Macro Theory B

Final exam (spring 2014)

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Details:

- Course number: 1011-4108-01
- Year/semester: 2013-2014/2
- The exam includes 4 questions
- The weight for each question is specified
- Duration: 3 hours
- Number of pages: 4 + 2 pages of formulas
- No other material is allowed

1 Incomplete markets (35 points)

Consider a stationary economy populated by a measure one continuum of infinitely lived, ex-ante identical agents with preferences over the discounted sequence of consumption and leisure. The per-period utility of an agent is given by:

$$U(c_{it},, h_{it}) = [u(c_{it}) + v(1 - h_{it})],$$

where $\beta \in (0,1)$ is the subjective discount factor, c_{it} is consumption and $h_{it} \in (0,1)$ is the time devoted to work out of a time endowment of 1. Assume that $u(\cdot)$ and $v(\cdot)$ are twice continuously differentiable concave increasing functions.

Agents can save a risk-free bond a, but face the borrowing constraint $a_{t+1} \geq -\phi$ for all t. The discount factor evolves according to a three-state Markov chain with transition probabilities $P_{i,j} = \Pr(\beta_{t+1} = \beta_j | \beta_t = \beta_i)$. Shocks are *iid* across individuals. Assume a law of large numbers holds, so that $\Pr(\beta, \beta')$ is also the fraction of agents in the population subject to this particular transition. Also assume that the Markov transition is well-behaved, so there is a unique invariant distribution.

Production takes place by a representative firm through the aggregate technology $C_t + K_{t+1} - (1-\delta)K_t = AK_t^{\alpha} H_t^{1-\alpha}$, where C_t, K_t and $H_t = \int_i (h_{it})$ are, respectively, aggregate consumption, aggregate capital, and aggregate time devoted to work at time t. To be clear, agents' asset is capital that they lend to the representative firm and the firm bears the depreciation of capital. Labor and asset markets are competitive and clear, every period, with prices w_t and r_t, respectively.

1. Write down the problem of the household in recursive form, making explicit the individual and the aggregate state variables. *10 points.*

2. Write down the individual first-order necessary conditions that characterize the optimal consumption-saving and labor-supply choices. Can the Lagrange multiplier associated with the resource constraint be zero? Explain. *10 points.*

3. The government needs to raise revenues for expenses which do not affect agents utility, at the fixed amount of G per period. The government considers taxing capital income at a fixed flat rate τ^c **or** taxing labor income at a fixed flat rate τ^l. Assuming that conditional on raising the required revenues the government is interested in maximizing the expected discounted sum of agents utility, write down the problem of the government in a recursive form. *15 points.*

2 The Shimer Puzzle (30 points)[1]

Consider an extension of the textbook discrete-time search-and-matching model studied in class where productivity follows some stochastic process. To make things concrete assume that productivity y take one of N values, e.g., $y_s \in \{y_1, y_2 \ldots y_N\}$ where a subscript s denotes the current state. Also assume that this process is persistent in the sense that $y_{s'}$ depends (stochastically) on y_s where a subscript s' denotes the state in the next period. All other parameters such as the flow utility in unemployment (b) and the separation rate (σ) are state independent.

1. Write down the value functions for the firm and the worker (two for each). Be explicit as to which values are state dependent by attaching the subscript s' in the right-hand-side of the value functions where appropriate. Also write down the Nash Bargaining problem and solve it. *10 points.*

2. Derive from those equations a set of N equations that relate the current market tightness (θ) to the expected value of the market tightness in the next period (θ'). *10 points.*

3. Describe the Shimer puzzle. In your answer include the relevant stylized facts of the economy, an explanation of the numerical exercise that Shimer conducts and a discussion on the counterfactual results. *10 points.*

3 McCall (15 points)

Consider the following version of the McCall model: an unemployed worker draws each period one offer w from some wage distribution $F(w) = \Pr(w < W)$. The worker has the option of rejecting the offer, in which case she receives b this period in unemployment compensation and waits until next period to draw another offer from F. Alternatively, the worker can accept the offer to work at w, in which case she receives a wage of w per period **until she is fired: after each working period, with probability α, $0 < \alpha < 1$, the worker is fired, in which case she needs to spend one period in unemployment before receiving the next wage offer.**

1. Formulate the Bellman equation for the worker's problem. *10 points.*

2. Is the reservation wage lower or higher than it would be in the original problem without firing? Explain (no need to solve). *5 points.*

[1] This question is based on "The cyclical behavior of equilibrium unemployment and vacancies", Shimer (2005).

4 Production with an Adjustment Cost (20 points)[2]

Consider the following problem of a firm under uncertainty in discrete time. At time t current profits net of adjustment costs of capital are given by $\pi(k_t, z_t)$ where $z_t \in Z$ is the current shock to profits and k_t is the amount of capital used during period t. The function $\pi(k, z)$ is assumed to be strictly concave in k. Assume that z_t follows some stochastic process.

At the beginning of the period the current shock z_t is realized and the firm observes it. The firm then decides whether or not to adjust its capital shock.

If the firm decides not to adjust the capital then its capital remains at the previous level k_{t-1}. Assume no depreciation. If instead the firm chooses to adjust its capital then it pays a fixed cost $c(z_t)$ and selects a new capital level k_t. The costs may depend on the shock z_t but do not depend on the new level of capital k_t chosen.

The firm's objective is to maximize expected discounted profits net of any costs incurred of changing its capital stock. The firm discounts profits using a constant interest rate r.

1. Argue that k_{t-1}, z_t is the relevant state variable for a firm trying to decide whether or not to adjust capital. Set up the Bellman equation for the value of the firm $v(k_-, z)$ where k_- represents the previous period's capital stock and z represents the current stock. *5 points.*

2. Use your Bellman equation from the previous section to argue that the optimal policy can be summarized by a region of inaction $K(z)$ and an optimal adjustment policy $k^*(z)$. For each $z \in Z$ the region of inaction $K(z)$ is a subset of $\Re_+$. If adjustment occurs then the firm chooses $k^*(z)$ as its new capital stock. Note: you are not asked to actually solve for $K(z)$ or $k^*(z)$ but just to argue that the solution takes this form. *5 points.*

3. Is $k^*(z)$ equal to the unconstrained optimum level of capital (without adjustment cost), i.e., is $k^*(z) = \arg\max_k \pi(k, z)$? *5 points.*

4. Can the region of inaction $K(z)$ always be summarized by an "sS rule", that is that for each state of the world the inaction region is a single closed interval so that there exists functions $\overline{k}(z)$ and $\underline{k}(z)$ such that

$$K(z) = \left\{k_- : \underline{k}(z) \leq k_- \leq \overline{k}(z)\right\}?$$

   (a) Why or why not? *5 points.*

[2]Credit: Adopted from a macro comp question by Ivan Werning, MIT.