Macro Theory B – 2013

Final Exam – Solution

Q1

1. The state variables for the household are the levels of assets saved from the previous period and the level of efficiency units.
   The household problem is:
   $$V(a_{t-1}, \varepsilon_t) = \max_{c_t, h_t, a_t} u(c_t) + v(1-h_t) + \beta \sum_{\varepsilon'} \pi(\varepsilon_t, \varepsilon')\, V(a_t, \varepsilon')$$
   s.t
   The borrowing constraint: $a' \geq 0$
   The budget constraint: $c_t + a_t \leq w_t \cdot (h_t \varepsilon_t) + a_{t-1} + (1-\tau) r_{t-1} a_{t-1} + b_t$

   The only aggregate state variable is K_t.

2. First order conditions:

w.r.t c_t: $u'(c_t) = \lambda_t$

w.r.t h_t: $v'(1-h_t) = \lambda_t w_t \varepsilon_t$

w.r.t a_t: $\beta \sum_{\varepsilon'} \pi(\varepsilon_t, \varepsilon')\, V_a'(a_t, \varepsilon') - \lambda_t + \beta \lambda_{t+1}[1 + (1-\tau) r_t] + \mu_t = 0$

using the envelope condition we get:

$-\lambda_t + \beta \lambda_{t+1}[1 + (1-\tau) r_t] + \mu_t = 0$

And the two constraints.

Euler conditions:

$u'(c_t) = \ \beta \lambda_{t+1}[1 + (1-\tau) r_t] + \mu_t$

$v'(1-h_t) = u'(c_t) w_t \varepsilon_t$

Note that μ_t is zero when the borrowing constraint is not binding

3. A stationary equilibrium is:

   a) A set of aggregate prices $w(K), r(K)$ that depends on the aggregate state.

b) A value function for the households $V(a_{t-1}, \varepsilon_t)$
c) A set of decision function for the households to determine c, h, a, based on each households state (a_{t-1}, ε_t)
d) The firm's choice for labor and capital demand
e) A low of motion G for the distribution

Such that:

a) Given the prices $w(K), r(K)$, the household decision functions solve the household problem and V is the associated value function
b) Given the prices $w(K), r(K)$, the firm optimally choose labor and capital demand
c) The labor market clears: Firm demand for labor matches labor supply $H = \int (h_i \varepsilon_i)$
d) The assets market clears: Firm demand for capital matches capital supply $K = \int a_i$
e) The goods market clear: Total production plus capital left (net of depreciation) matches total consumption plus total investment
f) Tax revenues matches households expected lump sum
g) The aggregate low of motion G is generated by the exogenous Markov process and the household's decision functions

4. The algorithm:

a) Guess r, b
b) From the firm's optimal decision for capital $r = \alpha A(K/H)^{\alpha-1}$, calculate the capital/effective labor ratio. Then use it to calculate the wage from the firms optimal decision for labor $w = (1-\alpha)A(K/H)^{\alpha}$
c) Solve the household problem. This includes:
    a. Defining a grid for the assets per each productivity type
    b. Guessing an initial value vector per type $V(a, \varepsilon)$
    c. For each type and for each assets level, find the best next asset level by solving the household problem defined in section 1. Note that this involves using the FOC for work hours, as if you have an asset level and want to move to a different asset level, there is a unique combination of c, h that will bring you there while obeying the FOC
    d. Iterating until convergence
d) Find a stationary distribution for the household assets. This includes:
    a. Using the household solution to build the decision rule for next period assets
    b. Building the transition matrixes from current level of assets to next period levels
    c. Iterating until convergence

e) Calculate total capital and effective labor supply from the stationary distribution and the household decision rule
f) Calculate total capital and effective labor demand using firm FOC
g) Update the guess for r using capital demand and supply, iterate (b)-(f) until convergence for r
h) Calculate tax revenues and derive lump sum b for next iteration
i) Iterate (b)-(h) until convergence for b

Q2

1. Steady state equations:
   Define market tightness $\theta \equiv v/u$
   The probability that an unmatched rancher finds a worker:
   $\frac{m}{v} = m\left(\frac{u}{v}, 1\right) = \bar{m}(u/v)^{\delta} = \bar{m}\theta^{-\delta} \equiv q(\theta)$
   The probability that an unmatched cowboy finds a job:
   $\frac{m}{u} = \frac{v}{u} m\left(\frac{u}{v}, 1\right) = \theta q(\theta)$

   Matched cowboy:
   $W = w + \beta[\lambda U + (1-\lambda)W]$
   Where $\beta = 1/(1+r)$

   Unmatched cowboy:
   $U = b + \beta\big[\big(1 - \theta q(\theta)\big)U + \theta q(\theta) W\big]$

   Matched rancher
   $J = y - w + \beta[\lambda V + (1-\lambda)J]$

   Unmatched rancher:
   $V = y_0 - k + \beta\big[\big(1 - q(\theta)\big)V + q(\theta) J\big]$

   Bargaining:
   $W - U = \gamma(J - V + W - U)$

   Flows in and out of unemployment:
   $(1-u)\lambda = \theta q(\theta) u$

Flows in and out of posting a vacancy:
$(1-v)\lambda = \ q(\theta)v$

2. Unemployment rate:
From the flows:
$u = \dfrac{\lambda}{\lambda + \theta q(\theta)}$
$v = \dfrac{\lambda}{\lambda + q(\theta)}$
As $\theta \equiv v/u$:
$\theta = \dfrac{\lambda + \theta q(\theta)}{\lambda + q(\theta)}$
$\theta\lambda + \ \theta q(\theta) = \lambda + \theta q(\theta)$
$\theta = 1$

