Macro Theory B
Final exam (spring 2017)

Lecturer: Ofer Setty, TA: Alon Rabinowicz
The Eitan Berglas School of Economics, Tel Aviv University

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Details:

- Course number: 1011-4108-01
- Year/semester: 2016-2017/2
- The exam includes: 3 questions
- The weight for each question is specified
- Duration: 3 hours
- Total number of pages: 6
- Any printed material is allowed

1 Search and matching (40 points)

1. Value functions:

$$\begin{aligned}
V &= -c + \beta q(\theta) J' \\
J &= y - w + \beta(1-\delta) J' \\
W &= w + \beta\left[\delta U_I' + (1-\delta) W'\right] \\
U_I &= b + \beta\left[\mu(\theta) W' + (1-\mu(\theta))\left((1-\lambda) U_I' + (\lambda) U_N'\right)\right] \\
U_N &= \beta\left[\mu(\theta) W' + (1-\mu(\theta)) U_N'\right]
\end{aligned}$$

2. flows / evolution equations:

$$\begin{aligned}
e &= \mu(\theta)(1-e) + (1-\delta)e = \mu(\theta)(u_N + u_I) + (1-\delta)(1 - u_N - u_I) \\
u_N' &= (1-\mu(\theta)) u_N + \lambda(1-\mu(\theta)) u_I \\
u_I' &= \delta(1 - u_N - u_I) + (1-\mu(\theta))(1-\lambda) u_I
\end{aligned}$$

3. difference between the two unemployment values:

$$\begin{aligned}
U_I - U_N &= b + (1-\mu(\theta))\left[(1-\lambda) U_I' + \lambda U_N' - U_N'\right] \\
&= b + (1-\mu(\theta))(1-\lambda)\left(U_I' - U_N'\right)
\end{aligned}$$

in steady state:

$$U_I - U_N = \frac{b}{1 - (1-\mu(\theta))(1-\lambda)} > 0$$

Now assume bargaining weight is $\frac{1}{2}$ and calculate the wage. Important: assume that the outside option is re-set upon matching, not upon employment (i.e. employers can't discriminate between the two types of unemployed). With this assumption, Nash bargaining:

$$\max_{w} (W - U_I)^{\frac{1}{2}} J^{\frac{1}{2}}$$

$$W - U_I = w - b + \beta \left[1 - \delta - \mu(\theta)\right] W' + \beta \left[\delta - (1 - \mu(\theta))(1 - \lambda)\right] U_I' - \beta\lambda(1 - \mu(\theta))U_N'$$

$$= w - b + \beta \left[1 - \delta - \mu(\theta)\right] W' + \beta \left[\delta - (1 - \mu(\theta))\right] U_I' - \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right]$$

$$= w - b + \beta \left[1 - \delta - \mu(\theta)\right] (W' - U_I') - \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right]$$

$$J = y - w + \beta(1 - \delta)J'$$

first order condition:

$$\frac{1}{2}(W - U_I)^{-\frac{1}{2}} J^{\frac{1}{2}} - \frac{1}{2}(W - U_I)^{\frac{1}{2}} J^{-\frac{1}{2}} = 0 \quad \Rightarrow \quad W - U_I = J$$

Substitute J and $W - U_I$ into the last euqation:

$$w - b + \beta \left[1 - \delta - \mu(\theta)\right] (W' - U_I') - \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right] = y - w + \beta(1 - \delta)J'$$

$$w - b + \beta \left[-\mu(\theta)\right] (W' - U_I') - \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right] = y - w$$

$$2w = y + b + \beta\mu(\theta)J' + \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right]$$

$$2w = y + b + \beta\theta q(\theta)J' + \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right]$$

$$2w = y + b + \theta c + \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right]$$

$$w = \frac{1}{2} \left(y + b + \theta c + \beta\lambda(1 - \mu(\theta)) \left[U_N' - U_I'\right]\right)$$

