Macro Theory B
Final exam (spring 2016) - Proposed Solution

Lecturer: Ofer Setty, TA: Avihai Lifschitz
The Eitan Berglas School of Economics, Tel Aviv University

July 3, 2016

1 Optimal Unemployment Insurance (25 points)

Consider the infinite-horizon optimal unemployment insurance problem discussed in class. A government (principal) insures an agent against unemployment.

Notice: I expect you to solve the problem with the infinite horizon but you will only be panelized by 8 points if you answer the same question using the one-period model.

The basic structure is as explained in class:

- The principal is committed to providing the agent with a utility U.
- The principal is risk neutral.
- The agent is risk averse with utility $u(c) - a$, where a is effort a $\in \{0, e\}$.
- The probability of finding a job is $\pi > 0$ if the agent exerts effort $a = e$ and zero otherwise.

Please answer the following questions:

a. Write down the principal's problem who insures the agent against unemployment and who is interested in inducing the agent to exert the high effort. You can assume that the constraints in the question are binding.

The problem for an employed worker is (not required in the question - brought here for completion)

$$\begin{aligned} W(U) = \max_{c,U^e} & \quad -c + w + \beta W(U^e) \\ s.t. & \quad : \\ U &= \log(c) - e + \beta U^e \end{aligned}$$

The problem for an unemployed worker is:

$$\begin{aligned} V(U) = \max_{c,U^e,U^n} & \quad -c + \beta\pi W(U^e) + \beta(1-\pi)V(U^n) \\ s.t. & \quad : \\ U &= \log(c) - e + \beta\pi U^e + \beta(1-\pi)U^n \\ U &= \log(c) + \beta U^n, \end{aligned}$$

where the constraints are written with equality (easy to prove).

Now suppose that the principal, can use upon unemployment a binary signal that is correlated with the agent's effort. The signal structure works as follows: the probability of a good signal given low effort ($a = 0$) is zero; the probability of a good signal given high effort ($a = e$) is θ. You can therefore think about the signal as a lottery whose outcomes are conditional on the agent's effort.

b. Write down the updated problem, allowing the principal to condition promised utility not just on the employment outcome, but also on the signal's outcome. Hint: there are now three possible outcomes instead of just two.

There is no change in the employment problem. The unemployment problem is:

The problem for an unemployed worker is:

$$\begin{aligned} V(U) = \max_{c,U^e,U^n} & \quad -c + \beta\pi W(U^e) + \beta(1-\pi)(\theta V(U^g + (1-\theta)V(U^g)) \\ s.t. & \quad : \\ U &= \log(c) - e + \beta\pi U^e + \beta(1-\pi)(\theta U^g + (1-\theta)U^b) \\ U &= \log(c) + \beta U^b, \end{aligned}$$

c. Answer the following questions:

i Is the signal useful? Why?

The signal is useful because it provides the planner with additional information on the agent's effort. The signal allows the planner to reduce consumption variation, hence a lower compensation for spread of consumption and lower consumption to the agent.

ii What can you tell about the value of the planner's value in the second problem, relative to the principal's value in the first best, when $\theta = 1$?

When $\theta = 1$ there is a 1-1 mapping between effort and the signal. This means that the effort is revealed to the planner and therefore the first best can be achieved.

2 Search (30 points)[1]

1. [a.]

2. In case the worker rejects the offer, the worker will be unemployed, and the her value is:

$$U = \beta \left[\pi_L W_L + \pi_H W_H\right]$$

The value for the worker of being offered a job of type i:

$$W_i = \max_{accept,reject} \left\{w_i - \gamma + \beta \left[\pi W_i + (1-\pi) W_{-i}\right], U\right\}$$

[b.]

3. Write down the firm's choice problem of whether to offer the job (conditional on the worker accepting it and taking the wage associated with the job as given) or to continue to search (post a vacancy).