So:

$$\boxed{u = \frac{\lambda}{\lambda + q(1)} = \frac{\lambda}{\lambda + \overline{m}}}$$

Wage:
$$S_c = W - U = w - b + \beta\big[\big(1-\lambda-\theta q(\theta)\big)W - \big(1-\lambda-\theta q(\theta)\big)U\big]$$
$$= w - b + \beta\big(1-\lambda-\theta q(\theta)\big)S_c$$
$$S_c = \frac{w-b}{1-\beta\big(1-\lambda-\theta q(\theta)\big)}$$

$$S_r = J - V = y - w - y_0 + k + \beta\big[\big(1-\lambda-q(\theta)\big)J - \big(1-\lambda-q(\theta)\big)V\big]$$
$$= y - w - y_0 + k + \beta\big(1-\lambda-q(\theta)\big)S_r$$
$$S_r = \frac{y-w-y_0+k}{1-\beta\big(1-\lambda-q(\theta)\big)}$$

From the barraging equation:

$$S_c = \frac{\gamma}{1-\gamma}S_r$$

$$\frac{w-b}{1-\beta\big(1-\lambda-\theta q(\theta)\big)} = \frac{\gamma}{1-\gamma}\frac{y-w-y_0+k}{1-\beta\big(1-\lambda-q(\theta)\big)}$$

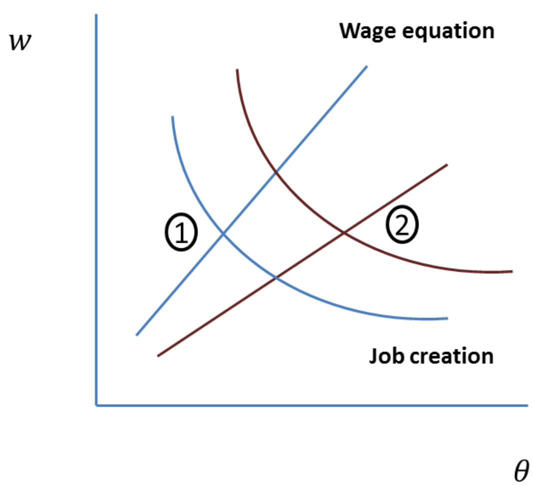
Using $\theta = 1$ from previous section:

$$\frac{w-b}{1-\beta(1-\lambda-\overline{m})}=\frac{\gamma}{1-\gamma}\frac{y-w-y_0+k}{1-\beta(1-\lambda-\overline{m})}$$

$$(w-b)(1-\gamma)=\gamma(y-w-y_0+k)$$

$$\boxed{w=b(1-\gamma)+\gamma(y-w-y_0+k)}$$

3. Decreasing the cost k decreases the wage as it decreases the surplus of the rancher from the match and thus the value of the match. It does not change the unemployment rate as in a setting with no free entry and the same number of cowboys and ranchers , the market tightness must be 1 by construction, locking the unemployment rate.
4. In the standard DMP model, we are looking at the wage and job creation equations:
$$w=b(1-\gamma)+\gamma(p+k\theta)$$
$$p-w=\big(1-\beta(1-\lambda)\big)\frac{k}{\beta q(\theta)}$$
Reducing the cost of posting a vacancy k moves the wage equation down as the value of the match is lower, and the job creation equation up as it is more profitable to open a new vacancy, so the wage change is undetermined and the market tightness goes up, pushing the unemployment rate down.



Q3

1. The worker has 3 possible choices – stay in both career and job, change job and change both:

$$V(\theta,\xi) = max\left\{\begin{array}{c} \theta+\xi+\beta V(\theta,\xi), \\ \theta+\int \xi'+\beta V(\theta,\xi')dG(\xi'), \\ \iint \theta'+\xi'+\beta V(\theta',\xi')dG(\xi')dF(\theta') \end{array}\right\}$$

2. Option #3, to change both, has a constant value, lets mark by V_3. Per each career, option #2 has a value which is a function of the career θ only, lets mark by $V_2(\theta)$. The value of option #1 if you choose it is obviously $V_1(\theta,\xi) = \frac{\theta+\xi}{1-\beta}$, as if you stay once you will always stay as the problem is stationary. Now, for a given θ, there is single job $\bar{\xi}$ for which the worker is indifferent, that satisfies

$$\frac{\theta+\bar{\xi}}{1-\beta} = max(V_2(\theta),V_3)$$

Inserting the explicit phrases we get

$$\bar{\xi}(\theta) = \max((1-\beta)V_2(\theta)-\theta,(1-\beta)V_3-\theta) \tag{1}$$

And the worker will stay put for any higher job in this career.

There is also a critical career level for which is indifferent between options 2 and 3, that satisfies:

$$V_2(\bar{\theta}) = V_3$$

So the worker never leaves any career higher than $\bar{\theta}$. Thus, the rule for staying in the current career is as follows: stay if $\theta > \bar{\theta}$ or $\xi > \bar{\xi}(\theta)$.

If $\theta > \bar{\theta}$, the worker will never leave his job, so:

$$V_2(\theta) = \frac{\theta}{1-\beta} + \int J(\xi')\ dG(\xi')$$

Where $J()$ is the expected discounted sum of the jb part only, given that the worker never switch careers. The Bellman equation for J is:

$$J(\xi) = max\left(\frac{\xi}{1-\beta}, \xi + \int J(\xi')\ dG(\xi')\right)$$

Which means that there is a cutoff job level for switching jobs, that is not dependent on θ, given that you are in the range where you never switch a career. And for a level for which you might switch a career, $\theta < \bar{\theta}$, equation 1 implies that

$$\bar{\xi}(\theta) = (1-\beta)V_3 - \theta$$

Meaning that the cut off level is linearly sloping down with θ.