In steady state, the wage function is:

$$
\begin{aligned}
w &= \frac{1}{2}\left(y + b + \theta c - \beta\lambda(1-\mu(\theta))\frac{b}{1-(1-\mu(\theta))(1-\lambda)}\right) \\
&= \frac{1}{2}\left(y + \theta c + b\left(1 - \frac{\beta\lambda(1-\mu(\theta))}{1-(1-\mu(\theta))(1-\lambda)}\right)\right) \\
&= \frac{1}{2}\left(y + \theta c + b\left(\frac{1-(1-\mu(\theta))(1-\lambda) - \beta\lambda(1-\mu(\theta))}{1-(1-\mu(\theta))(1-\lambda)}\right)\right) \\
&= \frac{1}{2}\left(y + \theta c + b\left(\frac{\mu(\theta)+\lambda(1-\mu(\theta)) - \beta\lambda(1-\mu(\theta))}{\mu(\theta)+\lambda(1-\mu(\theta))}\right)\right) \\
&= \frac{1}{2}\left(y + \theta c + b\left(\frac{\mu(\theta)+(1-\beta)\lambda(1-\mu(\theta))}{\mu(\theta)+\lambda(1-\mu(\theta))}\right)\right)
\end{aligned}
$$

Analytically:

$$
\frac{\partial w}{\partial \lambda} = \frac{1}{2}b\left[\frac{(1-\beta)(1-\mu(\theta))(\mu(\theta)+\lambda(1-\mu(\theta))) - (1-\mu(\theta))(\mu(\theta)+(1-\beta)\lambda(1-\mu(\theta)))}{(\mu(\theta)+\lambda(1-\mu(\theta)))^2}\right]
$$

And to sign the derivative we have to sign the numerator

$$
\begin{aligned}
&(1-\beta)(1-\mu(\theta))(\mu(\theta)+\lambda(1-\mu(\theta))) - (1-\mu(\theta))(\mu(\theta)+(1-\beta)\lambda(1-\mu(\theta))) = \\
&(1-\mu(\theta))\left[(1-\beta)(\mu(\theta)+\lambda(1-\mu(\theta))) - \mu(\theta) - (1-\beta)\lambda(1-\mu(\theta))\right] = \\
&(1-\mu(\theta))(1-\beta)\left[\mu(\theta)+\lambda(1-\mu(\theta)) - \frac{\mu(\theta)}{1-\beta} - \lambda(1-\mu(\theta))\right] = \\
&(1-\mu(\theta))(1-\beta)\left[\mu(\theta) - \frac{\mu(\theta)}{1-\beta}\right] = \\
&(1-\mu(\theta))(1-\beta)\mu(\theta)\left[-\frac{\beta}{1-\beta}\right] < 0
\end{aligned}
$$

When there is a possibility that benefits expire, it lowers the expected value of the outside option in the bargaining (i.e U_I is lower than what it is in a model without λ). As a result, the wage declines.

4. Steady state J:

$$
J = \frac{y-w}{1-\beta(1-\delta)} = \frac{y - \frac{1}{2}\left(y+\theta c + b\left(\frac{\mu(\theta)+(1-\beta)\lambda(1-\mu(\theta))}{\mu(\theta)+\lambda(1-\mu(\theta))}\right)\right)}{1-\beta(1-\delta)}
$$

It is clear that for a given θ, the steady state value of J increases because workers produce the same output but the wage is lower.

5. (a) The easiest way is to use the flow value for employment and rearrange (using

the flow values for u_I and u_N works too, but much more algebra):

$$\begin{aligned}
e &= \mu(\theta)(1-e) + (1-\delta)e = \mu(\theta) - \mu(\theta)e + e - \delta e \\
e(\mu(\theta)+\delta) &= \mu(\theta) \\
e &= \frac{\mu(\theta)}{\delta+\mu(\theta)} \\
u &= 1 - e = \frac{\delta}{\delta+\mu(\theta)}
\end{aligned}$$

(b) zero profit condition:

$$\begin{aligned}
c &= \beta q(\theta) J = \beta q(\theta) \frac{\frac{1}{2}y - \frac{1}{2}\left(\theta c + b\left(\frac{\mu(\theta)+(1-\beta)\lambda(1-\mu(\theta))}{\mu(\theta)+\lambda(1-\mu(\theta))}\right)\right)}{1-\beta(1-\delta)} \\
&= \beta \frac{\frac{1}{2}yq(\theta) - \frac{1}{2}\left(\mu(\theta)c + bq(\theta)\left(\frac{\mu(\theta)+(1-\beta)\lambda(1-\mu(\theta))}{\mu(\theta)+\lambda(1-\mu(\theta))}\right)\right)}{1-\beta(1-\delta)}
\end{aligned}$$

From the first part of the question we learn that changes in unemployment depend on changes in θ. Using the term in the second part we can derive $\frac{\partial\theta}{\partial\lambda}$ (using implicit function) and sign this derivative. If it is positive then we can say that when λ increases θ increase the the unemployment rate is lower.

