Macro Theory B
Final exam (spring 2016)

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Details:

- Course number: 1011-4108-01
- Year/semester: 2015-2016/2
- The exam includes: 4 questions
- The weight for each question is specified
- Duration: 3 hours
- Total number of pages: 6
- No other material is allowed

1 Optimal Unemployment Insurance (25 points)

Consider the infinite-horizon optimal unemployment insurance problem discussed in class. A government (principal) insures an agent against unemployment.

Notice: I expect you to solve the problem with the infinite horizon but you will only be panelized by 8 points if you answer the same question using the one-period model.

The basic structure is as explained in class:

- The principal is committed to providing the agent with a utility U.
- The principal is risk neutral.
- The agent is risk averse with utility $u(c) - a$, where a is effort a $\in \{0, e\}$.
- The probability of finding a job is $\pi > 0$ if the agent exerts effort $a = e$ and zero otherwise.

Please answer the following questions:

a. Write down the principal's problem who insures the agent against unemployment and who is interested in inducing the agent to exert the high effort. You can assume that the constraints in the question are binding.

Now suppose that the principal, can use upon unemployment a binary signal that is correlated with the agent's effort. The signal structure works as follows: the probability of a good signal given low effort ($a = 0$) is zero; the probability of a good signal given high effort ($a = e$) is θ. You can therefore think about the signal as a lottery whose outcomes are conditional on the agent's effort.

b. Write down the updated problem, allowing the principal to condition promised utility not just on the employment outcome, but also on the signal's outcome. Hint: there are now three possible outcomes instead of just two.

c. Answer the following questions:

   i. Is the signal useful? Why?

   ii. What can you tell about the value of the planner's value in the second problem, relative to the principal's value in the first best, when $\theta = 1$?

2 Search (30 points)[1]

Consider the following job search model. There is a unit measure of workers and a unit measure of firms. Workers can choose to work ($n_t = 1$) or not ($n_t = 0$). Firms each own a technology that takes as input one worker's time and yields output $Y_t = A_t \cdot n_t$. Productivity shocks $A_t \in \{A_H, A_L\}$ are distributed identically and independently across firms and follow a symmetric Markov process $\pi_{jk} = prob(A_{t+1} = A_j | A_t = A_k)$ where $\pi_{HH} = \pi_{LL} = \pi$ and $A_H > A_L$. Productivity shocks occur whether the firm is matched with a worker or not and are fully observable. Both workers and firms have linear preferences and discount the future at rate $\beta < 1 : c_t - \gamma n_t$ for workers and c_t for firms where c_t denotes consumption. Each period, workers and firms are either in a match (employed) or unmatched unemployed searching for a job opportunity. Each period an unemployed worker is matched with a firm who has a vacancy. Note that an "unemployed" firm of type i is matched with probability one with a worker. Workers and firms can choose to end a match.

The timing in a given period is:

i. At the beginning of the period the firm receives her technology shock;

ii. (all) Unmatched workers and (all) unmatched firms are matched;

iii. Wages are determined by a Nash bargaining game with equal weights where the choice not to participate means both parties enter the unemployment pool to be matched next period;

iv. Production and consumption occur in active matches.

To be clear:

- There is no vacancy cost, that is vacancies are free.
- Only unmatched firms can post a vacancy to be filled in the next period.
- Notice that there is a unit measure of firms, that is the usual DMP situation where there is a long queue of firms considering to open a vacancy does not hold here.
- Every worker is (randomly) matched with a firm at the beginning of the period

Please answer the following questions:

a. Write down the worker's choice problem of whether to accept a job of each type (conditional on the firms offering it and taking the wage associated with the job as given) or continue to search. Let π_i denote the probability of a match between a type i firm and a worker.

[1] Credit: Based on a macro comp question for PhD students at the University of Texas at Austin.

b. Write down the firm's choice problem of whether to offer the job (conditional on the worker accepting it and taking the wage associated with the job as given) or to continue to search (post a vacancy).

c. Write down the Nash wage bargaining problem in an active production match and write down the first order condition.

For the rest of the question assume $A_H > \gamma > A_L$.

d. Characterize (in words) the equilibrium under this condition, that is which type of firms will be producing. Explain.

e. What is the probability that an unemployed worker is matched with a high productivity project? Explain your answer.

f. For the solution of the problem, state the N equations and the N unknowns that will allow you to solve explicitly for the equilibrium. **There is no need to solve beyond stating the variables and the unknowns.**

3 Industry Equilibrium (25 points)[2]

Consider the following firm decision problem in a competitive market. Productivity shocks can take on two values $z_t \in \{z_L, z_H\}$ with $z_H = 1$ and $z_L = 0$. Productivity shocks follow a Markov process (i.e. $prob(z_{t+1} = z_i | z_t = z_i) = \theta, i \in \{H, L\}$). Any individual firm's production is given by $y(z) = 2zn^{\frac{1}{2}}$ where n is the quantity of labor. The price of a firm's good is denoted P and the wage is normalized to 1, both assumed independent of time and firms' decisions. A firm that does not exit commits to paying a fixed cost c_f in the next period. Firms discount the future at rate β. Firms can enter the market as well. The timing in any given period is as follows:

- A firm's productivity shock z is realized at the beginning of the period.
- A firm chooses how much labor to hire and receives the resulting profits.
- The firm chooses whether to exit or not. If it exits, it receives zero payoff in all future periods.

Please answer the following questions:

a. Write down and solve explicitly the firm's (static) profit function for $z_t \in \{z_L, z_H\}$.

b. Write down the firm's dynamic programming problem for $z_t \in \{z_L, z_H\}$.

c. Under what conditions on parameters does an equilibrium where all firms with low productivity shocks exit and all firms with high productivity shocks remain?

d. Now assume that to enter new firms must pay a fixed cost c_e and take a draw from a distribution which assigns equal probability to each of the two shocks. Assume that the conditions in the previous section (c) hold. Under what condition is there firm entry?

[2] Credit: Based on a macro comp question for PhD students at the University of Texas at Austin.

4 Incomplete markets (20 points)

Consider an economy populated by a continuum of size 1 of infinitely-lived consumers. Consumers maximize the discounted expected utility: $\sum_{t=0}^{\infty} \beta^t u(c_t)$. Consumers efficiency units (income levels) are denoted by $\epsilon \in \{\epsilon_L, \epsilon_H\}$ and there is a transition matrix π for the efficiency units. The labor income is $w\epsilon$ and consumers can save without limits using the interest rate r, and borrow up to $b > 0$. Assume exogenous values for the wage rate w and the interest rate r. All agents discount the future using the discount factor β.

1. Write, in recursive form, the consumer's problem. Make explicit the state variables and the choice variables.
2. Plot the decision functions of the consumers, on a graph with current assets on the x-axis and next period assets on the y-axis. Explain your graph.
3. How will the plot change if the utility function of the consumers was changed such that they were more risk-averse? plot the new decision functions on the graph using dashed (- -) lines.

Now assume also firms with a production function with constant return to scale in capital and labor, $F(K, L) = AK^{\alpha}L^{1-\alpha}$, and a general equilibrium setting were the wage rate and interest rate are determined endogenously. Consider only the aggregate steady state.

1. Explain why the endogenous interest level cannot be high enough such that $\beta(1 + r) > 1$.
2. Which change in the properties of the economy can allow the interest rate to be above $\frac{1}{\beta} - 1$?
3. Plot the demand and supply curves for capital in the economy on a graph with the amount of capital on the x-axis and the interest rate on the y-axis. Try to mark as many economically meaningful levels on the graph as you can.