Macro Theory B

Final exam (spring 2014) - Solution

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1 Incomplete markets

1. The state variables for the household are the levels of assets saved from the previous period and the discount factor.

The household problem is:

$$\begin{aligned} V(a_{t-1}, \beta_t) &= \max_{c_t, h_t, a_t} \left\{ u(c_t) + v(1-h_t) + \beta_t \sum_{\beta'} \pi(\beta, \beta') V(a_t, \beta') \right\} \\ & s.t. \\ a' &\geq 0 \\ c_t + a_t &\leq w_t h_t + a_{t-1} + r_{t-1} a_{t-1} + b_t \end{aligned}$$

And the only aggregate state variable is K_t.

2. The FOC are:

$$\begin{aligned} \text{w.r.t } c_t &: \quad u'(c_t) = \lambda_t \\ \text{w.r.t } h_t &: \quad v'(1-h_t) = \lambda_t w_t \\ \text{w.r.t } a_t &: \quad \beta_t \sum_{\beta'} \pi(\beta, \beta') V(a_t, \beta') - \lambda_t + \beta_t \lambda_{t+1} [1 + (1-\tau) r_t] + \mu_t = 0 \end{aligned}$$

The Euler conditions:

$$\begin{aligned} u'(c_t) &= \beta_t u'(c_{t+1}) [1 + r_t] + \mu_t \\ v'(1-h_t) &= u'(c_t) w_t \end{aligned}$$

Note that μ_t is zero when the borrowing constraint is not binding.

λ_t can not be zero. When a lagrange multiplier is zero, this indicates that the constraint is not binding. The resource constraint must bind, as otherwise that household can improve its condition by consuming a bit more, without violating any other constraint.

3. The government problem:

assume an invariant distribution, with B_i households in each state. Also assume that there is an asset distribution with a finite number k of possible states (the finite number is just for simplicity). so that there are $A_{i,j}$ households with discount factor β_i and asset level a_j

for capital taxes, the household problem would be:

$$
\begin{aligned}
V\left(a_{t-1},\beta_t\right) &= \max_{c_t,h_t,a_t}\left\{u\left(c_t\right)+v\left(1-h_t\right)+\beta_t\sum_{\beta'}\pi\left(\beta,\beta'\right)V\left(a_t,\beta'\right)\right\}\\
& s.t.\\
a' &\geq 0\\
c_t+a_t &\leq w_t h_t + a_{t-1} + \left(1-\tau^c\right) r a_{t-1} + b_t
\end{aligned}
$$

- Define:

$$
\begin{aligned}
W_{\tau^c} &= \max_{\tau^c}\left\{\sum_i \sum_{j=1,..,k}\left[A_{i,j}U(a_j,\beta_i)+\beta_i\sum_{\beta'}\pi\left(\beta,\beta'\right)V\left(a',\beta'\right)\right]\right\}\\
& s.t.\\
& a',h,w \text{ behave optimally according to household, firm FOC}\\
\sum_i\sum_{j=1,..,k}\tau^c A_{i,j} r_{t-1} a_j &> =G
\end{aligned}
$$

for labor taxes, the household problem would be:

$$
\begin{aligned}
V\left(a_{t-1},\beta_t\right) &= \max_{c_t,h_t,a_t}\left\{u\left(c_t\right)+v\left(1-h_t\right)+\beta_t\sum_{\beta'}\pi\left(\beta,\beta'\right)V\left(a_t,\beta'\right)\right\}\\
& s.t.\\
a' &\geq 0\\
c_t+a_t &\leq \left(1-\tau^l\right) w_t h_t + a_{t-1} + r a_{t-1} + b_t
\end{aligned}
$$

Define:

$$
\begin{aligned}
W_{\tau^l} &= \max_{\tau^c}\left\{\sum_i \sum_{j=1,..,k}\left[A_{i,j}U(a_j,\beta_i)+\beta_i\sum_{\beta'}\pi\left(\beta,\beta'\right)V\left(a',\beta'\right)\right]\right\}\\
& s.t.\\
& a',h,w \text{ behave optimally according to household FOC}\\
\sum_i\sum_{j=1,..,k}\tau^l A_{i,j} w h &> =G
\end{aligned}
$$

and the government problem is:

$$
W = \max_{c_t,h_t,a_t}\left\{W_{\tau^c},W_{\tau^l}\right\}
$$

2 Shimer puzzle

1. Steady state equations:

   Define market tightness

$$\theta = \frac{v}{u}$$

   The probability that a firm finds a worker:

$$\frac{m}{v} = m\left(\frac{u}{v}, 1\right) = \overline{m}\left(\frac{u}{v}\right)^{\delta} = \overline{m}\theta^{-\delta} = q\left(\theta\right)$$

   The probability that an unmatched worker finds a job:

$$\frac{m}{u} = \frac{v}{u} m\left(\frac{u}{v}, 1\right) = \theta q\left(\theta\right)$$

