n) = F_k k + F_n n \tag{15.2.4}$$

2.3 The government's constraint

The government finances its stream of purchases $\{g_t\}_{t=0}^{\infty}$ by levying a flat-rate, time varying tax on earnings from capital (k_t) at a rate τ_t^k and from labor (n_t) at a rate τ_t^n. The government can also trade one-period bonds (b_t) which can be equated with government indebtedness to the private sector (the HHs). We get that the government's budget constraint is:

$$g_t = \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{b_{t+1}}{R_t} - b_t \tag{15.2.5}$$

- r_t is the market-determined (endogenous) rental rate of capital.
- w_t is the market-determined (endogenous) wage rate.
- R_t is the gross rate of return on one-period bonds held from t to $t+1$. Interest earnings on bonds are assumed to be tax exempt.

The expression $r_t k_t$ can be thought of as the value of the capital used by the firms (and rented from the HHs) at time t, and $w_t n_t$ as the total amount of earnings on labor at time t. The government has to pay at time $t+1$ a sum of b_{t+1} (decided by the HH at time t when they solved their problem), and so at time t sells bonds equal to the sum of $\frac{b_{t+1}}{R_t}$.

3 Decentralized Solution

3.1 HH

$$max_{c_t,k_t,,n_t,b_{t+1}}\{\textstyle\sum_{t=0}^{\infty}\beta^t u(c_t,l_t)\}$$
$$s.t \quad c_t + k_{t+1} + \tfrac{b_{t+1}}{R_t} = (1-\tau_t^n)w_t n_t + (1-\tau_t^k)r_t k_t + (1-\delta)k_t + b_t$$
$$n_t + l_t = 1$$

(equation 15.2.6)

To get the expression for the RC here, we plugged equations (15.2.4) and (15.2.5) into equation (15.2.3):

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

$$c_t + \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{b_{t+1}}{R_t} - b_t + k_{t+1} = r_t k_t + w_t n_t + (1-\delta)k_t$$

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} - b_t = r_t k_t + w_t n_t - \tau_t^k r_t k_t - \tau_t^n w_t n_t + (1-\delta)k_t + b_t$$

$$c_{t+1} + k_{t+1} + \frac{b_{t+1}}{R_t} = (1-\tau_t^n)w_t n_t + (1-\tau_t^k)r_t k_t + (1-\delta)k_t + b_t$$

After building the lagrangian with multiplier $\beta^t\lambda^t$, we get the FOCs:

$$c_t : \ u_c(t) = \lambda_t \tag{15.2.7}$$

$$n_t : \ u_l(t) = \lambda_t(1-\tau_t^n)w_t \tag{15.2.8}$$

$$k_{t+1} : \ \lambda_t = \beta\lambda_{t+1}[(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta] \tag{15.2.9}$$

$$b_{t+1} : \ \lambda_t \cdot \frac{1}{R_t} = \beta\lambda_{t+1} \tag{15.2.10}$$

Inserting (15.2.7) into (15.2.8) and (15.2.9) we get:

$$u_l(t) = u_c(t)(1-\tau_t^n)w_t \tag{15.2.11a}$$

$$u_c(t) = \beta u_c(t+1)[(1-\tau^k_{t+1})r_{t+1} + 1 - \delta] \tag{15.2.11b}$$

And from (15.2.9) and (15.2.10) we get:

$$R_t = (1-\tau^k_{t+1})r_{t+1} + 1 - \delta \tag{15.2.12}$$

Notice that the HH can't adjust any of the values here for R_t.
This constraint is also enough to gaurantee that the is no arbitrage taking place, i.e that the rent costs of both capital and bonds are equal, and so there is no incentive to trade one and back to the other between periods.

3.2 Firms

In each period, the representative firms take r_t, w_t as given and also rents the capital k_t and labor n_t from the HH, and maximizes profits.

$$max_{k_t,n_t}\{\pi = F(k_t,n_t) - r_t k_t - w_t n_t\} \tag{15.2.7}$$

After building the lagrangian and taking the FOCs we get:

$$r_t = F_k(t) \tag{15.2.18a}$$

$$w_t = F_n(t) \tag{15.2.18b}$$

from CRTS we get that $\pi = 0$ in equilibrium.

4 Centralized Solution

4.1 Definitions

We use symbols without subscripts to denote the infinite sequences: $c \equiv \{c_t\}_{t=0}^{\infty}$, $k \equiv \{k_t\}_{t=0}^{\infty}$, $l \equiv \{n_t\}_{t=0}^{\infty}$, $g \equiv \{g_t\}_{t=0}^{\infty}$, $r \equiv \{r_t\}_{t=0}^{\infty}$, $w \equiv \{w_t\}_{t=0}^{\infty}$, $R \equiv \{R_t\}_{t=0}^{\infty}$, $b_t \equiv \{b_t\}_{t=0}^{\infty}$,

$\tau^n \equiv \{\tau_t^n\}_{t=0}^{\infty}$, $\tau^k \equiv \{\tau_t^n\}_{t=0}^{\infty}$.

