Macro Theory B
Final exam (spring 2017)

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Details:

- Course number: 1011-4108-01
- Year/semester: 2016-2017/2
- The exam includes: 3 questions
- The weight for each question is specified
- Duration: 3 hours
- Total number of pages: 6
- Any printed material is allowed

1 Search and matching[1] (40 points)

In this question we consider the effects of a potential expiry of unemployment benefits in the context of an otherwise standard DMP model. Assume a discrete time model with a discount factor $0 < \beta < 1$, and risk neutral workers and firms. There is free entry into vacancy posting and the cost to maintain a vacancy for a period is c. A worker and a firm match according to a constant returns to scale matching technology such that θ is labor market tightness, $\mu(\theta)$ is the probability that a worker finds a job, and $q(\theta)$ is the probability that a firm is able to find a worker (you may assume that both probabilities are positive, and that $\mu' > 0$ and $q' < 0$). Upon matching, a firm and a worker bargain over the wage w according to the standard Nash bargaining, where you can assume that the bargaining share for each party is $\frac{1}{2}$. As usual, matches that occur in the current period become productive in the next period. When employed, the worker produces y units of output. All existing matches are subject to an exogenous separation probability $0 < \delta < 1$.

Following a separation, a worker becomes unemployed and receives unemployment benefits $b > 0$. Following the first period of unemployment, and if the worker is not matched with a new employer, the worker may lose the eligibility for unemployment benefits with some probability $0 \leq \lambda \leq 1$. To simplify, assume that as soon as a worker is matched with a firm, his/her eligibility is restored (i.e. even if there is an immediate separation or if bargaining is unsuccessful). All unemployed workers, regardless of their benefits status, search for employment.

1. Denote the Bellman values for vacancies, filled jobs, employed workers, 'insured' unemployed workers, and 'non-insured' unemployed workers by V, J, W, U_I, U_N. Write the Bellman Values that are consistent with the assumptions of the model.

2. Denote the number of employed, insured unemployed, and uninsured unemployed by e, u_I, and u_N, and assume that $e + u_I + u_N = 1$ (so that e is the employment rate and $u_I + u_N$ is the unemployment rate). Write the flow equations for employment and the two types of unemployment.

3. Derive the steady state wage equation, i.e. the wage as a function of $y, b, c, \theta, \beta, \lambda$. (Note that along the way you will have to specify the Nash product, and also solve for $U_I - U_N$ in steady state).

4. Does the steady state wage increase/decrease/not changing when λ increases?

5. Write the steady state value of J as a function of $y, b, c, \theta, \beta, \lambda$. Does the steady state wage increase/decrease/not changing when λ increases?

6. What is the effect of λ on the unemployment rate? Specifically

   (a) Show that steady state unemployment rate is just like in the standard model $u = \frac{\delta}{\delta + \mu(\theta)}$

[1]Credit: Yaniv Yedid-Levy, UBC, comp PhD macro.

(b) Write the zero profit condition as a function of $y, b, c, \theta, \beta, \lambda$ and discuss how you would use it to find the effect of λ on the unemployment rate. (You don't have to actually derive this result. Just discuss how you would and what you can conclude regarding the unemployment rate).

7. What would be a problem for the current setup if workers preferences were represented by a concave utility function that satisfies the Inada conditions? Briefly describe two different ways to resolve this issue.

2 McCall search model[2] (35 points)

Assume a McCall - like model where workers can be either employed or unemployed, and job offers are exogenous. Assume that time is discrete and that future values are discounted by a discount factor $0 < \beta < 1$.

Unemployed workers get unemployment benefits $b > 0$ for each period of unemployment. With some positive probability $0 < \phi < 1$ an unemployed worker receives a job offer. The offer is a wage w that remains constant as long as the match exists. w is drawn from some distribution with a CDF $F(w)$ (assume that $w > 0$). To simplify, you may assume that if a worker accepts an offer, then he/she becomes employed starting the next period.

Employed workers earn the wage w that they accepted as long as they keep the job. Workers may receive a competing wage offer w', drawn from the same exogenous wage distribution. The probability of receiving a competing offer is $0 < \alpha < 1$. If the worker accepts the competing offer, he/she starts working at the new job in the following period.

1. Denote the Bellman values for unemployed and employed workers by U and $V(w)$, respectively. Write the expressions for U and $V(w)$.

2. Explain why the workers' optimal policy involves thresholds / reservations wages? Denote the reservation wage for an unemployed worker by w_R, and re-write the Bellman values to incorporate this optimal policy.

3. Show that the following condition holds:

$$w_R = b + \beta [\phi - \alpha] \int_{w_R} (V(w') - V(w_R))\, dF(w')$$

   (hint: evaluate $V(w_R)$. Then, explain why $V(w_R) = U$ and subtract the Bellman values)

4. Explain the equation that you derived in the previous part. You can start by thinking about the configuration of parameters (ϕ, α) that results in a standard McCall model, and then think about how this version of the model may be different.

