Macro Theory B

A question on the islands model

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June 9, 2015

Consider the following simple islands economy. There are only two islands and there are only two periods. Moreover, the productivity is deterministic and permanent, i.e., it is known at time 0 and does not change over the two periods. Workers are risk-neutral and discount the future at rate β. Denote by θ_1, θ_2 the (deterministic) productivity in islands 1 and 2 respectively. There is a continuum of measure 1 workers that is distributed equally across the two islands, i.e., the fraction of workers in each island, denoted by x_0^1, x_0^2, where the superscript denotes the island's identity and the subscript denotes the time, is 0.5. Production in each island follows x^α where x is the number of workers and $\alpha = 0.5$ is a parameter. For the following sections assume that $\beta = 1$ and that $\theta_1 = 1$ and $\theta_2 = 2$.

1 An economy without frictions

Assume that workers can move instantaneously across islands, i.e., they can move and start producing already in period 1. Find the distribution of agents across the two islands in each period.

2 An economy with frictions

Now assume instead that it takes one period to move across the islands, i.e., workers who move become productive only in period 2.

a. Write down the value functions for agents in each possible state.

b. Argue that if there are labor movements then they will happen at time 0.

c. Find the distribution of agents across the two islands in each period.

d. Find the lowest discount rate for which there would be movements across the islands.

3 An economy with frictions and utility from moving

Continue to assume that it takes one period to move across the islands. Also assume that now a worker receives a utility of $b = \frac{5}{12}$ from crossing to another island.

Repeat the steps of the last section (An economy with frictions). you will need to solev a non-linear equation whose solution is $x_1 = 0.36, x_2 = 0.64$.

4 Efficiency versus utility

a. For each economy calculate the total productivity of the economy.

b. Discuss the differences.

c. Discuss (no need to solve) whether workers in the economy with $b > 0$ are worse off or better off that those with $b = 0$.

5 An economy with a lower discount rate

Solve under the assumptions of section 2 again but now assuming that $\beta = 0.2$, i.e., that workers heavily discount the future.