The value of a firm from type i:

$$J_i = \max_{offer,search} \left\{A_i - w_i + \beta \left[\pi J_i + (1-\pi) J_{-i}\right], \beta \left[\pi J_i + (1-\pi) J_{-i}\right]\right\}$$

As regardless of current matching, the firm will need to renegotiate with a worker next period. This means that the value for the firm of successfully being matched with worker, above its outside option, is always $(A_i - w_i)$

[c.]

4. With Nash bargaining with equal weights, the wage maximizes:

$$w_i = \arg\max \left(W_i - U\right)^{\frac{1}{2}} \left(A_i - w_i\right)^{\frac{1}{2}}$$

The FOC is:

$$\frac{1}{2}\left(W_i - U\right)^{-\frac{1}{2}} \left(A_i - w_i\right)^{\frac{1}{2}} - \frac{1}{2}\left(W_i - U\right)^{\frac{1}{2}} \left(A_i - w_i\right)^{-\frac{1}{2}} = 0$$

[1]Credit: Based on a macro comp question for PhD students at the University of Texas at Austin.

Or:

$$W_i - U = A_i - w_i$$

For the rest of the question assume $A_H > \gamma > A_L$.

1. [d.]

2. First, note that for a firm there is no option value in matching, as even if it is "unemployed" this period, it will be matched with a worker next period with probability one, and anyway it needs to renegotiate. Hence, a firm will never offer a wage higher than current period productivity so it has to be that if a firm with low productivity offers a job, $w_L < \gamma$.

   Consider the possible equilibriums:

   1. Both type of firms are offering jobs and producing. In this case, as we saw, $w_L < \gamma$, so the worker who is matched with a low-productivity firm has negative current-period consumption. If this worker deviates and rejects the offer, she will have current-period consumption of zero (which is better), and will not lose any option value as she will surely be matched next period with the same firm, which will be the only "unemployed" firm, so this cannot be an equilibrium.

   2. Only low-productivity firms are producing. Here also $w_L < \gamma$, so the worker who is matched has negative current-period consumption. As workers who are matched with high-productivity firms do not produce and earn zero, there cannot be any positive option value for being matched with a low-productivity firm (even if $\pi < \frac{1}{2}$), so workers will surely reject the offer and this cannot be an equilibrium.

   3. Both type of firms are not offering jobs and both are not producing. In this case the value of being unemployed is zero. Consider the case of a high-productivity firm considering to deviate. In case of such a deviation, the Nash-bargaining FOC is:

$$\begin{aligned} w_H - \gamma &= A_H - w_H \\ w_H &= \frac{A_H + \gamma}{2} > \gamma \end{aligned}$$

   As $A_H > w_H > \gamma$, the firm has incentive to deviate and the worker will accept the offer, so this also cannot be an equilibrium

   4. The only equilibrium left is that only high-productivity firms to produce

[e.]

3. If only high-productivity firms produce, unemployed firms are always from type low. So, the probability of being matched with a high productivity firm is $\pi_H = 1 - \pi$

[f.]

4. Assuming the equilibrium with only high-productivity firms producing, the 6 unknowns are: $U, w_H, W_H, W_L, J_H, J_L$. The equations are:

$$
\begin{aligned}
U &= \beta\left[\pi W_L + (1-\pi) W_H\right] \\
W_L &= w_H - \gamma + \beta\left[\pi W_H + (1-\pi) W_L\right] \\
W_L &= \beta\left[\pi W_L + (1-\pi) W_H\right] \\
J_H &= A_H - w_H + \beta\left[\pi J_H + (1-\pi) J_L\right] \\
J_L &= U \\
W_H - U &= A_H - w_H
\end{aligned}
$$

3 Industry Equilibrium (25 points)[2]

1. The firm's static problem is:

$$\max \Pi = \max_{n_t} P \cdot y(z_t) - w_t n_t - c_f$$

The FOC is:

$$\begin{aligned} P z_t (n_t)^{-\frac{1}{2}} &= w_t \\ n_t &= \left(\frac{P z_t}{n_t}\right)^2 \\ z &= z_L : n_t = 0, \Pi = -c_f \\ z &= z_H : n_t = P^2, \Pi = 2P^2 - P^2 - c_f = P^2 - c_f \end{aligned}$$