5. Explain the effect of b on the reservation wage. (No need to explicitly derive the reservation wage)

6. In a standard McCall model, is it possible that the worker accepts a job offer with $w < b$? How about this version of the model? Explain.

7. Now assume that there is a probability $0 < \delta < 1$ that workers exogenously lose their jobs and go back to unemployment. Write the Bellman values to incorporate this probability. (You may assume that only workers that do not separate may draw competing offers).

[2]Credit: Yaniv Yedid-Levy, UBC, comp PhD macro.

8. With separations the reservation wage is characterized by

$$w_R = b + \beta \left[\phi - (1-\delta)\alpha\right] \int_{w_R} \left(V(w') - V(w_R)\right) dF(w')$$

Taking into account the effect of separation in a baseline McCall model (like the one we discussed in class), and in this model, describe the effect of δ on the reservation wage. (You don't have to derive anything, just discuss the potential channels)/

3 Optimal Unemployment Insurance (25 points)

Consider a variant of the one-period optimal unemployment insurance problem discussed in class.

A risk-averse student is taking a class with a risk-neutral teacher. The goal of the student is to get a recommendation from the teacher. Assume that the recommendation has some continuous and unbounded support (such as the real line) of quality that is denoted by x, over which the student has an increasing and concave utility $u(x)$. The student does not like to study. Assume that the study decision is binary - she either studies at high intensity (denoted H), in which case she has disutility d, or she studies at low intensity (denoted L), in which case she has disutility of 0.

The outcome of study could be B (bad knowledge), G (good knowledge), or E (excellent knowledge).

The teacher wants to use the recommendation's quality to encourage the student to study as the student only cares about the recommendation's quality and not on the level of knowledge. The teacher is not generous and would like to minimize the expected quality of the recommendation.[3] (You can therefore assume that the teacher minimizes expected x). The teacher, however, is committed to delivering to the student an expected utility U.

The mapping between effort and outcomes is stochastic. Let the probability of outcome $o \in \{B, G, E\}$ conditional on effort $e \in \{L, H\}$ be π_o^e. So for example π_G^H is the probability of achieving good knowledge conditional on high-intensity (or simply high) effort.

Please answer the following questions:

a. Formulate the problem as a principal-agent problem in the "optimal unemployment insurance" framework studied in class.

b. Assume that the following two conditions on parameters holds: $\pi_B^L - \pi_B^H = \pi_E^H - \pi_E^L > 0, \pi_G^H = \pi_G^L$. Derive a condition on parameters for the difference in utilities between the utility of the recommendation conditional on outcome E and the utility of the recommendation conditional on outcome B. Discuss the role of parametric values for the size of that difference, explaining the qualitative role of each parameter using economic intuition.

[3]To be clear, the attitude for risk in this question (the student is risk-averse and the teacher is risk-neutral) is with regard to the recommendation's quality.

Solution of question 3

a. Denote by x^o the recommendation conditional on outcome o. The lecturer's problem is:

$$min_{x^B,x^G,x^E}\{\pi_B^H * x^B + \pi_G^H * x^G + \pi_E^H * x^E\}$$
$$s.t.$$
$$\pi_B^H * u(x^B) + \pi_G^H * u(x^G) + \pi_E^H * u(x^E) - d \geq U$$
$$\pi_B^H * u(x^B) + \pi_G^H * u(x^G) + \pi_E^H * u(x^E) - d \geq \pi_B^L * u(x^B) + \pi_G^L * u(x^G) + \pi_E^L * u(x^E)$$

Notice that standard arguments can be used to determine equality in both PK (otherwise the cost can reduced) and the IC (by decreasing the spread the lecturer can relax the PK).

b. Denote $\pi_\Delta = \pi_B^L - \pi_B^H = \pi_E^H - \pi_E^L$.

From the IC (the second constraint) we have that:

$$\begin{aligned}
\pi_B^H * u(x^B) + \pi_G^H * u(x^G) + \pi_E^H * u(x^E) - d &\geq \pi_B^L * u(x^B) + \pi_G^L * u(x^G) + \pi_E^L * u(x^E) \\
\pi_B^H * u(x^B) + \pi_E^H * u(x^E) - d &\geq \pi_B^L * u(x^B) + \pi_E^L * u(x^E) \\
\pi_\Delta u(x^E) - d &\geq \pi_\Delta u(x^B) \\
\pi_\Delta(u(x^E) - u(x^B)) &\geq d \\
u(x^E) - u(x^B) &\geq \frac{d}{\pi_\Delta}
\end{aligned}$$

Intuition: The G outcome does not appear in the IC because its probability is the same regardless of effort. The difference between the remaining two outcomes, increases with d (the source of moral hazard) and decreases with the probability difference, which is informative for the lecturer as a signal to high effort. Notice that when $\pi_\Delta = 0$ there is no way to support the contract, and when $\pi_\Delta = 1.0$ (which implies that $\pi_G^H = \pi_G^L = 0$) the lecturer only compensates the student for her effort.