   The value of a matched worker:

$$W = w + \beta\left[\lambda U'_{s'} + (1-\lambda) W'_{s'}\right]$$

   Where $\beta = \frac{1}{1+r}$

   The value of an unmatched worker:

$$U = b + \beta\left[(1-\theta q(\theta)) U'_{s'} + \theta q(\theta) W'_{s'}\right]$$

   The value of a matched firm:

$$J = y - w + \beta\left[\lambda V' + (1-\lambda) J'_{s'}\right]$$

   The value of a vacancy:

$$V = -c + \beta\left[(1-q(\theta)) V' + q(\theta) J'_{s'}\right]$$

   Free entry condition implies:

$$V = 0$$

   The bargaining problem:

$$W = \arg\max (W-U)^{\xi} (J-V)^{1-\xi}$$

   Solution to the bargaining problem, using FOC:

$$
\begin{aligned}
\xi(W-U)^{\xi-1}(J)^{1-\xi}\frac{\partial(W-U)}{\partial W}+(1-\xi)(W-U)^{\xi}(J)^{-\xi}\frac{\partial(J)}{\partial W} &= 0 \\
\frac{J}{W-U}\frac{\xi}{1-\xi}+\frac{\frac{\partial(J)}{\partial W_t}}{\frac{\partial(W-U)}{\partial W}} &= 0 \\
\frac{J}{W-U}\frac{\xi}{1-\xi}+\frac{-1}{1} &= 0 \\
(1-\xi)(W-U) &= \xi J
\end{aligned}
$$

2. for each one of current N states:
define $S=W+J-U$
combing the equations:

$$
\begin{aligned}
S &= y-b+\beta\left[(1-\lambda)\,S_{s'}^{'}+\theta q\left(\theta\right)\left(W_{s'}^{'}-U_{s'}^{'}\right)\right] \\
S &= y-b+\beta\left[(1-\lambda)\,S_{s'}^{'}+\theta q\left(\theta\right)\xi S_{s'}^{'}\right] \\
S &= y-b+\beta\left[\left(1-\lambda+\theta q\left(\theta\right)\xi\right)S_{s'}^{'}\right]
\end{aligned}
$$

Using the value of a vacancy:

$$
S_{s'}^{'}=(1-\xi)J_{s'}^{'}=\frac{c}{\beta q\left(\theta\right)}
$$

And pluging in, you get for each one of n possible levels of current value of θ :

$$
\frac{c}{\beta q\left(\theta\right)}=y-b+\beta\left[\left(1-\lambda+\theta q\left(\theta\right)\xi\right)\frac{c}{\beta E\left[q\left(\theta^{'}\right)\right]}\right]
$$

3. From the paper

3 McCall

1. The value of accepting a job offer w is:

$$
v^e(w)=w+\beta\left[\alpha\left(b+\beta v\right)+(1-\alpha)v^e(w)\right]
$$

or:

$$
v^e(w)=\frac{w}{1-(1-\alpha)\beta}+\frac{\beta\alpha\left(b+\beta v\right)}{1-(1-\alpha)\beta}
$$

and the value of recieving an offer w is:

$$v(w) = \max \{v^e(w), b + \beta v\}$$

where:

$$v = \int v(w^{'})dF(w^{'})$$

2. The reservation wage will be lower. As the value of holding a particular job is lower since it is expected to hold for a shorter time, there is less value in waiting out for a better offer, so lower offers will be accepted

4 Production with an adjustment cost

1. the value of the firm is:

$$v(k_{-1}, z) = \max \left\{ \pi(k_{-1}, z) + \beta E v(k_{-1}, z^{'}), \max_k \pi(k, z) + \beta E v(k, z^{'}) - c(z) \right\}$$

with the first option when the firm does not adjust the capital and the second option when it does.

2. As the cost of adjustment is only dependent on the shock z and not on the level of capital k_{-1}, it is clear from the Bellman equation that for a given z, if the firm adjusts, it will adjust to the same level k regardless of k_{-1}. Thus, the second option in the Bellman equation is a const $V^*(z)$, given the optimal choice of adjustment that we designate $k^*(z)$. Obviously the firm will not adjust whenever $\pi(k_{-1}, z) + \beta E v(k_{-1}, z^{'}) > V^*(z)$ (the region of inaction) and adjust to $k^*(z)$ otherwise.

3. No. The unconstrained max does not take into account the expectation for $z^{'}$ given z. If the expectation is for a higher level, for example, this might yield an optimal level of capital which is higher than $\arg\max(\pi(k, z))$.

4. Given that we assume that π is strictly concave (i.e. single picked), it is still possible that given an erratic enough process for z, $\pi(k_{-1}, z) + \beta E v(k_{-1}, z^{'})$ will not be single picked and thus there will not be necessarily a single region of inaction.