DEFINITION 1: A *feasible allocation* is a sequence (k, c, l, g) that satisfies equation (15.2.3) - the production constraint.
DEFINITION 2: A *price system* is a 3-tuple of nonnegative bounded sequences (w, r, R).
DEFINITION 3: A *government policy* is a 4-tuple of sequences (g, τ^k, τ^n, b).
DEFINITION 4: A *competitive equilibrium* is a feasible allocation, a price system, and a government policy such that

- Given the price system and the government policy, the allocation solves both the firms' problem and the HH's problem.
- Given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (15.2.5).

There are many competitive equilibria, indexed by different government policies. This multiplicity motivates the Ramsey Problem.

DEFINITION 5: Given k_0 and b_0, the *Ramsey Problem* is to choose a competitive equilibrium that maximizes expression (15.2.1) - the HH's problem.

Remark: To make the Ramsey Problem interesting we usually impose a restriction on τ_0^k, for example:

- $\tau_0^k = \mathit{small\,const}$, like $\tau_0^k = 0$. This approach rules out taxing k_0 by capital levy that would constitute a lump-sum tax, since $k_0$is fixed (and any positive τ_0^k would also be fixed). We aren't interested in lump-sum taxes since by definition they are *not distortive*, and the whole point of the Ramsey Problem is to find the optimal distortive tax.
- $\{\tau_t^k\}_{t=1}^{\infty}$is bounded from above by some arbitrarily given numbers.

4.2 The Solution

We formulate the problem as if the government chooses the after-tax rental rate of capital $\tilde{r}_t$ and the after-tax wage rate $\tilde{w}_t$.

$$\tilde{r}_t \equiv (1 - \tau_t^k) r_t$$

$$\tilde{w}_t \equiv (1-\tau_t^n)w_t$$

Using (15.2.18) and (15.2.4) we get an expression for the government's tax revenue:

$$\tau_t^k r_t k_t + \tau_t^n w_t n_t = (r_t - \tilde{r}_t)k_t + (w_t - \tilde{w}_t)n_t = F_k(t)k_t + F_n(t)n_t - \tilde{r}_t k_t - \tilde{w}_t n_t = F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t$$

We plug this into (15.2.5) and something which combines the firm's FOC's with the government budget constraint:

$$g_t = F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t$$

We now formulate the central planners problem, and he has to maximize the HH's utility and take into account:

- The reformulated government budget constraint:

$$g_t = F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t$$

- The aggregate resource constraint (15.2.3):

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

- The HH's FOCs (system 15.11):

$$u_l(t) = u_c(t)\tilde{w}_t$$

$$u_c(t) = \beta u_c(t+1)[(\tilde{r}_{t+1} + 1 - \delta]$$

- The no arbitrage constraint onR_t from equation (15.2.12):

$$R_t = \tilde{r}_{t+1} + 1 - \delta$$

Remark: The HH's budget constraint doesn't need to be satisfied explicitly since it is redundant when the government satisfies its budget constraint and the production constraint holds.

Building the lagrangian we get:

$$
\begin{aligned}
L \quad = \quad & \sum_{t=0}^{\infty} \beta^t \{ u(c_t, 1-n_t) \\
& + \Psi_t [F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t - g_t] \\
& + \theta_t [F(k_t, n_t) + (1-\delta) k_t - c_t - g_t - k_{t+1}] \\
& + \mu_{1t} [u_l(t) - u_c(t) \tilde{w}_t] \\
& + \mu_{2t} [u_c(t) - \beta u_c(t+1)(\tilde{r}_{t+1} + 1 - \delta)] \}
\end{aligned}
$$

$$
s.t: \quad R_t = \tilde{r}_t + 1 - \delta
$$

(equation 15.4.1)

We take an FOC with respect to k_{t+1} (since we are interested in the optimal capital tax τ_t^k):

$$
\theta_t = \beta \{ \Psi_{t+1} [F_k(t+1) - \tilde{r}_{t+1}] + \theta_{t+1} [F_k(t+1) + 1 - \delta] \} \qquad (15.4.2)
$$

We are looking for a long-term steady state, i.e we assume that after some time $t > T$ we get constant values for all t-subscript values. Using also $F_k = r_t$ (in equilibrium) we get:

$$
\theta = \beta [\Psi(r - \tilde{r}) + \theta(r + 1 - \delta)] \qquad (15.4.3)
$$

From (15.2.11b) we get:

$$
1 = \beta(\tilde{r} + 1 - \delta) \qquad (15.4.4)
$$

Inserting this into (15.4.3) we get:

$$
(\theta + \Psi)(r - \tilde{r}) = 0 \qquad (15.4.5)
$$

Since $\theta > 0$ (this is the coefficient in the lagrangian for the production constraint, so we consider this to be the 'marginal social value of goods') and $\Psi \geq 0$ (this is the coefficient in the lagrangian for the resource constraint, so we consider this to be the 'marginal social value of reducing government debt or taxes') we get that $\tilde{r} = r$ and therefore $\tau^k = 0$.