Note that a simple reasoning based on the graphs with a downward sloping job creation curve and a linear upward sloping wage line cannot be used directly because the wage function is now non-linear in θ, and so if we were to use it, we still have to argue that it is generally increasing in θ.

6. If workers have the described utility function then having zero consumption cannot be an option because the marginal utility from consumption is infinity. Two ways to resolve: first is purely technical - allowing some minimum level of benefits. Second is more complex - allow workers to save so that they may accumulate sufficient assets and never hit the zero consumption level.

2 McCall search model (35 points)

1.

$$
\begin{aligned}
U &= b + (1-\phi)\beta U + \beta\phi \int \max\{U, V(w)\} dF(w) \\
V(w) &= w + (1-\alpha)\beta V(w) + \alpha \int \max\{V(w), V(w')\} dF(w')
\end{aligned}
$$

2. Starting with employed workers, it is clear that any offer that has a wage that is higher than the current wage will be accepted. This implies that, as in the standard McCall model, the value $V(w)$ is monotonically increasing in w, and the worker receives unemployment benfits $b > 0$ etc. Therefore we expect to see a reservation wage policy such that any offer below w_R is rejected, and any offer $w \geq w_R$ is accepted. With this, we can rewrite the Bellman values:

$$
\begin{aligned}
U &= b + (1-\phi)\beta U + \beta\phi \int_0^{w_R} U dF(w) + \beta\phi \int_{w_R} V(w) dF(w) \\
V(w) &= w + (1-\alpha)\beta V(w) + \alpha \int_0^{w} V(w) dF(w') + \alpha \int_{w} V(w') dF(w')
\end{aligned}
$$

3. First note that the reservation wage policy implies that the worker is indifferent between working and unemployment at $w = w_R$. Therefore it must be that $V(w_R) =$

U. Using this we can subtract the Bellman values $V(w_R) - U$ so that we have:

$$\begin{aligned}
0 &= w_R - b + \beta V(w_R) - \alpha\beta V(w_R) + \alpha \int_0^{w_R} V(w_R) dF(w') \\
&\quad + \alpha \int_{w_R} V(w')dF(w') - (1-\phi)\beta U - \beta\phi \int_0^{w_R} U dF(w') - \beta\phi \int_{w_R} V(w')dF(w') \\
&= w_R - b + \beta U - \alpha\beta U + \alpha\beta \int_0^{w_R} U dF(w') + \alpha\beta \int_{w_R} V(w')dF(w') \\
&\quad -(1-\phi)\beta U - \beta\phi \int_0^{w_R} U dF(w') - \beta\phi \int_{w_R} V(w')dF(w') \\
&= w_R - b + \beta U - \alpha\beta U + \alpha\beta U F(w_R) + \alpha\beta \int_{w_R} V(w')dF(w') \\
&\quad -(1-\phi)\beta U - \beta\phi U F(w_R) - \beta\phi \int_{w_R} V(w')dF(w') \\
&= w_R - b + \beta U \left[1 - \alpha + \alpha F(w_R) - (1-\phi) - \phi F(w_R)\right] + \beta \left[\alpha - \phi\right] \int_{w_R} V(w')dF(w') \\
&= w_R - b + \beta U \left[\phi(1 - F(w_R)) - \alpha(1 - F(w_R))\right] + \beta \left[\alpha - \phi\right] \int_{w_R} V(w')dF(w') \\
&= w_R - b + \beta U \left[\phi - \alpha\right] (1 - F(w_R)) + \beta \left[\alpha - \phi\right] \int_{w_R} V(w')dF(w') \\
&= w_R - b + \beta \left[\alpha - \phi\right] \int_{w_R} \left(V(w') - U\right) dF(w') \\
&= w_R - b + \beta \left[\alpha - \phi\right] \int_{w_R} \left(V(w') - V(w_R)\right) dF(w') \\
w_R &= b + \beta \left[\phi - \alpha\right] \int_{w_R} \left(V(w') - V(w_R)\right) dF(w')
\end{aligned}$$

(note that my algebra may have some unnecessary steps, and this is probably not the only way to show).