2. The Dynamic problem, given that z_t is the only state variable:

$$\begin{aligned} V(z_t) &= \max_{n_t} \Pi(z_t) + \max_{stay,exit} \{\beta E V(z_{t+1}|z_t), 0\} \\ V_L &= -c_f + \max_{stay,exit} \{\beta [\theta V_L + (1-\theta)V_H], 0\} \\ V_H &= P^2 - c_f + \max_{stay,exit} \{\beta [\theta V_H + (1-\theta)V_L], 0\} \end{aligned}$$

3. Under such an equilibrium, if firms with low productivity shocks exit:

$$\beta [\theta V_L + (1-\theta)V_H] < 0$$

and:

$$V_L = -c_f$$

and if firms with high productivity shocks remain:

$$\beta [\theta V_H + (1-\theta)V_L] \geqslant 0$$

[2] Credit: Based on a macro comp question for PhD students at the University of Texas at Austin.

and:

$$
\begin{aligned}
V_H &= P^2 - c_f + \beta\left[\theta V_H + (1-\theta)V_L\right] \\
V_H\left(1-\beta\theta\right) &= P^2 - c_f + \beta(1-\theta)V_L \\
V_H &= \frac{P^2 - c_f\left(1+\beta(1-\theta)\right)}{1-\beta\theta}
\end{aligned}
$$

Inserting back into the conditions:

$$
\begin{aligned}
\beta\left[\theta V_L + (1-\theta)V_H\right] &< 0 \\
-\theta c_f + (1-\theta)\frac{P^2 - c_f\left(1+\beta(1-\theta)\right)}{1-\beta\theta} &< 0 \\
\frac{(1-\theta)}{\theta}\frac{P^2 - c_f\left(1+\beta(1-\theta)\right)}{1-\beta\theta} &< c_f
\end{aligned}
$$

and:

$$
\begin{aligned}
\beta\left[\theta V_H + (1-\theta)V_L\right] &\geqslant 0 \\
\theta\frac{P^2 - c_f\left(1+\beta(1-\theta)\right)}{1-\beta\theta} - (1-\theta)c_f &\geqslant 0 \\
\frac{\theta}{(1-\theta)}\frac{P^2 - c_f\left(1+\beta(1-\theta)\right)}{1-\beta\theta} &\geqslant c_f
\end{aligned}
$$

4. In order for firms to enter, the value of entering has to be non-negative:

$$
\begin{aligned}
-c_e + \beta EV(z_{t+1}) &\geqslant 0 \\
\beta\left(\frac{1}{2}\frac{P^2 - c_f\left(1+\beta(1-\theta)\right)}{1-\beta\theta} - \frac{1}{2}c_f\right) &\geqslant c_e
\end{aligned}
$$

4 Incomplete markets (20 points)

1. The consumer's problem:

$$\begin{aligned} V(b_t, \epsilon_t) &= \max_{c_t, b_{t+1}} u(c_t) + \beta EV(b_{t+1}, \epsilon_{t+1}|\epsilon_t) \\ & \quad s.t. \\ c_t + b_{t+1} &= w_t \epsilon_t + (1+r) b_t \\ b_{t+1} &> 0 \end{aligned}$$

2. The graph - the plot for the H type is above the 45 degrees line, starting from some positive number and falls below the 45 degrees line at some point, the plot for the L type is below the 45 degrees line, and equal to zero for some range $[0, a]$

3. If consumers are more risk averse, they will save more if they are of type H and consume less if they are of type L, so both lines will be above the lines in section 2.

For the aggregate part:

1. if $\beta(1+r) > 1$ than consumers with H type will always save, so their decision function will always be above the 45 degrees line, and the model will not have a steady state with a final amount of capital.

2. Any change that will not allow consumers to save infinite amount of capital - hard limit, 100% tax above a threshold, etc

3. The graph - provided in section 3 of the notes on incomplete markets