4. First note that if we set $\phi = 1$ and $\alpha = 0$ we get the standard McCall model. In this case, taking a job that offers the reservation wage delivers a benefit w_R but comes at a cost of giving up the unemployment compensation as well as the opportunity to receive a better offer in the future.
   The key difference in this model is when $\alpha > 0$, because it reduces the cost of accepting a job offer as it introduces a positive probability that the worker will be able to improve the wage *while working*. The introduction of ϕ complicates the analysis a bit, but in an intuitive way. If $\phi < 1$ then it reduces the benefit from waiting (or the cost of accepting a job offer) because it may take longer until the worker receives a good enough offer.

5. A higher b just increases the benefit from waiting, and so it is expected to increase

the reservation wage (keeping everything else equal).

6. In a standard McCall model this is not possible because once a worker accepts a job the job lasts forever, so the worker is clearly better off remaining unemployed forever. The difference $\phi - \alpha$ may be negative, in which case the worker may be better off taking a job at a lower reservation wage because the chances of getting a better offer are better when employed.

7. The modified Bellman values with the separation probability δ:

$$\begin{aligned} U &= b + (1-\phi)\beta U + \beta\phi \int_0^{w_R} U dF(w) + \beta\phi \int_{w_R} V(w) dF(w) \\ V(w) &= w + \delta\beta U + (1-\delta)(1-\alpha)\beta V(w) + (1-\delta)\alpha \int_0^{w} V(w) dF(w') + (1-\delta)\alpha \int_{w} V(w') dF(w' \end{aligned}$$

8. In the standard McCall model the positive separation probability implied that the value from a job at any w is lower, so there is a weaker incentive to wait for a "great job" because there is a probability of losing it. In other words, the cost of accepting some offer w is lower, and so the reservation wage is lower. Here we see another effect, which is that a separation probability effectively lowers the probability of receiving a new job offer while employed. As a result, the cost of accepting some offer w when unemployed is higher, and the reservation wage should be higher. Overall effects is theoretically ambiguous.

3 Optimal Unemployment Insurance (25 points)

a. Denote by x^o the recommendation conditional on outcome o. The lecturer's problem is:

$$min_{x^B,x^G,x^E}\{\pi_B^H * x^B + \pi_G^H * x^G + \pi_E^H * x^E\}$$
$$s.t.$$
$$\pi_B^H * u(x^B) + \pi_G^H * u(x^G) + \pi_E^H * u(x^E) - d \geq U$$
$$\pi_B^H * u(x^B) + \pi_G^H * u(x^G) + \pi_E^H * u(x^E) - d \geq \pi_B^L * u(x^B) + \pi_G^L * u(x^G) + \pi_E^L * u(x^E)$$

Notice that standard arguments can be used to determine equality in both PK (otherwise the cost can reduced) and the IC (by decreasing the spread the lecturer can relax the PK).

b. Denote $\pi_\Delta = \pi_B^L - \pi_B^H = \pi_E^H - \pi_E^L$.

From the IC (the second constraint) we have that:

$$\begin{aligned}
\pi_B^H * u(x^B) + \pi_G^H * u(x^G) + \pi_E^H * u(x^E) - d &\geq \pi_B^L * u(x^B) + \pi_G^L * u(x^G) + \pi_E^L * u(x^E) \\
\pi_B^H * u(x^B) + \pi_E^H * u(x^E) - d &\geq \pi_B^L * u(x^B) + \pi_E^L * u(x^E) \\
\pi_\Delta u(x^E) - d &\geq \pi_\Delta u(x^B) \\
\pi_\Delta(u(x^E) - u(x^B)) &\geq d \\
u(x^E) - u(x^B) &\geq \frac{d}{\pi_\Delta}
\end{aligned}$$

Intuition: The G outcome does not appear in the IC because its probability is the same regardless of effort. The difference between the remaining two outcomes, increases with d (the source of moral hazard) and decreases with the probability difference, which is informative for the lecturer as a signal to high effort. Notice that when $\pi_\Delta = 0$ there is no way to support the contract, and when $\pi_\Delta = 1.0$ (which implies that $\pi_G^H = \pi_G^L = 0$) the lecturer only compensates the student for her effort.